Mathematica Labs | Denis Shubleka

Subject: Calculus

Topic: Rates of Change

Goal: Explore average and instantaneous rates of change.

Task 1

Goal: Introduce the Difference Quotient as a function

Below we define the difference quotient algebraically, and function g(x):

```
differencequotient[f_] := (f[x + h] - f[x]) / h
g[x_] := 1 / x
```

Note that h is often called 'delta x' and represents the change in x.
Now we ask Mathematica to compute the difference quotient of g(x):

```
differencequotient[g]
```

Select the resulting output, and then click on the Simplify[] button in the Algebraic Manipulations palette. The resulting output is the simplified difference quotient.

Now try a new function s(x) of your own. First define it, then compute the difference quotient with Mathematica:

```
s[x_] := ...
```

```
differencequotient[s]
```

Task 2

Goal: Compute the average rate of change of a Function

First, we clear the variables that may have been used before.

```
Clear[f, x, h];
```

$$f[x\_] := \frac{\text{Cos}[x]}{x}$$

```
differencequotient[f]
```

Next, we compute average rate of change of f(x) from x=2 to =5.
The following command replaces x with 2 and h with 3 (to get to 5).

```
differencequotient[f] /. {x → 2, h → 3}
```

To convert the answer to a decimal expression, type and execute:

```
N[%]
```

Your turn: compute the average rate of change from x=1.9 to x=2. (Note that if you set x=2, then h is -0.1.)

Numerically, we can investigate the average rates of change near x=2, for small values of h:

```
myData = Table[{h, differencequotient[f]} /. {x → 2, h → N[10^-n]}, {n, 1, 5}];
myDataWithHeadings = Prepend[myData, {"h", "(f (2 + h) - f (2))/h"}];
Text@Grid[myDataWithHeadings, Alignment → Left, Dividers → {Center, 2 → True}]
```

Note that one could similarly compute and construct a table of average rates for negative values of h, close to zero. Feel free to try it!

Task 3

Goal: Investigate Instantaneous Rate of Change at a point

The instantanous rate of change is the limit of the average rate of change as h approaches zero, whenever it exists. We first compute the average rate of change, then find its limit After executing the following:

```
differencequotient[f] /. x → 2
```

, compute the limit with:

```
Limit[%, h → 0]
```

The result is the precise value of the instantanous rate of change of f(x)
at x=2. Graphically, it is the slope of the tangent line at (2, f(2)). To express it in three decimal places, enter:

```
N[%, 3]
```

How does this result compare with the table values we found earlier?

Related Exercises/Notes: