Mathematica Labs | Denis Shubleka

Subject: Calculus

Topic: Discovering FTC

Goal: Use *Mathematica* to introduce the Fundamental Theorem of Calculus.

Task 1

a) Draw or plot the line y = 2t + 1, and use geometry to find the area under the
line, above the t-axis, and between the vertical lines t=1 and t=3.

```
Plot[2 t + 1, {t, 1, 3}, PlotRange → {-1, 8}, Filling → Axis]
```

b) If x > 1, let A(x) be the area of the region that lies under the line y = 2t + 1,
between t=1 and t=x. Sketch this region on paper, and use geometry to find an
expression for A(x). The Manipulate command below varies x from 1.1 to 3.

```
Manipulate[Plot[2 t + 1, {t, 1, x}, PlotRange → {-1, 8}, Filling → Axis], {x, 1.1, 3}]
```

c) Differentiate the area function A(x) with respect to x. What do you notice? If using *Mathematica*, execute the following commands in the given order:

$$A[x\_] := \frac{(3 + (2\,x + 1)) * (x - 1)}{2};$$

$$D\left[\frac{(3 + (2\,x + 1)) * (x - 1)}{2}, x\right]$$

```
Simplify[%]
```

Task 2

a) If x ⩾ -1, define $A(x)=\int_{-1}^{x}(1+t^2)\,dt$. A(x) represents the area of a region. Sketch
that region on paper.

b) Use *Mathematica* to find an expression for A(x)

$$\text{Integrate}\left[1 + t^2, \{t, -1, x\}\right]$$

c) Differentiate the answer using:

```
D[%, x]
```

What do you notice?

d) If x ⩾ -1 and h is a small positive number, then A(x+h) - A(x) represents the area of a region. Describe and sketch the region on paper.

e) Draw a rectangle that approximates the region above. By comparing the areas of these
two regions, argue that:

$\frac{A(x+h)-A(x)}{h} \approx 1+x^2$

f) Give an intuitive explanation for the result of part c). [Hint: Consider the limit
of the left hand side in the part e), as h approaches 0.]

Task 3

a) Draw the graph of the function $f(x)=\cos(x^2)$in the viewing rectangle [0, 2] by [-1.25, 1.25].

```
f[t_] := Cos[t^2]
```

```
Plot[f[x], {x, 0, 2}, PlotRange → {-1.25, 1.25}]
```

c)Define a new function, g(x), using f(x) as the integrand:

$$g[x\_] := \int_0^x f[t]\, dt$$

Test its functionality:

```
g[1] // N
```

d)Plot g′(x) and f(x) in the same window:

```
Plot[{g'[x], f[x]}, {x, 0, 2}]
```

What do you notice? Ask *Mathematica* to onfirm your observation:

```
g'[x] == f[x]
```

e) Now it is time to generalize. Suppose that f is a continuous function on
a closed interval [a, b]. If we define a new function:
$g(x)=\int_{t=a}^{t=x} f(t)\, dt$, conjecture an expression for g′(x).

The result is one of the two parts of the Fundamental Theorem of Calculus.

Related Exercises/Notes: