Mathematica Labs | Denis Shubleka

Subject: Calculus

Topic: Chain Rule

Goal: Use *Mathematica* to find partial derivatives of compositions of several variables.

Task 1

In *Mathematica* we define three function z(x, y), x(s, t), and y(s, t).

```
z[x_, y_] := E^(x + 2 y);
x[s_, t_] := s / t;
y[s_, t_] := t / s;
```

To compute the partial derivatives $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$, we use the derivative command.

```
D[z[x[s, t], y[s, t]], t]
D[z[x[s, t], y[s, t]], s]
```

To evaluate the partial derivative $\frac{\partial z}{\partial s}$, which we computed last, at {s=1, t=2} use the substitution rule:

```
% /. {s → 1, t → 2}
```

Related Exercises/Notes:

1. Use the Chain Rule to find the indicated partial derivatives. Use *Mathematica* to confirm your answers by first finding the derivative(s) and then evaluating it.

a) $P = \sqrt{u^2 + v^2 + w^2},\ u = x\,e^{y},\ v = y\,e^{x},\ w = e^{x\,y};\ \frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}$ when $x = 0,\ y = 2.$

b) $u = x\,e^{t\,y},\ x = \alpha^2\,\beta,\ y = \beta^2\,\gamma,\ t = \gamma^2\,\alpha;\ \frac{\partial u}{\partial \alpha}, \frac{\partial u}{\partial \beta}, \frac{\partial u}{\partial \gamma}$ when $\alpha = -1,\ \beta = 2, \gamma = 1.$
