Present neatly ~~on separate paper~~. Justify for full credit. No Calculators.

Name SHUBLEKA/KEY Score ______ ~10 minutes

1.

Show that the triangle that is formed by any tangent line to the graph of $y = 1/x$, $x > 0$, and the coordinate axes has an area of 2 square units. [5 points]

2.

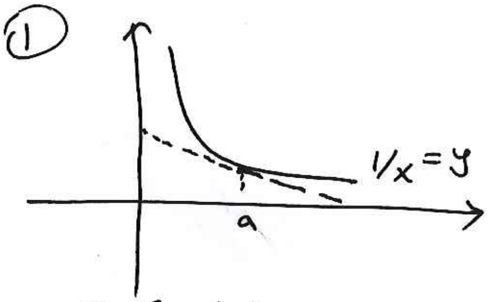
Find conditions on a, b, c, and d so that the graph of the polynomial $f(x) = ax^3 + bx^2 + cx + d$ has

(a) exactly two horizontal tangents

(b) exactly one horizontal tangent

(c) no horizontal tangents. [5 points]

1



@ $\left(a, \frac{1}{a}\right)$ $\left.\frac{dy}{dx}\right|_{x=a} = \left.\frac{-1}{x^2}\right|_{x=a} = \frac{-1}{a^2}$

Tangent: $y - \frac{1}{a} = \frac{-1}{a^2}(x-a)$

x-int: $y=0$, $0 - \frac{1}{a} = \frac{-1}{a^2}(x-a) \Rightarrow x - a = a \Rightarrow \boxed{x = 2a}$

y-int: $x=0$, $y - \frac{1}{a} = \frac{-1}{a^2}(-a) \Rightarrow y - \frac{1}{a} = \frac{1}{a} \Rightarrow \boxed{y = \frac{2}{a}}$

Area $= \frac{\text{base} * \text{height}}{2} = \left(2a \cdot \frac{2}{a}\right)/2 = \frac{4}{2} = 2.$

2 $f'(x) = 3ax^2 + 2bx + c$

a) $D = (2b)^2 - 4 \cdot 3a \cdot c > 0 \Rightarrow 4b^2 - 12ac = 4(b^2 - 3ac) > 0$ $\boxed{b^2 > 3ac}$

b) $D = 4b^2 - 12ac = 4(b^2 - 3ac) = 0 \Rightarrow \boxed{b^2 = 3ac}$

c) $D = 4b^2 - 12ac = 4(b^2 - 3ac) < 0 \Rightarrow \boxed{b^2 < 3ac}$