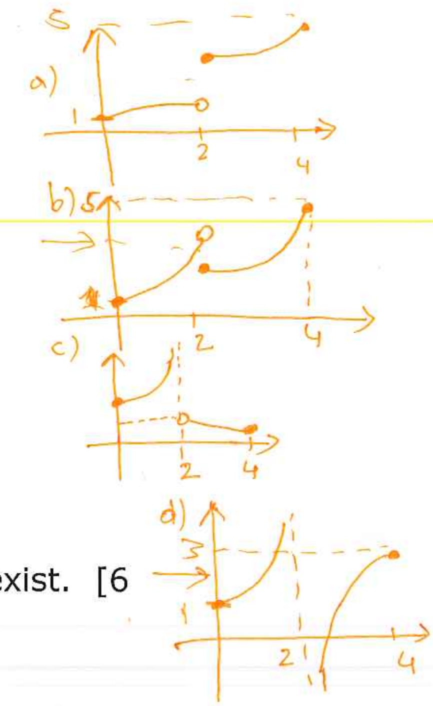
AP Calculus AB   Mr. Shubleka   Quiz: 10

Present neatly on separate paper. Justify for full credit. No Calculators.

Name Key / SHUBLEKA.   Score _______   ~15 minutes

1. In each case draw a graph of f(x) on the interval [0, 4]. [4 points]



a)

Jump discontinuity at $x = 2$ and does not satisfy the conclusion of the IVT.

b)

Jump discontinuity at $x = 2$ and satisfies the conclusion of the IVT on [0, 4].

c)

Infinite one-sided limits at $x = 2$ and does not satisfy the conclusion of the IVT.

d)

Infinite one-sided limits at $x = 2$ and satisfies the conclusion of the IVT on [0, 4].

2.

For a) through d), find the limit or explain why it does not exist. [6 points]

a)

$$\lim_{h\to 0} \frac{\sin(2h)(1-\cos h)}{h^2} = \lim_{h\to 0} 2\left(\frac{\sin 2h}{2h}\right)\cdot\left(\lim_{h\to 0}\frac{1-\cos h}{h}\right) = 2\cdot 1\cdot 0 = 0$$

b)

$$\lim_{x\to\frac{\pi}{3}} \frac{2\cos^2 x + 3\cos x - 2}{2\cos x - 1} = \lim_{x\to\frac{\pi}{3}} \frac{\cancel{(2\cos x - 1)}(\cos x + 2)}{\cancel{(2\cos x - 1)}} = \lim_{x\to\frac{\pi}{3}} [\cos x + 2] = \frac{1}{2} + 2 = \frac{5}{2}$$

c)

$$\lim_{x\to -2} \frac{x^3+8}{x^2+6x+8} = \lim_{x\to -2} \frac{(x+2)(x^2+2x+4)}{(x+2)(x+4)} = \lim_{x\to -2} \frac{x^2+2x+4}{x+4} = \frac{12}{2} = 6.$$

d)

$$\lim_{\theta\to\frac{\pi}{4}} \left(\frac{1}{\tan\theta - 1} - \frac{2}{\tan^2\theta - 1}\right) = \lim_{\theta\to\pi/4} \frac{(\tan\theta + 1 - 2)}{(\tan^2\theta - 1)} = \lim_{\theta\to\pi/4} \frac{1}{\tan\theta + 1} = \frac{1}{1+1} = \frac{1}{2}$$

e) Find the value(s) of c for which the limit exists.

$$\lim_{x\to 1} \frac{x^2+3x+c}{x-1}$$ $\longrightarrow f(x) = 0$ when $x = 1$ so that $(x-1)$ can be a factor

$f(1) = 1^2 + 3\cdot 1 + c = 4 + c = 0 \Rightarrow \boxed{c = -4.}$

f) Find the value(s) of c for which the limit exists.

$$\lim_{x\to 1} \left(\frac{1}{x-1} - \frac{c}{x^3-1}\right) = \lim_{x\to 1} \frac{x^2+x+1 - c}{x^3-1} \Rightarrow$$

the numerator must be zero when $x=1$: $1^2 + 1 + 1 - c = 0$

$3 - c = 0$

$\boxed{c = 3}$

$\frac{7}{0}$ $\frac{\cancel{(x-3)}}{\cancel{(x-3)}}$