AP Calculus BC Mr. Shubleka Quiz: 16

Present neatly. Justify for full credit. No Calculators.

Name SHUBLEKA/KEY Score /10 ~10 minutes / A

Find the coordinates of the point in the first quadrant at which the tangent line to the curve $x^3 - xy + y^3 = 0$ is parallel to the x-axis. $\longrightarrow$ Slope = 0.

$$x^3 - xy + y^3 = 0$$

$$\frac{d}{dx}\left(x^3 - xy + y^3\right) = \frac{d}{dx}(0)$$

$$3x^2 - y - x\frac{dy}{dx} + 3y^2 \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[3y^2 - x\right] = y - 3x^2$$

$$\frac{dy}{dx} = \frac{y-3x^2}{3y^2-x} = 0 \Rightarrow y - 3x^2 = 0$$

$$y = 3x^2$$

Plug this into original equation:

$$x^3 - x(3x^2) + (3x^2)^3 = 0$$

$$\Leftrightarrow x^3 - 3x^3 + 27x^6 = 0$$

$$x^3\left[27x^3 - 2\right] = 0$$

$x^3 = 0$

$\boxed{x = 0}$ not in QI

$27x^3 - 2 = 0$

$x^3 = \frac{2}{27}$

$\boxed{x = \frac{\sqrt[3]{2}}{3}}$

$$y = 3x^2 = 3 \cdot \frac{2^{2/3}}{3^2} =$$

$$y = \frac{2^{2/3}}{3}$$

$$\left(\frac{\sqrt[3]{2}}{3}, \frac{\sqrt[3]{4}}{3}\right)$$

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Prove that if P and Q are two distinct points on the rotated ellipse $x^2 + xy + y^2 = 4$ such that P, Q, and the origin are collinear, then the tangent lines to the ellipse at P and Q are parallel.

$P(a, m) \longrightarrow m_{OP} = \frac{m-0}{a-0} = \frac{m}{a} = \frac{\cancel{n}-m}{b-a}$

$Q(b, n) \longrightarrow m_{OQ} = \frac{n-0}{b-0} = \frac{n}{b} = \frac{\cancel{n}-m}{b-a}$

tangent lines: $2x + y + xy' + 2yy' = 0$

@P: $2a + m + ay' + 2my' = 0$

$y' = \frac{-2a-m}{a+2m} = \text{slope}.$

@Q: $2b + n + by' + 2ny' = 0$

$y' = \frac{-2b-n}{b+2n} = \text{slope}$

Show: $\frac{-(2a+m)}{a+2m} = -\left(\frac{2b+n}{b+2n}\right)$

$(2a+m)(b+2n) = (2b+n)(a+2m)$

$\cancel{2ab} + 4an + mb + \cancel{2mn} = \cancel{2ab} + 4bm + an + \cancel{2mn}$

$3an = 3bm$

$an = bm$

$\boxed{\frac{n}{b} = \frac{m}{a}}$ true.