AP Calculus AB   Mr. Shubleka   Quiz: 13

Present neatly ~~on separate paper~~. Justify for full credit. No Calculators.

Name SHUBLEKA / KEY.   Score ______   ~10 minutes

1. Use the definition of the derivative to find a slope-measuring rule for the function $y=\sqrt{9-4x}$. [5 points]

2.

Suppose that $f(x)=\begin{cases} x^2-1, & x\le 1 \\ k(x-1), & x>1. \end{cases}$

For what values of k is f

(a) continuous?   (b) differentiable?   [5 points]

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① $f(x)=\sqrt{9-4x}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{9-4(x+h)}-\sqrt{9-4x}}{h}$$

$$=\lim_{h\to 0}\frac{\left(\sqrt{9-4(x+h)}-\sqrt{9-4x}\right)}{h}\,\frac{\left(\sqrt{9-4(x+h)}+\sqrt{9-4x}\right)}{\text{'' \quad '' \quad ''}}$$

$$=\lim_{h\to 0}\frac{(9-4x-4h)-(9-4x)}{h\left[\sqrt{9-4(x+h)}+\left(\sqrt{9-4x}\right)\right]}=$$

$$=\lim_{h\to 0}\frac{-4\cancel{h}}{\cancel{h}\left(\sqrt{9-4(x+h)}+\sqrt{9-4x}\right)}=\frac{-4}{2\sqrt{9-4x}}=\frac{-2}{\sqrt{9-4x}}.$$

② a) $\lim_{x\to 1^+}f(x)=\lim_{x\to 1^+}k(x-1)=0=\lim_{x\to 1^-}f(x)=\lim_{x\to 1^-}x^2-1=0$

$=f(1)=0$ Cont. for all k.

b) $f'(x)=\begin{cases} 2x & x<1 \\ k \text{ (slope)} & x>1 \end{cases}$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-1-(x^2-1)}{h}$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}=2x.$$

near $x=1$: $2x\big|_{x=1}=k\big|_{x=1}\Rightarrow 2\cdot 1=k$

$\boxed{k=2}\longrightarrow$ makes it differentiable.