Name_______________

Label and present your answers neatly on separate paper.

1. a) Draw the line $y = 2t + 1$ and use geometry to find the area under this line, above the $t-$axis, and between the vertical lines $t = 1$ and $t = 3$.

b) If $x > 1$, let $A(x)$ be the area of the region that lies under the line $y = 2t + 1$ between $t = 1$ and $t = x$. Sketch this region and use geometry to find an expression for $A(x)$.

c) Differentiate the area function $A(x)$. What do you notice?

2. a) If $x \geq -1$, let

$$A(x) = \int_{-1}^{x} (1 + t^2)\,dt$$

$A(x)$ represents the area of a region. Sketch that region.

b) Use the result below to find an expression for $A(x)$.

$$\int_{a}^{b} x^2 dx = \frac{b^3}{3} - \frac{a^3}{3}$$

c) Find $A'(x)$. What do you notice?

d) If $x \geq -1$ and h is a small positive number, then $A(x+h) - A(x)$ represents the area of a region. Describe and sketch the region.

e) Draw a rectangle that approximates the region in part d). By comparing the areas of these two regions, show that:

$$\frac{A(x+h) - A(x)}{h} \approx 1 + x^2$$

f) Use part e) to give an intuitive explanation for the result of part c).

3. a) Draw the graph of the function $f(x) = \cos(x^2)$ in the viewing rectangle [0, 2] by [-1.25, 1.25].

b) If we define a new function g by

$$g(x) = \int_0^x \cos(t^2)dt$$

then $g(x)$ is the area under the graph of f from 0 to x [until $f(x)$ becomes negative, at which point $g(x)$ becomes a difference of areas]. Use part a) to determine the value of x at which $g(x)$ starts to decrease.

c) Use the definite integral command on your calculator to estimate $g(0.2), g(0.4), g(0.6), \ldots, g(1.8), g(2)$. Then use these values to sketch a graph of g.

d) Use your graph of g from part c) to sketch the graph of g' using the interpretation of $g'(x)$ as the slope of the tangent line. How does the graph of g' compare with the graph of f?

4. Suppose f is a continuous function on the interval [a, b] and we define a new function

$$g(x) = \int_a^x f(t)\, dt$$

Based on your results from problems 1 – 3, conjecture an expression for $g'(x)$.