AP Calculus – Final Review Sheet

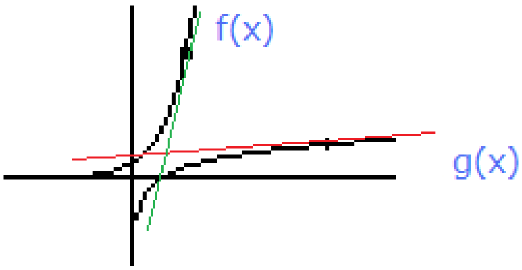
When you see the words ….	This is what you think of doing
1. Find the zeros of a function.	
2. Find equation of the line tangent to f(x) at (a,f(a)).	
3. Find equation of the line normal to f(x) at (a,f(a)).	
4. Show that f(x) is even.	
5. Show that f(x) is odd.	
6. Find the interval where f(x) is increasing.	
7. Find the interval where the slope of f(x) is increasing.	

8. Find the relative minimum value of a function f(x).	
9. Find the absolute minimum slope of a function f(x) on [a,b].	
10. Find critical values for a function f(x).	
11. Find inflection points of a function f(x).	
12. Show that $\lim_{x \to a} f(x)$ exists.	
13. Show that f(x) is continuous.	
14. Show that a piecewise function is differentiable at the point *a* where the function rule splits such as $h(x) = \begin{cases} f(x) \ \ for \ \ x \leq a \\ g(x) \ \ for \ \ x > a \end{cases}$	
15. Find vertical asymptotes of a function f(x).	

16. Find horizontal asymptotes of function f(x).	
17. Find the average rate of change of f(x) on [a,b].	
18. Find instantaneous rate of change of f(x) on [a,b].	
19. Find the average value of f(x) on [a,b].	
20. Find the absolute maximum of f(x) on [a,b].	
21. Show that a piecewise function is differentiable at the point *a* where the function rule splits	
22. Given s(t), the position function, find v(t), the velocity function.	

23. Given v(t), the velocity function, find how far a particle travels on [a,b].	
24. Find the average velocity of a particle on [a,b] given s(t), the position function. Find the average velocity of a particle on [a,b] given v(t), the velocity function.	
25. Given v(t), the velocity function, determine the intervals where a particle is speeding up.	
26. Given v(t), the velocity function, and s(0), the initial position, find s(t), the position function as a function of t.	
27. Show that Rolle's Theorem holds for a function f(x) on [a,b].	

28. Show that the Mean Value Theorem holds for a function f(x) on [a,b].	
29. Find domain of f(x).	
30. Find range of f(x) on [a,b].	
31. Find range of f(x) on $(-\infty, \infty)$.	
32. Find f ‘ (x), the derivative of f(x), by definition	

33. Given two functions f and f^{-1} are inverse functions ($f(a)=b$ and $f^{-1}(b)=a$) and $f'(a)$, find derivative of inverse function f^{-1} at $x=b$. Suppose that $g^{-1}(x)=f(x)$ and $g(x)=f^{-1}(x)$. Suppose a tangent line is drawn at (a,b) on the function f. Find the slope of the function g at the point (b,a).  	
34. Given $\dfrac{dy}{dt}$ is increasing proportionally to *y*, find a family of functions that describe the population as a function of time.	

35. Find the line x=c that divides the area under f(x) on [a,b] to two equal areas.

36. $\frac{d}{dx}\int_a^x f(t)\ dt =$

37. Given that u is some function of x

find $\frac{d}{dx}\int_a^u f(u)\ dt =$

38. Find the area bounded by f(x), the x-axis, x=1 and x = 10 using 3 trapezoids, where Δx=3.

39. Approximate the area bounded by f(x), the x-axis, x=0 and x = 7 using left Reimann sums from information about f(x) given in tabular data.

x	0	1	5	7
y	1	13	16	5

40. Approximate the area bounded by f(x), the x-axis, x=0 and x = 7 using right Reimann sums from information about f(x) given in tabular data

x	0	1	6	7
y	-1	-13	-16	-5

41. Approximate the area bounded by f(x), the x-axis, x = 0, and x = 14 using two subintervals and midpoint rectangles from information about f(x) x: 0, 3, 6, 10, 14 y: 1, 7, 12, 11, 3	
42. Approximate the area bounded by f(x), the x-axis, x = 0, and x = using three trapezoids from information about f(x) given in tabular data. x: 1, 5, 6, 10 y: 2, 7, 12, 15	
43. Given the graph of f ‘ (x) >0 between x=0 and x = a and f(0) =8, find f(a).	
44. Solve the differential equation $\frac{dy}{dx} = \frac{1+x}{y}$.	
45. Describe the meaning of $\int_a^x f(t)\ dt$	
46. Given a base is bounded by x = a, x = b, f(x) and g(x), where f(x) < g(x) for all a<x<b, find the volume of the solid whose cross section, perpendicular to the x-axis are squares.	

Table for 41:

x	0	3	6	10	14
y	1	7	12	11	3

Table for 42:

x	1	5	6	10
y	2	7	12	15

47. Find where the tangent line to f(x) is horizontal.	
48. Find where the tangent line to f(x) is vertical.	
49. Find the minimum acceleration given v(t), the velocity function.	
50. Approximate the value of f(1.1) by using the tangent line to *f* at x=1.	
51. Given the value of F(a) and the fact that the anti-derivative of *f* is *F*, find F(b).	
52. Find the derivative of f(g(x)).	
52. Given $\int_a^b f(x)\,dx$, find $\int_a^b \left[f(x)+k\right]\,dx$	

53. Given a graph of f '(x), find where f(x) is increasing.	
54. Given v(t), the velocity function, and s(0), the initial position, find the greatest distance from the origin of a particle on [0,b].	
55. Given a water tank with *g* gallons initially, is being filled at the rate of F(t) gallons/min and emptied at the rate of E(t) gallons/min on $\left[t_1, t_2\right]$, find the amount of water in the tank at *m* minutes where $t_1 < m < t_2$.	
56. Given a water tank with *g* gallons initially, is being filled at the rate of F(t) gallons/min and emptied at the rate of E(t) gallons/min on $\left[t_1, t_2\right]$, find the rate the water amount is changing at *m*.	
57. Given a water tank with *g* gallons initially, is being filled at the rate of F(t) gallons/min and emptied at the rate of E(t) gallons/min on $\left[t_1, t_2\right]$, find the time when the water is at a minimum.	
58. Given a chart of *x* and f(x) on selected values between *a* and *b*, estimate f '(c) where *c* is between *a* and b.	

59. Given $\frac{dy}{dx}$, draw a slope field	
60. Given that f(x) < g(x). find the area between curves f(x) and g(x) between x = a and x = b on [a,b].	
61. Given that f(x) > g(x). Find the volume of the solid created if the region between curves f(x) and g(x) between x = a and x = b on [a,b]. is revolved about the x-axis.	
62. Find a limit in the form $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$. Find the limit: $\lim_{x \to 0} \frac{\cos(x)-1}{x-1}$	
63. Given information about f(x) for x in [a,b], show that there exists a c in the interval [a,b], where .	
64. Given f " (x) and all critical values of x in (a,b) where f '(x)=0, determine the location of all relative extrema for f.	

65. Given f '(x) in graphical form on a domain (a,b), determine the location of all relative extrema for f.	
66. Given that functions f and g are twice differentiable, find h '(x) if h(x) = f(x)g(x) +k.	

AP Calculus – Final Review Sheet

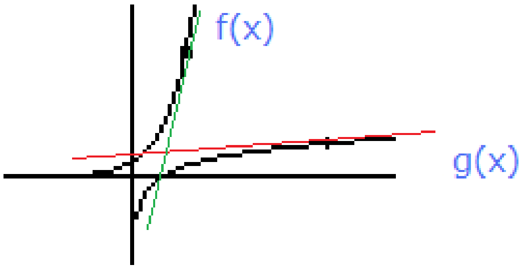
When you see the words ….	This is what you think of doing
1. Find the zeros of a function.	Set the function equal to zero and solve for x.
2. Find equation of the line tangent to f(x) at (a,f(a)).	Find f '(x),the derivative of f(x). Evaluate f '(a). Use the point and the slope to write the equation: y= f '(a)(x-a)+f(a)
3. Find equation of the line normal to f(x) at (a,f(a)).	Find f '(x),the derivative of f(x). Evaluate f '(a). The slope of the normal line is $-\frac{1}{f'(a)}$. Use the point and the slope to write the equation:
4. Show that f(x) is even.	Evaluate f at x = -a and x = a and show they are equal.
5. Show that f(x) is odd.	Evaluate f at x = -a and x = a and show they are opposite.
6. Find the interval where f(x) is increasing.	Find f '(x) and find all intervals in the domain of f and f ' where f '(x) > 0.
7. Find the interval where the slope of f(x) is increasing.	Find f "(x) and find all intervals in the domain of f, f ', and f " where f "(x) >0.

8. Find the relative minimum value of a function f(x).	Find all the critical points for f, where f '(x)=0 or f '(x) does not exist. Find all locations where f ' changes from negative to positive or where f changes from decreasing to increasing.
9. Find the absolute minimum slope of a function f(x) on [a,b].	Find all critical points of f ', where f "(x)=0 or f " (x) does not exist. Evaluate f '(x) at all critical points of f ' and the endpoints. From these values find where f ' is minimum.
10. Find critical values for a function f(x).	Find f '(x) and then locate all points where f '(x)=0 or f '(x) does not exists.
11. Find inflection points of a function f(x).	Find f "(x) and then find all locations where f "(x) changes sign.
12. Show that $\lim_{x\to a} f(x)$ exists.	Find $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ and show they are equal.
13. Show that f(x) is continuous.	Show that $\lim_{x\to a} f(x)$ exists and that $\lim_{x\to a} f(x) = f(a)$
14. Show that a piecewise function is differentiable at the point *a* where the function rule splits such as $h(x) = \begin{cases} f(x) \text{ for } x \le a \\ g(x) \text{ for } x > a \end{cases}$	Find f ' (x) and g ' (x). Then show that $\lim_{x\to a^-} f'(x) = \lim_{x\to a^+} g'(x)$.
15. Find vertical asymptotes of a function f(x).	Look at the definition of the function f(x). If f is written in a ratio, first check that the function cannot be simplified. Then locate all places where the denominator of the function equals zero.

16. Find horizontal asymptotes of function f(x).	Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. If either of these limits exists then the function has at least one or two horizontal asymptotes. Their form would be y = k where k is the limit.
17. Find the average rate of change of f(x) on [a,b].	This is the slope of the secant line between (a,f(a)) and (b, f(b)) or $\dfrac{f(b)-f(a)}{b-a}$.
18. Find instantaneous rate of change of f(x) on [a,b].	This is another name for f ′(a), or the derivative the function evaluated at x = a.
19. Find the average value of f(x) on [a,b].	This means to find the average value that f takes on between (a, f(a)) and (b, f(b)). It is found by find the area of the function bounded by x=a. x=b, x=0, and y=f(x). Then divide this by the width of the interval b-a. It is written as $\dfrac{\int_a^b f(x)dx}{b-a}$
20. Find the absolute maximum of f(x) on [a,b].	Find all the critical points for f, where f ‘(x)=0 or f ‘(x) does not exist. Evaluate the function at all critical points of f and endpoints. From these values find where f is maximum.
21. Show that a piecewise function is differentiable at the point *a* where the function rule splits	Find the derivative of each piece of the function. Show that the $\lim_{x\to a} f'(x)$ exists or is equal from the left and the right.
22. Given s(t), the position function, find v(t), the velocity function.	Find the derivative of s(t).

23. Given v(t), the velocity function, find how far a particle travels on [a,b].	Evaluate $\int_a^b \lvert v(t) \rvert \, dt$. Remember that $\int_a^b v(t)dt$ only finds the net distance traveled.
24. Find the average velocity of a particle on [a,b] given s(t), the position function. Find the average velocity of a particle on [a,b] given v(t), the velocity function.	The average velocity of a particle, given s(t), is the slope of the secant line: $\frac{s(b)-s(a)}{b-a}$. The average velocity of a particle, given v(t), iw the same as finding the average value of a
25. Given v(t), the velocity function, determine the intervals where a particle is speeding up.	Evaluate v(t) for its sign. Find the derivative of v(t) to determine a(t). Determine when the particle in stationery (v(t)=0). Determine when a(t)=0. Study the intervals where the particle is initially at rest and then shows positive or negative velocity, which means it will move left or right. The particle will have to speed up until it reaches point where a(t)=0. Locate the point where the particle will have an a(t)=0. (Now it will begin to slow down and eventually come to rest again.
26. Given v(t), the velocity function, and s(0), the initial position, find s(t), the position function as a function of t.	To write s(t) you will need to write a function using an integral: $s(t) = s(0) + \int_0^t v(x)dx$
27. Show that Rolle's Theorem holds for a function f(x) on [a,b].	Verify that f(x) is continuous on [a,b] and differentiable on (a,b). Verify that f(a)=0 and f(b)=0. Then you are guaranteed that there exists a point c (a<c<b) where f '(c)=0.

28. Show that the Mean Value Theorem holds for a function f(x) on [a,b].	Verify that f(x) is continuous on [a,b] and differentiable on (a,b). Then you are guaranteed that there exists a point c (a<c<b) where $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.
29. Find domain of f(x).	Analyze the function f. Look for radical expressions in the description. Determine values of x that cannot be used within the radical. Exclude these from the domain. Look at the denominator. If the denominator contains a polynomial, find the zeros for this polynomial and exclude these x values from the domain.
30. Find range of f(x) on [a,b].	If f is continuous on [a,b], then the range of f will between [minimum value of f, maximum value of f].
31. Find range of f(x) on $(-\infty,\infty)$.	If f is continuous on $(-\infty,\infty)$ then you will need to consider $\lim\limits_{x\to+\infty} f(x)=k_1$ and $\lim\limits_{x\to-\infty} f(x)=k_2$. If these limits are above the local maximum or below the local minimum the range will be $[k_1,k_2]$. Otherwise you will have to adjust the range. If the limits go to infinity then the range is $(-\infty,\infty)$.
32. Find f ‘ (x), the derivative of f(x), by definition	Use $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

33. Given two functions f and f^{-1} are inverse functions ($f(a)=b$ and $f^{-1}(b)=a$) and $f'(a)$, find derivative of inverse function f^{-1} at $x=b$. Suppose that $g^{-1}(x)=f(x)$ and $g(x)=f^{-1}(x)$. Suppose a tangent line is drawn at (a,b) on the function f. Find the slope of the function g at the point (b,a). 	$\left(f^{-1}\right)'(b) = \dfrac{1}{f'(a)}$ The slope of the function g at the point (b,a) is simply the reciprocal of the slope of the f function at (a,b) or $g'(b) = \dfrac{1}{f'(a)}$.
34. Given $\dfrac{dy}{dt}$ is increasing proportionally to *y*, find a family of functions that describe the population as a function of time.	First begin with $\dfrac{dy}{dt} = ky$, then separate the variables, integrate each side and add a constant of integration to one side. Continue to solve for the population as a function of t. If you are told that the graph will pass through a point, substitute this point into the equation to solve for the constant. After replacing the constant continue to write the equation as a function, by considering any restrictions placed on the problem by the point used in the substitution step. $\dfrac{dy}{dt} = ky \Rightarrow \dfrac{dy}{y} = kdt$ $\int \dfrac{dy}{y} = \int kdt$ $\ln\lvert y\rvert = kt + C$ $y = e^{kt+C}$ $y = Ce^{kt}$

35. Find the line x=c that divides the area under f(x) on [a,b] to two equal areas.

Find a point c such that $\int_a^c f(x)dx = \dfrac{\int_a^b f(x)dx}{2}$

36. $\dfrac{d}{dx}\int_a^x f(t)\ dt =$

f(x)

37. Given that u is some function of x

find $\dfrac{d}{dx}\int_a^u f(u)\ dt =$

$f(u)\dfrac{du}{dx}$

38. Find the area bounded by f(x), the x-axis, x=1 and x = 10 using 3 trapezoids, where Δx=3.

Find f(1), f(4), f(7), and f(10). Use these for the bases in finding the area of three trapezoids with heights of 3:

$\frac{1}{2}(3)(f(1)+f(4))+\frac{1}{2}(3)(f(4)+f(7))+\frac{1}{2}(3)(f(7)+f(10))$

39. Approximate the area bounded by f(x), the x-axis, x=0 and x = 7 using left Reimann sums from information about f(x) given in tabular data.

x	0	1	5	7
y	1	13	16	5

Find the base, difference between x values, and height (at left hand end) of the three rectangles.

$(1)(1)+(4)(13)+(2)(16)$

40. Approximate the area bounded by f(x), the x-axis, x=0 and x = 7 using right Reimann sums from information about f(x) given in tabular data.

x	0	1	6	7
y	-1	-13	-16	-5

Find the base, difference between x values, and height (at right hand end) of the three rectangles.

$(1)(-13)+(5)(-16)+(1)(-5)$

41. Approximate the area bounded by f(x), the x-axis, x = 0, and x = 14 using two subintervals and midpoint rectangles from information about f(x)

x	0	3	6	10	14
y	1	7	12	11	3

Find the intervals for the two rectangles: (0,6) and (6,14). The midpoints are 3 and 10. Find the height of the rectangles: 7 and 11 respectively.

Find the area: $(6)(7)+(8)(11)$

42. Approximate the area bounded by f(x), the x-axis, x = 0, and x = using three trapezoids from information about f(x) given in tabular data.

x	1	5	6	10
y	2	7	12	15

Find the height of the three trapezoids: 4, 1, and 4. Find the bases: 2 and 7, 7 and 12, and 12 and 15. Find the areas:

$$\frac{1}{2}(4)(2+7)+\frac{1}{2}(1)(7+12)+\frac{1}{2}(4)(12+15)$$

43. Given the graph of f ' (x) >0 between x=0 and x = a and f(0) =8, find f(a).

$f(a)=f(0)+\int_0^a f'(x)dx$ To find the integral you can find the area under the f ' graph between x=0 and x=a.

44. Solve the differential equation $\frac{dy}{dx}=\frac{1+x}{y}$.

Separate the variables and then integrate each side. Remember to include a constant of integration. If possible find the constant through substitution.

45. Describe the meaning of $\int_a^x f(t)\ dt$

Suppose f(x) is a rate equation for F(t). Then this integral represent the net change in F(t) from time a to time x.

46. Given a base is bounded by x = a, x = b, f(x) and g(x), where f(x) < g(x) for all a<x<b, find the volume of the solid whose cross section, perpendicular to the x-axis are squares.

Volume of the solid = $\int_a^b (g(x)-f(x))^2\ dx$

47. Find where the tangent line to f(x) is horizontal.	Find f '(x) and then set f '(x) =0 and solve for x.
48. Find where the tangent line to f(x) is vertical.	Find f '(x) and then analyze f '(x) to determine where f '(x) is undefined because of a denominator.
49. Find the minimum acceleration given v(t), the velocity function.	Find a(t) or the derivative of v(t) and a'(t). Find the critical points for a(t) from a'(t). Find where a'(t) is changing from negative to positive (a(t) changing from decreasing to increasing). These are locations for the local minimum accelerations.
50. Approximate the value of f(1.1) by using the tangent line to *f* at x=1.	Write the tangent line at x=1. $y = f'(1)(x-1) + f(1)$. Use x = 1.1 in this tangent line to find the approximate value of f(1.1).
51. Given the value of F(a) and the fact that the anti-derivative of *f* is *F*, find F(b).	$F(b) = F(a) + \int_a^b f(x)dx$
52. Find the derivative of f(g(x)).	$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$
52. Given $\int_a^b f\left(x\right)\,dx$, find $\int_a^b \left[f\left(x\right)+k\right]\,dx$	$\int_a^b \left[f\left(x\right)+k\right]\,dx = \int_a^b f(x)dx + \int_a^b kdx =$ $\int_a^b f(x)dx + k(b-a)$

53. Given a graph of f '(x), find where f(x) is increasing.	From the graph of f '(x) find where the graph is below the x-axis. This means f '(x) is negative. Describe these intervals.
54. Given v(t), the velocity function, and s(0), the initial position, find the greatest distance from the origin of a particle on [0,b].	Find when v(t) is zero. This means the function is at rest at these values. Write s(t). $s(t) = s(0) + \int_0^t v(x)dx$. Evaluate s(t) at each place v(t) is zero. Pick out the greatest distance from the origin.
55. Given a water tank with *g* gallons initially, is being filled at the rate of F(t) gallons/min and emptied at the rate of E(t) gallons/min on $\left[t_1, t_2\right]$, find the amount of water in the tank at *m* minutes where $t_1 < m < t_2$.	$\int_{t_1}^{m} \left(F(t) - E(t)\right) dt$
56. Given a water tank with *g* gallons initially, is being filled at the rate of F(t) gallons/min and emptied at the rate of E(t) gallons/min on $\left[t_1, t_2\right]$, find the rate the water amount is changing at *m*.	F(t)-E(t)
57. Given a water tank with *g* gallons initially, is being filled at the rate of F(t) gallons/min and emptied at the rate of E(t) gallons/min on $\left[t_1, t_2\right]$, find the time when the water is at a minimum.	Differentiate the integral in question 55 with respect to t. This will give you a rate equation or the equation in question 56. Find the zeros for F(t)-E(t). Evaluate the integral from question 55 at these zero's and the endpoints. Pick out the minimum value.
58. Given a chart of *x* and f(x) on selected values between *a* and *b*, estimate f '(c) where *c* is between *a* and b.	Use two sets of points (a, f(a)) and (b, f(b)) near c to evaluate $f'(c) \approx \dfrac{f(b) - f(a)}{b - a}$

59. Given $\frac{dy}{dx}$, draw a slope field	Identify points on the graph. Name the coordinates of these points. Evaluate $\frac{dy}{dx}$ at these points. Draw a short line that represents the given slope at that point. The slope field should model the slope of a family of functions whose derivative
60. Given that f(x) < g(x). find the area between curves f(x) and g(x) between x = a and x = b on [a,b].	$\int_a^b (f(x) - g(x))dx$
61. Given that f(x) > g(x). Find the volume of the solid created if the region between curves f(x) and g(x) between x = a and x = b on [a,b]. is revolved about the x-axis.	$\pi\int_a^b \left((f(x))^2 - (g(x))^2\right)dx$
62. Find a limit in the form $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$. Find the limit: $\lim_{x\to 0}\frac{\cos(x)-1}{x-1}$	Determine the value of a and the function f. Differentiate f and evaluate at a. Notice that f(x)=cos x and that x is approaching 0 so this expression is of the form $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$. The limit can be found by taking the derivative of cos x at the point x = 0.
63. Given information about f(x) for x in [a,b], show that there exists a c in the interval [a,b], where $f'(c) = \frac{f(b)-f(a)}{b-a}$.	Check to see that f(x) is continuous on [a,b] and differentiable on (a,b). Then the Mean Value Theorem guarantees that there exists a c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.
64. Given f " (x) and all critical values of x in (a,b) where f '(x)=0, determine the location of all relative extrema for f.	Check the concavity of f at each critical value where f '(x) = 0. If f"(x)>0 you have found the location of a minimum. If f"(x)<0 you have found the location of a maximum.

65. Given f '(x) in graphical form on a domain (a,b), determine the location of all relative extrema for f.	Find locations where the graph of f ' is changing from being below the x-axis to being above the x-axis. This is a location of a relative minimum. Find locations where the graph of f ' is changing from being above the x-axis to being below the x-axis. This is a location of a relative maximum.
66. Given that functions f and g are twice differentiable, find h '(x) if h(x) = f(x)g(x) +k.	h '(x)=f(x)g '(x) +g(x)f '(x)