Present neatly. Justify for full credit. No Calculators.

Name ________________________ Score _______ ~10 minutes / A

1. Find the points on the curve $y=(\cos x)/(2+\sin x)$ at which the tangent line is horizontal. [5 points]
2. Evaluate or explain why it does not exist. [5 points]
   a)
   $$\lim_{x\to 1}\frac{\sin(x-1)}{x^2+x-2}$$
   b)
   $$\lim_{x\to \pi/4}\frac{1-\tan x}{\sin x-\cos x}$$

1.

$$f(x)=\frac{\cos x}{2+\sin x}$$

$$f'(x)=\frac{(2+\sin x)(-\sin x)-\cos x*\cos x}{(2+\sin x)^2}$$

$$f'(x)=\frac{-2\sin x-\sin^2 x-\cos^2 x}{(2+\sin x)^2}=\frac{-2\sin x-(\sin^2 x+\cos^2 x)}{(2+\sin x)^2}=\frac{-2\sin x-1}{(2+\sin x)^2}=0$$

$$\to -2\sin x-1=0$$

$$\to \sin x=\frac{-1}{2}$$

$$\to x=2k\pi-\frac{\pi}{6},x=2k\pi-\frac{5\pi}{6}$$

$$f\left(\frac{-\pi}{6}\right)=\frac{1}{\sqrt{3}}$$

$$f\left(\frac{-5\pi}{6}\right)=-\frac{1}{\sqrt{3}}$$

$$\text{Points: }\left(2k\pi-\frac{\pi}{6},\frac{1}{\sqrt{3}}\right),\left(2k\pi-\frac{5\pi}{6},\frac{-1}{\sqrt{3}}\right)$$

Below we confirm our findings with Mathematica:

```
In[1]:= Plot[{Cos[x] / (2 + Sin[x]), 1 / Sqrt[3], -1 / Sqrt[3]}, {x, -10, 10}]
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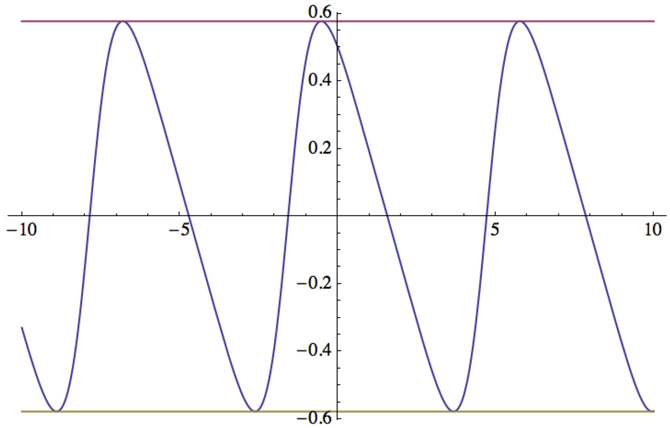
Out[1]=



2.

a)

$$\lim_{x\to 1}\frac{\sin(x-1)}{x^2+x-2}=\lim_{x\to 1}\frac{\sin(x-1)}{(x-1)(x+2)}=\lim_{x\to 1}\left(\frac{\sin(x-1)}{(x-1)}\frac{1}{x+2}\right)$$

$$=\lim_{x\to 1}\left(\frac{\sin(x-1)}{(x-1)}\right)*\lim_{x\to 1}\left(\frac{1}{x+2}\right)=1*\frac{1}{3}=\frac{1}{3}$$

b)

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\lim_{x\to\frac{\pi}{4}}\frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x}=\lim_{x\to\frac{\pi}{4}}\frac{\frac{\cos x-\sin x}{\cos x}}{\sin x-\cos x}=$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos x(\sin x-\cos x)}=\lim_{x\to\frac{\pi}{4}}\frac{-(\sin x-\cos x)}{\cos x(\sin x-\cos x)}=$$

$$\lim_{x\to\frac{\pi}{4}}\frac{-1}{\cos x}=\frac{-1}{\frac{\sqrt{2}}{2}}=-\sqrt{2}$$

Present neatly. Justify for full credit. No Calculators.

Name ________________________ Score _______ ~10 minutes / F

1.

For what values of x does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent?

[5 points]

2.Find

$$\frac{d^{35}}{dx^{35}}(x\sin x)$$

[2 points]

3.

Suppose $f(\pi/3) = 4$ and $f'(\pi/3) = -2$, and let $g(x) = f(x)\sin x$ and $h(x) = (\cos x)/f(x)$. Find

(a) $g'(\pi/3)$ (b) $h'(\pi/3)$

[3 points]

1.

$$f(x) = x + 2\sin x$$

$$f'(x) = 1 + 2\cos x = 0 \to \cos x = \frac{-1}{2} \to x = \frac{2\pi}{3} + 2k\pi, x = \frac{-2\pi}{3} + 2k\pi$$

2.

$$f^{(0)}(x) = x\sin x$$

$$f^{(1)}(x) = \sin x + x\cos x$$

$$f^{(2)}(x) = \cos x + \cos x - x\sin x = 2\cos x - x\sin x$$

$$f^{(3)}(x) = -2\sin x - (\sin x + x\cos x) = -3\sin x - x\cos x$$

$$f^{(4)}(x) = -3\cos x - (\cos x + x(-\sin x)) = -4\cos x + x\sin x$$

$$f^{(5)}(x) = 4\sin x + \sin x + x\cos x = 5\sin x + x\cos x$$

$$f^{(6)}(x) = 5\cos x + \cos x - x\sin x = 6\cos x - x\sin x$$

....

$$f^{(35)}(x) = -35\sin x - x\cos x$$

3.

a)

$$g'(x) = f'(x)\sin x + f(x)\cos x \to g'(\pi/3) = (-2) * \frac{\sqrt{3}}{2} + 4 * \frac{1}{2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

b)

$$h'(x) = \frac{f(x)(-\sin x) - f'(x)\cos x}{(f(x))^2} \to h'(\pi/3) = \frac{4 * (\frac{-\sqrt{3}}{2}) - (-2) * \frac{1}{2}}{(4)^2} = \frac{1 - 2\sqrt{3}}{16}$$