Mathematica Labs | Denis Shubleka

Subject: Calculus

Topic: The Lagrange Multiplier Method

Goal: Use *Mathematica* to optimize f(x, y) subject to a constraint g(x,y) = 0.

Task 1

Find the maximum and minimum of $f(x,y) = 2x - y$ on the circle of radius 5, centered at the origin.
We first define the objective function and the contraint, and then compute the two gradients:

```
F[x_, y_] := 2 x - y
G[x_, y_] := x^2 + y^2 - 25
gradientf = {D[F[x, y], x], D[F[x, y], y]};
gradientg = {D[G[x, y], x], D[G[x, y], y]};
Print["Gradient of f:", gradientf]
Print["Gradient of g:", gradientg]
```

Next, we find solutions to the equation: $\nabla f(x,y) = \lambda \nabla g(x,y)$. The solutions are candidates for optimal points.

```
candidates = Solve[{gradientf == l * gradientg, G[x, y] == 0}, {x, y, l}]
```

Next, compute each pair of (x, y) coordinates and print the resulting function values:

```
{x, y, N[F[x, y]]} /. candidates
```

Where does f(x,y) the maximum attain a maximum point? A minimum?

Related Exercises/Notes:

1. A rectangular box without a lid is to be made from 12 square meters of cardboard. Find the maximum volume of such a box.

2. Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

3. Find all the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3, 1, -1)$.

4. Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$

on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$. [Hint: Two Lagrange Multipliers are needed.

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z) + \gamma \nabla h(x, y, z)$$]