AP Calculus BC　　Mr. Shubleka　　Quiz: 13

Present neatly. Justify for full credit. No Calculators.

Name SHUBLEKA / KEY　Score ______　~10 minutes / A

1. Find the first derivative of the given function[6 points]

a)

$$y = \sqrt{1 + \sqrt{1 + \sqrt{x}}}$$

b)

$$y = \sec(\sqrt{t^2 - 9})$$

2. [4 points]

Compute the second derivative of $\sin(g(x))$ at $x = 2$, assuming that $g(2) = \frac{\pi}{4}$, $g'(2) = 5$, and $g''(2) = 3$.

1) a) $y = \left[1 + (1 + x^{1/2})^{1/2}\right]^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}\left[1 + (1 + x^{1/2})^{1/2}\right]^{-1/2} \cdot \left[0 + \frac{1}{2}(1 + x^{1/2})^{-1/2} \cdot \left(0 + \frac{1}{2\sqrt{x}}\right)\right]$$

$$= \frac{1}{2}\left[1 + \sqrt{1 + \sqrt{x}}\right]^{-1/2} \frac{1}{2}(1 + \sqrt{x})^{-1/2} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{8} \frac{1}{\sqrt{1 + \sqrt{1 + \sqrt{x}}}} \cdot \frac{1}{\sqrt{1 + \sqrt{x}}} \cdot \frac{1}{\sqrt{x}}$$

b) $y = \sec(\sqrt{t^2 - 9})$ $\quad \frac{dy}{dt} = \sec(\sqrt{t^2 - 9}) \tan\sqrt{t^2 - 9} \cdot \frac{2 \cdot t}{2\sqrt{t^2 - 9}}$

$$\frac{dy}{dt} = \frac{t \sec(\sqrt{t^2 - 9}) \tan(\sqrt{t^2 - 9})}{\sqrt{t^2 - 9}}$$

2) $y = \sin(g(x))$

$\frac{dy}{dx} = \cos(g(x)) \cdot g'(x)$ $\quad \frac{d^2y}{dx^2} = \cos(g(x))\, g''(x) - \sin(g(x))\left[g'(x)\right]^2$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2} = \cos(g(2)) \cdot g''(2) - \sin(g(2)) \cdot \left[g'(2)\right]^2$$

$$= \cos\left(\frac{\pi}{4}\right) \cdot 3 - \sin\left(\frac{\pi}{4}\right) \cdot 5^2$$

$$= \frac{3\sqrt{2}}{2} - 25\frac{\sqrt{2}}{2} = -11\sqrt{2}$$

Present neatly. Justify for full credit. No Calculators.

Name SHUBLEKA/KEY. Score ______ ~10 minutes / F

1. Find the first derivative of the given function[6 points]

a)

$$y = \sqrt{\sqrt{x+1}+1} = \left[(x+1)^{1/2}+1\right]^{1/2}$$

b)

$$y = \cot^7(x^5)$$

2. Calculate:

$$\frac{d}{dpenguin}\left(\tan^2\left(\frac{penguin}{penguin+k}\right)\right)$$ [4 points]

① a) $\frac{dy}{dx} = \frac{1}{2}\left[(x+1)^{1/2}+1\right]^{-1/2} \cdot \frac{1}{2}(x+1)^{-1/2} = \frac{1}{4}\frac{1}{\sqrt{\sqrt{x+1}+1}} \cdot \frac{1}{\sqrt{x+1}}$

b) $\frac{dy}{dx} = 7\cot^6(x^5)\left(-\csc^2(x^5)\right) \cdot 5x^4$

$= -35x^4 \cdot \cot^6(x^5)\csc^2(x^5).$

② $\frac{d}{dp}\left(\tan^2\left[\frac{p}{p+k}\right]\right) = 2 \cdot \tan\left(\frac{p}{p+k}\right) \cdot \sec^2\left[\frac{p}{p+k}\right] \cdot \frac{k}{(p+k)^2}$