AP Calculus BC Mr. Shubleka Quiz: 14

Present neatly. Justify for full credit. No Calculators.

Name SHUBLEKA / KEY. Score ______ ~10 minutes / A

1.

Find all values of x for which the line that is tangent to $y = 3x - \tan x$ is parallel to the line $y - x = 2$. $\longrightarrow$ slope = 1

2.

Suppose that $f(x) = M \tan x + N \sec x$ for some constants M and N. If $f(\pi/4) = 2$ and $f'(\pi/4) = 0$, find an equation for the tangent line to $y = f(x)$ at $x = 0$.

(1) $\frac{dy}{dx} = 3 - \sec^2 x = 1$

$\sec^2 x = 2$

$1/\cos x = \pm\sqrt{2}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4} + k\pi/2$ (k integer) ✓

(2) Tangent Line @ $x = 0$?

$f(\frac{\pi}{4}) = M \cdot \tan(\pi/4) + N \sec(\pi/4)$

$f(\frac{\pi}{4}) = M \cdot 1 + N \cdot \sqrt{2} = \underbrace{M + \sqrt{2} N = 2}_{*}$

$f'(x) = M \sec^2 x + N \sec x \tan x$

$f'(\pi/4) = M \cdot [\sqrt{2}]^2 + N \cdot \sqrt{2} \cdot 1 = 0 \iff \cancel{\sqrt{2}(M+N) = 0}$

$\iff \underbrace{2M + N\sqrt{2} = 0}_{**}$

$(* - **)$ gives : $-M = 2$

$\boxed{M = -2}$

$\boxed{N = \frac{2 - M}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}}$

@ $x = 0$ $f(0) = M \tan 0 + N \sec 0$

$f(0) = 0 + 2\sqrt{2}$

$f'(0) = M \sec^2 0 + N \sec 0 \tan 0$

$f'(0) = (-2) \cdot 1 + N \cdot 1 \cdot 0$

$f'(0) = -2$.

tangent : $y - 2\sqrt{2} = -2(x - 0)$

$\boxed{y = -2x + 2\sqrt{2}}$

AP Calculus BC Mr. Shubleka Quiz: 14

Present neatly. Justify for full credit. No Calculators.

Name SHUBLEKA/KEY. Score ______ ~10 minutes / F

1.

Suppose that $f(x) = M \sin x + N \cos x$ for some constants M and N. If $f(\pi/4) = 3$ and $f'(\pi/4) = 1$, find an equation for the tangent line to $y = f(x)$ at $x = 3\pi/4$.

2.

Suppose that $f'(x) = 2x \cdot f(x)$ and $f(2) = 5$.

(a) Find $g'(\pi/3)$ if $g(x) = f(\sec x)$.

(b) Find $h'(2)$ if $h(x) = [f(x)/(x-1)]^4$.

① $f(\pi/4) = M \cdot \sin(\pi/4) + N \cos(\pi/4) = \frac{\sqrt{2}}{2}(M+N) = 3 \Rightarrow M+N = \frac{6}{\sqrt{2}}$

$f'(\pi/4) = M \cos(\pi/4) - N \sin(\pi/4) = \frac{\sqrt{2}}{2}(M-N) = 1 \Rightarrow M-N = \frac{2}{\sqrt{2}}$

$+$ $2M = \frac{8}{\sqrt{2}}$

$M = 2\sqrt{2}$

$N = \sqrt{2}$

@ $x = \frac{3\pi}{4}$

$f\left(\frac{3\pi}{4}\right) = 2\sqrt{2} \cdot \frac{\sqrt{2}}{2} + \sqrt{2} \cdot \left(-\frac{\sqrt{2}}{2}\right) = 1$

$f'\left(\frac{3\pi}{4}\right) = 2\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) - \sqrt{2} \cdot \frac{\sqrt{2}}{2} = -3$

tangent: $y - 1 = -3\left(x - \frac{3\pi}{4}\right)$

$y = -3x + 1 + \frac{9\pi}{4}$.

②

a) $g(x) = f(\sec x) \quad g'(x) = f'(\sec x) \cdot \sec x \tan x$

$g'\left(\frac{\pi}{3}\right) = f'\left(\sec \frac{\pi}{3}\right) \cdot \sec \frac{\pi}{3} \cdot \tan \frac{\pi}{3}$

$= f'(2) \cdot 2 \cdot \sqrt{3} = 2 \cdot 2 \cdot f(2) \cdot 2\sqrt{3} = 40\sqrt{3}$

b) $h(x) = \left(\frac{f(x)}{x-1}\right)^4 \quad h'(x) = 4\left(\frac{f(x)}{x-1}\right)^3 \cdot \frac{(x-1)f'(x) - f(x)}{(x-1)^2}$

$h'(2) = 4\left(\frac{5}{2-1}\right)^3 \cdot \frac{(2-1) \cdot 20 - 5}{(2-1)^2} =$

$= \frac{4 \cdot 125}{1^3} \cdot 15 = \frac{4 \cdot 125 \cdot 15}{1} = 60 \cdot 125 = 7500$