AP Calculus AB | Mr. Shubleka | Quiz: 16

Present neatly ~~on separate paper~~. Justify for full credit. No Calculators.

Name KEY/SHUBLEKA Score _______ 10 minutes

1.

Find dy/dx if $y = \dfrac{\sin x}{1+\cos x}$.

2.

Use a derivative to evaluate each limit.

(a) $\displaystyle\lim_{h\to 0} \frac{\sin\left(\frac{\pi}{2}+h\right)-1}{h}$ (b) $\displaystyle\lim_{h\to 0} \frac{\csc(x+h)-\csc x}{h}$

3.

Find $f''(\pi/4)$ if $f(x) = \sec x$.

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① $\dfrac{dy}{dx} = \dfrac{(1+\cos x)\cos x - (-\sin x)\sin x}{(1+\cos x)^2} = \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \dfrac{\cos x + 1}{(\cos x + 1)^2} = \dfrac{1}{\cos x + 1}$

② $\displaystyle\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = f'(a)$

a) $\displaystyle\lim_{h\to 0} \frac{\sin\left(\frac{\pi}{2}+h\right)-1}{h} = \lim_{h\to 0} \frac{\sin\left(\frac{\pi}{2}+h\right)-\sin\left(\frac{\pi}{2}\right)}{h} = \frac{d}{dx}(\sin x)\Big|_{x=\frac{\pi}{2}}$

$= \cos\frac{\pi}{2} = 0$

b) $\displaystyle\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$

$\displaystyle\lim_{h\to 0} \frac{\csc(x+h)-\csc x}{h} = \frac{d}{dx}(\csc x) = -\csc x \cot x$

③ $f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f''(x) = \sec x \underbrace{\sec^2 x} + \underbrace{\sec x \tan x} \cdot \tan x$

@$_{x=\frac{\pi}{4}}$ $f''(\pi/4) = \left(\sec\frac{\pi}{4}\right)^3 + \sec\frac{\pi}{4}\tan^2\frac{\pi}{4} = 2\sqrt{2} + \sqrt{2}\cdot 1$

$= 3\sqrt{2}.$