Mathematica Labs | Denis Shubleka

Subject: Calculus

Topic: Riemann Sums

Goal: Use *Mathematica* to compute Riemann Sums

Task 1

Recall that a Riemann Sum is an approximation of the definite integral, often
set up as a sum of areas of rectangles (left, right, midpoint etc.) or trapezoids.
The Sum function makes the computation easy. For example, the following command
adds all the squares of integers from 1 to 10, including 1 and 100.

```
Sum[i^2, {i, 1, 10}]
```

Alternatively, we can also use the Sigma operation from the palette:

$$\sum_{i=1}^{10} i^2$$

The definition of a right-hand Riemann sum using n rectangles on a closed
interval [a, b], is given by: $\sum_{i=1}^{n} f[x_i]\Delta x, d$ where:
n = number of rectangles

$$\Delta x = \frac{b-a}{n}$$

$$x_i = a + i * \Delta x$$

In the following example we are going to estimate the definite integral of Sin[x]
on the interval [1, 4].

```
f[x_] := Sin[x]
a = 1;
b = 4;
```

Next, we introduce functions for Δx and x_i, and use them to define functions for
the left and right hand Riemann sums:

```
Δx[n_] := (b - a) / n;
```

```
x[i_, n_] = a + i * Δx[n];
```

$$\text{rightRiemann}[n\_] := \sum_{i=1}^{n} f[x[i, n]] * \Delta x[n] \;//\; N$$

$$\text{leftRiemann}[n\_] := \sum_{i=0}^{n-1} f[x[i, n]] * \Delta x[n] \;//\; N$$

We compute the two sums for f(x)=Sin[x] on [1, 4], using fifty subdivisions:

```
rightRiemann[50]
leftRiemann[50]
```

, and then compare them with the definite integral, after converting to
decimal form, as shown below.

$$\int_1^4 \text{Sin}[x]\, dx$$

```
N[%, 6]
```

To obtain a plot of the net area, type and execute the following:

```
Plot[f[x], {x, 1, 4}, Filling → Axis]
```

Are the approximations close to the definite integral's value? Explain how
one can improve the estimate.

We conclude this Task with a table summary of right-hand Riemann Sums, for
several values of n:

```
mydata = Table[{n, rightRiemann[n]}, {n, 10, 200, 10}];
mydataWithHeadings = Prepend[mydata, {"n", "Right Riemann Sum"}];
Text@Grid[mydataWithHeadings, Alignment → Left, Dividers → {Center, 2 → True}]
```

Your turn: write two new functions that compute the Midpoint and Trapezoidal
Sums. Feel free to use a new function as an example.

Related Exercises/Notes: