Name_______________

Solve the following neatly on separate paper. Use your calculator on problem 1 only.

1. Let f be a function that has derivatives of all orders for all real numbers x Assume that

$$f(5)=6,\ f'(5)=8,\ f''(5)=30,\ f'''(5)=48, \text{ and } \left|f^{(4)}(x)\right|\le 75$$

for all x in the interval $[5, 5.2]$.

(a) Find the third-degree Taylor polynomial about $x = 5$ for $f(x)$.

(b) Use your answer to part (a) to estimate the value of $f(5.2)$. What is the maximum possible error in making this estimate? Give three decimal places.

(c) Find an interval $[a, b]$ such that $a \le f(5.2) \le b$. Give three decimal places.

(d) Could $f(5.2)$ equal 8.254? Show why or why not.

2. Let f be the function given by $f(x)=\cos\left(2x+\frac{\pi}{6}\right)$ and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$.

(a) Find $P(x)$.

(b) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right)-P\left(\frac{1}{10}\right)\right|<\frac{1}{12,000}$.

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3. Find the first four nonzero terms of the power series for $f(x)=\sin x$ centered at $x=\frac{3\pi}{4}$.

Find the first four nonzero terms and the general term for the Maclaurin series for:

4. $f(x)=x\cos\left(x^3\right)$

5. $g(x)=\frac{x^2}{1+x}$

Find the radius and interval of convergence for:

6. $\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{3^n n^2}$

7. $\sum_{n=1}^{\infty}(2n)!(x-5)^n$

Multiple Choice

8. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x)=\sin\left(x^2\right)$ is

$(A)\ -\frac{1}{6}$ $(B)\ 0$ $(C)\ \frac{1}{120}$ $(D)\ \frac{1}{6}$ $(E)\ 1$

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9. If f is a function such that $f'(x)=\sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x = 0$ is

$(A)\ \frac{1}{7!}$ $(B)\ \frac{1}{7}$ $(C)\ 0$ $(D)\ -\frac{1}{42}$ $(E)\ -\frac{1}{7!}$

Answers to Worksheet on Power Series and Lagrange Error Bound

1. (a) $6+8(x-5)+15(x-5)^2+8(x-5)^3$

   (b) $f(5.2)\approx P_3(5.2)=8.264$

   $|R_3(5.2)|\le 0.005$

   (c) $8.259\le f(5.2)\le 8.269$

   (d) No, 8.254 does not lie in the interval found in part (c).

2. (a) $\frac{\sqrt{3}}{2}-x-\frac{2\sqrt{3}}{2!}x^2+\frac{4}{3!}x^3$

   (b) $\left|R_3\left(\frac{1}{10}\right)\right|\le\left|\frac{16\left(\frac{1}{10}\right)^4}{4!}\right|=\frac{2^4\left(\frac{1}{2^4\cdot 5^4}\right)}{4!}=\frac{1}{5^4\cdot 4!}=\frac{1}{625\cdot 24}=\frac{1}{15{,}000}<\frac{1}{12{,}000}$

3. $\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\left(x-\frac{3\pi}{4}\right)-\frac{\sqrt{2}}{2\cdot 2!}\left(x-\frac{3\pi}{4}\right)^2+\frac{\sqrt{2}}{2\cdot 3!}\left(x-\frac{3\pi}{4}\right)^3+\ldots$

4. $x-\frac{x^7}{2!}+\frac{x^{13}}{4!}-\frac{x^{19}}{6!}+\ldots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{6n+1}}{(2n)!}$

5. $x^2-x^3+x^4-x^5+\ldots=\sum_{n=0}^{\infty}(-1)^n x^{n+2}$

6. Radius = 3; interval: $-1\le x\le 5$

7. Converges only if $x = 5$

8. A

9. D