#1

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Name________________________________

Date________________ Period____

Evaluate each indefinite integral. Use the provided substitution.

1) $\int -15x^4(-3x^5-1)^5\,dx;\ u=-3x^5-1$

2) $\int -16x^3(-4x^4-1)^{-5}\,dx;\ u=-4x^4-1$

3) $\int -\frac{8x^3}{(-2x^4+5)^5}\,dx;\ u=-2x^4+5$

4) $\int (5x^4+5)^{\frac{2}{3}}\cdot 20x^3\,dx;\ u=5x^4+5$

5) $\int \frac{(5+\ln x)^5}{x}\,dx;\ u=5+\ln x$

6) $\int 4\sec 4x\cdot \tan 4x\cdot \sec^4 4x\,dx;\ u=\sec 4x$

7) $\int 36x^3(3x^4+3)^5\,dx;\ u=3x^4+3$

8) $\int x(4x-1)^4\,dx;\ u=4x-1$

Evaluate each indefinite integral.

9) $\int -9x^2(-3x^3+1)^3\,dx$

10) $\int 12x^3(3x^4+4)^4\,dx$

11) $\int -12x^2(-4x^3+2)^{-3}\,dx$

12) $\int (3x^5-3)^{\frac{3}{5}} \cdot 15x^4\,dx$

13) $\int (-2x^4-4)^4 \cdot -32x^3\,dx$

14) $\int \left(e^{4x}-4\right)^{\frac{1}{5}} \cdot 8e^{4x}\,dx$

15) $\int x(4x+5)^3\,dx$

16) $\int 5x\sqrt{2x+3}\,dx$

#3

Name________________________________

Substitution for Definite Integrals

Date________________ Period____

Express each definite integral in terms of u, but do not evaluate.

1) $\int_{-1}^{0} \frac{8x}{(4x^2+1)^2}\,dx;\ u = 4x^2 + 1$

2) $\int_{0}^{1} -12x^2(4x^3-1)^3\,dx;\ u = 4x^3 - 1$

3) $\int_{-1}^{2} 6x(x^2-1)^2\,dx;\ u = x^2 - 1$

4) $\int_{0}^{1} \frac{24x}{(4x^2+4)^2}\,dx;\ u = 4x^2 + 4$

Evaluate each definite integral.

5) $\int_{-3}^{0} -\frac{8x}{(2x^2+3)^2}\,dx;\ u = 2x^2 + 3$

6) $\int_{0}^{1} \frac{16x}{(4x^2+4)^2}\,dx;\ u = 4x^2 + 4$

7) $\int_{-1}^{0} 18x^2(3x^3+3)^2\,dx;\ u = 3x^3 + 3$

8) $\int_{0}^{1} -\frac{8x}{(4x^2+2)^2}\,dx;\ u = 4x^2 + 2$

#4

Name: ____________________

Integrate each:

$$\int (3-x)^{10}\,dx$$

$$\int \sqrt{7x+9}\,dx$$

$$\int \frac{x^3}{(1+x^4)^{1/3}}\,dx$$

$$\int e^{5x+2}\,dx$$

$$\int 4\cos(3x)\,dx$$

$$\int \frac{\sin(\ln x)}{x}\,dx$$

$$\int \frac{3x+6}{x^2+4x-3}\,dx$$

$$\int x\,3^{x^2+1}\,dx$$

$$\int \frac{3}{x\ln x}\,dx$$

$$\int \frac{\cos(5x)}{e^{\sin(5x)}}\,dx$$

$$\int_0^{\sqrt{\pi}} x\sin(x^2)\,dx$$

$$\int (2x+5)(x^2+5x)^7\,dx$$

#5

u-Substitution - Classwork

When you take derivatives of more complex expressions, you frequently have to use the chain rule to differentiate. The integration equivalent of the chain rule is called *u-substitution*. *u-substitution* allows you integrate expressions which do not appear integratable.

1) $\int x(x^2-1)^5\,dx$ Set up a u = __________ Find $\dfrac{du}{dx}$ = __________. Solve for du = __________

You need to manufacture your du in the original expression. So you will have to multiply by ______ on the inside and thus multiply by ________ on the outside. Now change everything to u.

Now integrate in terms of u.

Finally, change back to the variaable x and add C.

2) $\int (3x-2)^4\,dx$

3) $\int \sqrt{5x-2}\,dx$

4) $\int 4(6x-1)^{2/3}\,dx$

5) $\int x\sqrt{x^2-2}\,dx$

6) $\int x^2\sqrt{1-4x^3}\,dx$

7) $\int \dfrac{x}{\sqrt[3]{2x^2-1}}\,dx$

8) $\int x^{1/2}\left(x^{3/2}+2\right)^9\,dx$

9) $\int (x+2)\sqrt{x^2+4x-3}\,dx$

#6

10) $\int (x+2)\sqrt{x-4}\, dx$

11) $\int \frac{x-5}{\sqrt{x-6}}\, dx$

12) $\int \frac{x^2}{\sqrt{x+1}}\, dx$

13) $\int \cos 4x\, dx$

14) $\int 3\sin(1-3x)\, dx$

15) $\int \sin^3 x \cos x\, dx$

16) $\int \tan 10x \sec 10x\, dx$

17) $\int \tan^2 x \sec^2 x\, dx$

18) $\int \sin x \sqrt{\cos x}\, dx$

19) $\int \frac{\cos x}{\sqrt{1-\sin x}}\, dx$

u-Substitution - Homework

1. $\int \sqrt{x-2}\, dx$

2. $\int (2x+3)^{11}\, dx$

3. $\int \sqrt{5x-1}\, dx$

4. $\int \sqrt[3]{6x+1}\, dx$

5. $\int 5(3-4x)^{2/3}\, dx$

6. $\int \frac{dx}{(8x-1)^3}$

7. $\int x(x^2+2)^6\, dx$

8. $\int 6x^2\sqrt{3x^3-1}\, dx$

9. $\int \left(1+\frac{1}{x}\right)^3\left(\frac{1}{x^2}\right) dx$

10. $\int x^{1/3}\left(x^{4/3}+9\right)^8\, dx$

11. $\frac{2}{3}\int \sqrt{4-\frac{3}{5}x}\, dx$

12. $\int (3x+15)\sqrt{x^2+10x+4}\, dx$

13. $\int (x+2)\sqrt{x-2}\,dx$

14. $\int \frac{x^2}{\sqrt{x-4}}\,dx$

15. $\int \sin 5x\,dx$

16. $\int \cos\frac{x}{2}\,dx$

17. $\int \frac{1}{3}\sec^2 8x\,dx$

18. $\int \sin 4x \cos 4x\,dx$

19. $\int \cos^3 x \sin x\,dx$

20. $\int \tan x \sec^2 x\,dx$

21. $\int \sqrt{\cos 6x}\,\sin 6x\,dx$

22. $\int \frac{\sin x}{(4-\cos x)^3}\,dx$

#9

Integration by Substitution Worksheet

Name:________________

Common Integral formulas to remember:

$\int u^n du = \frac{u^{n+1}}{n+1} + C$ $\int \sin u du = \cos u + C$ $\int \cos u du = -\sin u + C$ $\int \csc^2 u du = -\cot u + C$

$\int \sec^2 u du = \tan u + C$ $\int \sec u \tan u = \sec u + C$ $\int \csc u \cot u du = -\csc u + C$

$\int \tan u du = -\ln|\cos u| + C$ or $\ln|\sec u| + C$ $\int \cot u du = \ln|\sin u| + C$ $\int \frac{du}{u} = \ln|u| + C$

$\int e^{ku} du = \frac{e^{ku}}{k} + C$ $\int a^{ku} du = \frac{a^{ku}}{k \ln|a|} + C$

Evaluate the Integrals:

1. $\int (x-1)^{243} dx$
2. $\int \sqrt{1-x} dx$
3. $\int \frac{1}{\sqrt{1-x}} dx$
4. $\int x\sqrt{2x^2 - 1} dx$
5. $\int (1+x^3)3x^2 dx$
6. $\int x(x^2+9)^{10} dx$
7. $\int \frac{x^2}{\sqrt{1+x^3}} dx$
8. $\int \frac{dt}{2\sqrt{1+t}}$
9. $\int 2xe^{x^2} dx$
10. $\int \frac{\sin x}{\cos^2 x} dx$
11. $\int \frac{dx}{x\sqrt{25x^2 - 2}}$
12. $\int \frac{dx}{\sqrt{1-4x^2}}$

Evaluate the integrals:

13. $\int_{1}^{0} \frac{3}{3x-2} dx$

14. $\int_{0}^{\pi/4} \tan x \sec^2 x dx$

15. $\int_{0}^{3} \frac{1}{x+1} dx$

16. $\int_{1}^{2} \frac{2 \ln x}{x} dx$

17. $\int_{1}^{4} \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$

18. $\int_{\pi/6}^{\pi/2} \sin^2 x \cos x dx$

19. $\int_{0}^{\pi/2} 2^{\cos x} \sin x dx$

20. $\int_{0}^{1} \frac{4x dx}{\sqrt{1-x^4}}$ Hint: use $u = x^2$

21. $\int_{0}^{1} \frac{x}{1+x^4} dx$

22. $\int_{\ln 4}^{\ln 7} \frac{e^x}{1+e^x} dx$

Objective: The objective of this worksheet is to get automatic in solving integrals with "u" substitution

1. For each of the integral use some form of U-substitution and solve the problems.
2. These problems are meant to be solved at home and questions regarding any of these problems should be asked in the discussion class.

1.

$$\int (\sin^5(x) + 3\sin^3(x) - \sin(x))\cos(x)\,dx$$

3.

$$\int \sin(x)\sec^8(x)\,dx$$

2.

$$\int x^2(x^3+1)^{40}\,dx$$

4.

$$\int \frac{e^{2x}}{e^{2x}+1}\,dx$$

5.

$$\int_1^{e^2} \frac{\ln(x)}{x}\,dx$$

6.

$$\int_0^2 x^3\sqrt{16-x^4}\,dx$$

7.

$$\int_0^{\pi/4} \frac{\sin(x)}{\cos^3(x)}\,dx$$

8.

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$$