AP Calculus AB　　　　Mr. Shubleka　　　　Quiz: 11

Present neatly ~~on separate paper~~. Justify for full credit. No Calculators.

Name KEY / SHUBLEKA　　Score ______　　~10 minutes

1. Consider the function $f(x)=\frac{9}{\sqrt{x}}$. Use the definition of slope to determine the equation of the tangent line at the point on the curve where $x=9$. [8 points]

2. Writing: Briefly discuss how the tangent line to the graph of a function $y=f(x)$ at a point $P(x_0, f(x_0))$ is defined in terms of secant lines to the graph through point $P$. [2 points]

---

① $f(x)=\frac{9}{\sqrt{x}}$　　$f(9)=\frac{9}{\sqrt{9}}=3$　　$P(9,3)$

$$m=\text{slope}=\lim_{h\to 0}\frac{f(9+h)-f(9)}{h}=\lim_{h\to 0}\frac{\frac{9}{\sqrt{9+h}}-\frac{9}{\sqrt{9}}}{h}=$$

$$=9\lim_{h\to 0}\frac{\frac{1}{\sqrt{9+h}}-\frac{1}{\sqrt{9}}}{h}=9\lim_{h\to 0}\frac{\sqrt{9}-\sqrt{9+h}}{h\sqrt{9}\sqrt{9+h}}$$

$$=9\lim_{h\to 0}\frac{\sqrt{9}-\sqrt{9+h}}{3h\sqrt{9+h}}\cdot\frac{(\sqrt{9}+\sqrt{9+h})}{(\sqrt{9}+\sqrt{9+h})}=$$

$$=\frac{9}{3}\lim_{h\to 0}\frac{9-(9+h)}{h\sqrt{9+h}(\sqrt{9}+\sqrt{9+h})}=3\lim_{h\to 0}\frac{-h}{h\sqrt{9+h}(3+\sqrt{9+h})}$$

$$=3\lim_{h\to 0}\frac{-1}{\sqrt{9+h}(3+\sqrt{9+h})}=\frac{3\cdot(-1)}{3(3+3)}=\frac{-1}{6}$$

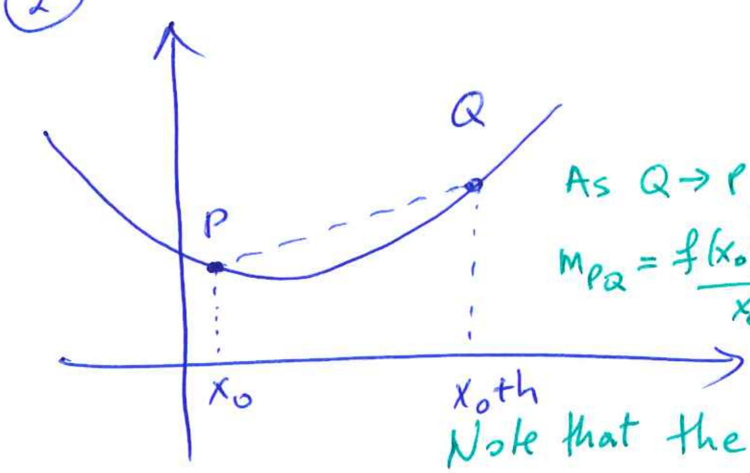
Point: $(9,3)$　Slope: $\frac{-1}{6}$　Equation: $y-3=\frac{-1}{6}(x-9)$

or

$$y=\frac{-1}{6}x+\frac{3}{2}+3$$

$$\boxed{y=\frac{-1}{6}x+\frac{9}{2}}$$

②




As $Q\to P$, $m_{PQ}=\frac{f(x_0+h)-f(x_0)}{x_0+h-x_0}$ approaches the true slope at $P$.

Note that the slopes of the secant lines are approximations of the true slope.