

Present neatly on separate paper. Justify for full credit. No Calculators.

Name SHUBLEKA/KEY Score _____ ~10 minutes

1.

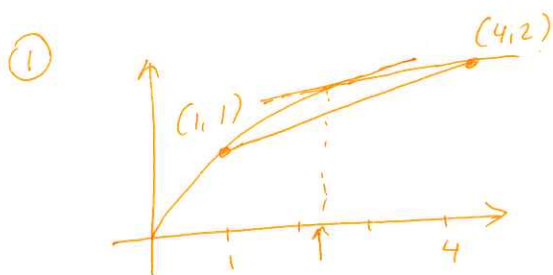
Find the x -coordinate of the point on the graph of $y = \sqrt{x}$ where the tangent line is parallel to the secant line that cuts the curve at $x = 1$ and $x = 4$.

[5 points]

2.

Find k if the curve $y = x^2 + k$ is tangent to the line $y = 2x$.

[5 points]



$$y = \sqrt{x}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \quad \text{at } x = a, \text{ slope}$$

$$\text{has to be equal to } \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{4 - 1}$$

$$= \frac{1}{3}$$

$$\left. \frac{dy}{dx} \right|_{x=a} = \frac{1}{2\sqrt{a}} = \frac{1}{3} \Rightarrow 2\sqrt{a} = 3 \Rightarrow \sqrt{a} = \frac{3}{2} \Rightarrow a = \frac{9}{4} = 2.25$$

$$\boxed{x = \frac{9}{4}}$$

② $y = x^2 + k$ is tangent to $y = 2x$ @ $x = a, y = 2a$

The slopes are the same, so: $\left. \frac{d}{dx}(x^2 + k) \right|_{x=a} = \left. \frac{d}{dx}(2x) \right|_{x=a}$

$$\Leftrightarrow 2x \Big|_{x=a} = 2 \Big|_{x=a} \Leftrightarrow 2a = 2$$

$$\Leftrightarrow a = 1$$

If $a = 1$, the $y = 2 \cdot a = 2$, so we plug in $(1, 2)$ in

$$2 = 1^2 + k \Rightarrow \boxed{k = 1}$$

