AP Calculus | Worksheet | L'Hospital's Rule | Shubleka   NAME ____________________

Solve neatly on separate paper.

Part I

Evaluate the following limits:

(a) $\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$

(b) $\lim_{x \to 0+} x^x$

(c) $\lim_{x \to \infty} \frac{x}{\sqrt{x^2+1}}$

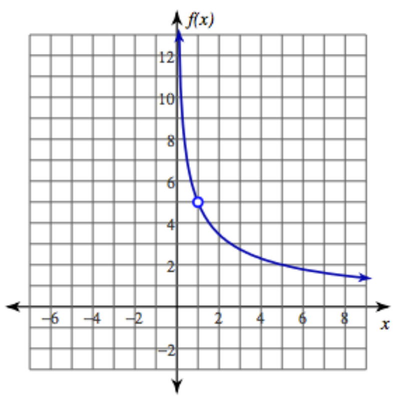
(d) $\lim_{x \to 0+} \left(\frac{1}{x^3} - \frac{1}{x^2}\right)$

(e) $\lim_{x \to \infty} x \tan(1/x)$

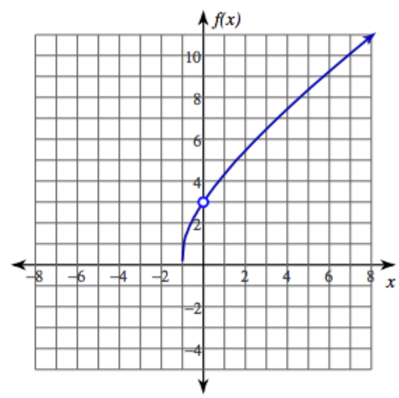
(f) $\lim_{x \to 0} \frac{x^2 \cos\left(\frac{1}{x}\right)}{x}$

Part II

1) $\lim_{x \to 1} \frac{5 \ln x}{x - 1}$



2) $\lim_{x \to 0} \frac{3x}{\ln(x+1)}$



3) $\lim_{x \to 0^+} 5x^2 \ln x$

4) $\lim_{x \to \infty} 4x \cdot e^{-x}$

5) $\lim_{x \to \frac{\pi}{2}} (3\sec x - 3\tan x)$

6) $\lim_{x \to \infty} \left(\frac{x^2}{x-1} - \frac{x^2}{x+1}\right)$

7) $\lim_{x\to 0^+} 5\cdot(\tan x)^{\sin x}$

8) $\lim_{x\to 0^+} 3x^x$

Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.

9) $\lim_{x\to 0} \frac{e^x - e^{-x}}{x}$

10) $\lim_{x\to 0^+} \frac{e^x + e^{-x}}{\sin(2x)}$

Part III

Use l'Hôpital's rule, if applicable, to find the limit.

1) $\lim_{x\to 5} \frac{x^2 - 9x + 20}{x - 5}$

2) $\lim_{x\to\infty} \frac{\ln(x)}{e^{2x}}$

3) $\lim_{y\to 0^+} \frac{\ln(7y^2 + 15y)}{\ln y}$

4) $\lim_{x\to 1} \frac{x - x^5}{\ln x}$

5) $\lim_{x\to\infty} \frac{(\ln x)^3}{x}$

6) $\lim_{x\to\infty} \frac{8x}{\ln(e^x + 1)}$

Key/Solutions

Part I

(a) $\lim_{x\to(\frac{\pi}{2})^-}(\sec x - \tan x) = \lim_{x\to(\frac{\pi}{2})^-}\left(\frac{1}{\cos(x)} - \frac{\sin(x)}{\cos(x)}\right) = \lim_{x\to(\frac{\pi}{2})^-}\frac{1-\sin(x)}{\cos(x)} = \lim_{x\to(\frac{\pi}{2})^-}\frac{-\cos(x)}{-\sin(x)} = \frac{0}{1} = 0$

(b) $\lim_{x\to0^+} x^x = \lim_{x\to0^+} e^{x\ln x} = \exp(\lim_{x\to0^+}\frac{\ln x}{1/x}) = \exp(\lim_{x\to0^+}\frac{1/x}{-1/x^2}) = \exp(\lim_{x\to0^+} -x) = e^0 = 1$

(c) $\lim_{x\to\infty}\frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty}\frac{1}{\frac{1}{x}\sqrt{x^2+1}} = \lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x^2}}} = \frac{1}{\sqrt{1+0}} = 1$

(d) $\lim_{x\to0^+}\left(\frac{1}{x^3} - \frac{1}{x^2}\right) = \lim_{x\to0^+}\left(\frac{1-x}{x^3}\right) = \infty$

(e) $\lim_{x\to\infty} x\tan(1/x) = \lim_{x\to\infty}\frac{\tan(1/x)}{1/x} = \lim_{x\to\infty}\frac{\sec^2(1/x)\frac{-1}{x^2}}{-1/x^2} = \lim_{x\to\infty}\sec^2(1/x) = 1$

(f) This is a great example of when we cannot use l'Hospital's theorem. By the squeeze theorem the top goes to 0 in the limit, so we could try l'Hospital's theorem:

$$\lim_{x\to0}\frac{x^2\cos(\frac{1}{x})}{x} = \lim_{x\to0}\frac{\sin(\frac{1}{x}) + 2x\cos(\frac{1}{x})}{1} = \text{DNE}$$

Since the limit of the quotient of derivatives does not exist, one may be tempted to say that the original limit does not exist, but l'Hospital's theorem *does not say this*. Instead, let's revisit the original limit: $\lim_{x\to0}\frac{x^2\cos(\frac{1}{x})}{x} = \lim_{x\to0} x\cos(\frac{1}{x})$. Instead we can just apply the squeeze theorem to this term to get that the limit is 0.

Part II

1) 5
2) 3
3) 0
4) 0
5) 0
6) 2
7) 5
8) 3
9) 2
10) infinity, *

Part III

1) 1
2) 0
3) 1
4) -4
5) 0
6) 8