Mathematica Labs | Denis Shubleka

Subject: Calculus

Topic: The Gradient; Directional Derivatives

Goal: Use *Mathematica* to compute gradient vectors and directional derivatives.

Task 1

In *Mathematica* we define a function f(x, y):

```
f[x_, y_] := Cos[x^2 + y^2];
```

To compute the gradient vector as a general rule, type and execute:

```
{∂x f[x, y], ∂y f[x, y]}
```

Or alternatively:

```
D[f[x, y], {{x, y}}]
```

To evaluate the gradient at a given point, let's say (1, 3), use the substitution rule:

```
% /. {x → 1, y → 3}
```

If we plan to use this particular gradient vector for more computations, we can assign the result to a variable called 'gradient'.

```
gradient = %;
```

Suppose that at the given point (1, 3) we want to compute the directional derivative in the direction of v = <-1, 5>. Recall that the directional derivative is the dot product between the gradient and the unit vector in the direction of v.

```
Dot[gradient, {-1, 5}] / Norm[{-1, 5}]
```

To plot a set of gradient vectors for a particular rectangle in the domain space (i.e. xy plane), type and execute the following. Can you identify the gradient vector that we computed above, at point (1,3)?

```
D[f[x, y], {{x, y}}]
VectorPlot[%, {x, -1, 3}, {y, 1, 5}]
```

Task 2

In *Mathematica* we define a function g(x, y, z):

```
g[x_, y_, z_] := 80 / (1 + x^2 + 2 y^2 + 3 z^2);
```

Suppose we want the gradient at the point (1, 1, -2). In *Mathematica*, type and execute:

```
D[g[x, y, z], {{x, y, z}}] /. {x → 1, y → 1, z → -2}
```

To compute the gradient vector as a general formula and a 3-dimensional plot of gradient vectors for a particular cuboid in the domain space (x-y-z), type:

```
D[g[x, y, z], {{x, y, z}}]
VectorPlot3D[%, {x, 0, 2}, {y, 0, 2}, {z, -3, -1}]
```

Related Exercises/Notes:

1. Suppose f(x, y, z) = x sin(yz). Find the gradient of f and the directional derivative of f at the point (1, 3, 0) in the direction of v = <1, 2, -1>.

2. Find the directional derivative of the function $f(x, y) = x^2 y^3 - 4y$ at the point (2, -1) in the direction of v = <2, 5>.