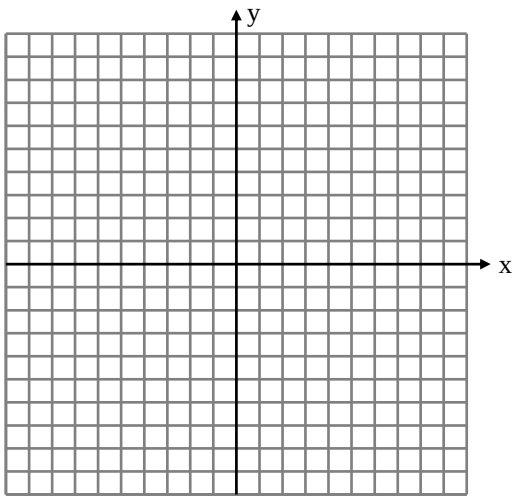
Derivatives of Inverse Functions
Discovery Activity



1. Given $f(x) = 2x + 5$.

(a) Graph $f(x)$. Is f a one-to-one function?
(Why does it matter?)

(b) The point (1, 7) lies on f. Write this point in function notation: $f(\quad) =$

(c) Find $f'(x)$.

(d) Evaluate $f'(x)$ at the point (1, 7).

(e) Find $f^{-1}(x)$ and graph it.

(f) Is f^{-1} a one-to-one function?

How would you know without graphing it?

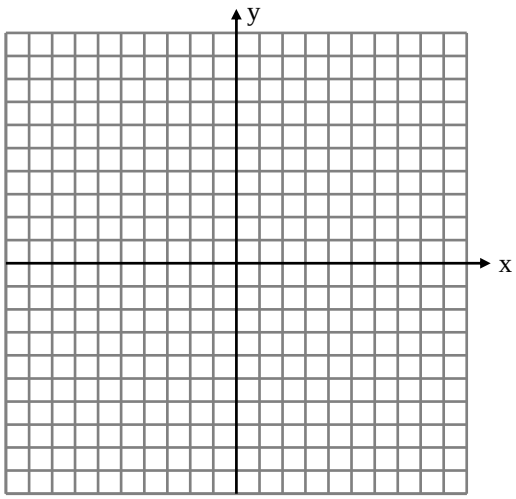
(g) Find $\left(f^{-1}\right)'(x)$.

(h) What point on f^{-1} corresponds to the point (1, 7) that lies on f?
Write this point in function notation: $f^{-1}(\quad) =$

(i) Evaluate $\left(f^{-1}\right)'(x)$ at the point on $f^{-1}(x)$ that you found in (h).

(j) What do you notice about your answers to (d) and (i)?

2. Given $f(x) = x^2$ for $x \geq 0$.



(a) Graph $f(x)$. Is f a one-to-one function?

(b) There is a point on the graph of f where the *x*-coordinate is 3. What is the *y*-coordinate of this point?
Write this point in function notation: $f(\quad) =$

(c) Find $f'(x)$.

(d) Evaluate $f'(x)$ at the point (3, 9).

(e) Find $f^{-1}(x)$, restricting it if necessary, and graph it.

(f) Is f^{-1} a one-to-one function?

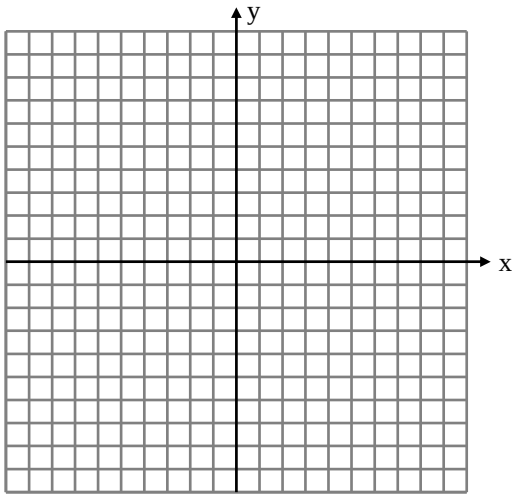
(g) Find $\left(f^{-1}\right)'(x)$.

(h) What point on f^{-1} corresponds to the point (3, 9) that lies on f?
Write this point in function notation: $f^{-1}(\quad) =$

(i) Evaluate $\left(f^{-1}\right)'(x)$ at the point on $f^{-1}(x)$ that you found in (h).

(j) What do you notice about your answers to (d) and (i)?

3. Given $f(x) = \sqrt[3]{x}$.



(a) Graph $f(x)$. Is f a one-to-one function?

(b) There is a point on the graph of f where the y-coordinate is 2. What is the x-coordinate of this point?

Write this point in function notation: $f(\quad) =$

(c) Find $f'(x)$.

(d) Evaluate $f'(x)$ at the point (8, 2).

(e) Find $f^{-1}(x)$, restricting it if necessary, and graph it.

(f) Is f^{-1} a one-to-one function?

How would you know without graphing it?

(g) Find $\left(f^{-1}\right)'(x)$.

(h) What point on f^{-1} corresponds to the point (8, 2) that lies on f?

Write this point in function notation: $f^{-1}(\quad) =$

(i) Evaluate $\left(f^{-1}\right)'(x)$ at the point on $f^{-1}(x)$ that you found in (h).

(j) What do you notice about your answers to (d) and (i)?

4. Given $f(x) = x^3 + 3x - 5$.

(a) Is f a one-to-one function?

How can you tell without graphing it?

(b) There is a point on f in which the y-coordinate is 9. What is the x-coordinate at this point?

Write this point in function notation: $f(\quad) =$

(c) What point on f^{-1} corresponds to the point you found in (b) that lies on f?

Write this point in function notation: $f^{-1}(\quad) =$

(d) Find $\left(f^{-1}\right)'(9)$ **without** finding a function for f^{-1}. Do this below, and list the steps you used.

Can you write a rule for finding the derivative of the inverse of a function without actually finding the inverse?

$\left(f^{-1}\right)'(a) =$ _____