Mathematica Labs | Denis Shubleka

Subject: Calculus

Topic: Partial Derivatives, Tangent Planes

Goal: Use *Mathematica* to differentiate functions in several variables and find the equation of a tangent plane.

Task 1

In *Mathematica* we define a real-valued function in two variables:

```
f[x_, y_] := Cos[x^2] + Sin[x * y];
```

Or more conveniently:

```
Clear[f, x, y];
f = Cos[x^2] + Sin[x * y];
```

To differentiate f(x,y) with respect to x, we can use the $\frac{\partial}{\partial \square}$ from the Basic Math Assistant palette, or type and execute:

```
D[f, x]
```

As always, if the answer needs simplification, use the Simplify command:

```
Simplify[%]
```

If you need to differentiate the first partial derivative with respect y (or x again), simply refer to the previous answer:

```
D[%, y]
```

For higher order partial derivatives, we can also use the following command, which takes the third derivative in the direction of x:

```
D[f, {x, 3}]
```

To evaluate a certain derivative at a particular point, use the replacement rule as shown below:

```
% /. {x → 0, y → 1}
```

Task 2

The equation of the tangent plane to a surface z= f(x, y) at a point (a, b) is given by $L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$.

Use *Mathematica* to find the equation of the plane tangent $f(x, y) = x^2 + y^2$ at (1, 1, 2). Plot the original surface and the plane in the same window to verify your work. After defining f(x,y) as a function, we compute the partial derivatives slopex and slopey at the given point, and then plot the tangent plane and the surface in the same viewing box:

```
Clear[x, y, f]
f[x_, y_] := x^2 + y^2;

slopex = D[f, x] /. {x → 1, y → 1}

slopey = D[f, y] /. {x → 1, y → 1}

Plot3D[{f[x, y], f[1, 1] + slopex * (x - 1) + slopey (y - 1)}, {x, -5, 5}, {y, -5, 5}]
```

Related Exercises/Notes:

1. Find an equation of the tangent plane to the given surface at the given point. Verify your work with *Mathematica*.

$f(x, y) = 2^{\sin(y)} - x^2 \qquad (3, \pi, -8)$