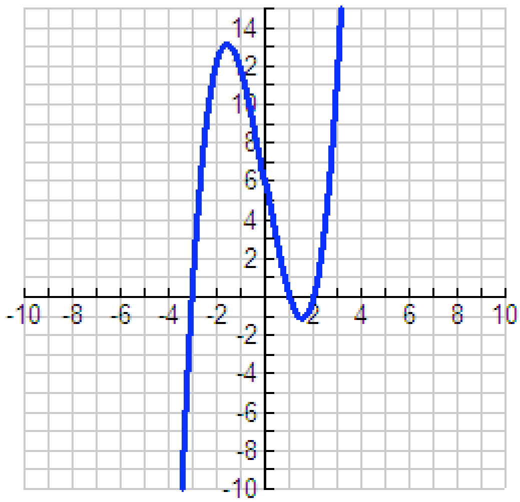
Worksheet: Extrema, Mean Value Theorem, Increasing Decreasing Behavior

1. The graph of the derivative, $f'(x)$, of a continuous function f is shown below.



graph of $f'(x)$

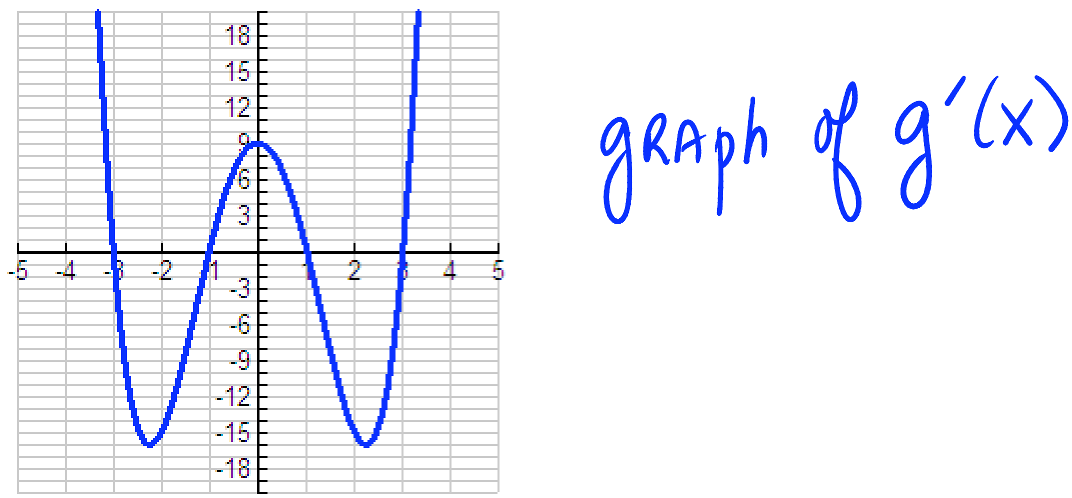
(a) Based on the graph above, where does the graph of $f(x)$ have critical values?

(b) On what interval(s) is the graph of f increasing? Explain fully.

(c) On what interval(x) is the graph of f decreasing? Explain fully.

(d) At what $x-$value(s) does the graph of f have a relative minimum value? Explain fully.

2. The graph of the derivative, $g'(x)$, of a continuous function g is shown below.



(a) Based on the graph above, where does the graph of g have critical values?

(b) Locate all relative extrema of the graph of g. Explain fully.

3. Find the value of c that satisfies the Mean Value Theorem for the function $f(x) = \sqrt{1-x}$ for the interval $[-8,\ 1]$

4. A police officer clocks a commuter's speed at 50 mph as he enters a tunnel whose length is exactly 0.75 miles. A second officer measures the commuter's speed at 45 mph as he exits the tunnel 43 seconds later and tickets the driver for exceeding the posted speed limit of 50 mph. Use the Mean Value Theorem to justify the speeding charge levied by the officer, even though the driver was neither exceeding the posted speed limit while entering nor while exiting the tunnel.

5. A particle is moving along the $x-$axis with velocity $v(t) = (2t-3)^2(t-5)$. At what time(s) in the open interval (0, 6) does the particle change direction? Explain fully.

6. Consider the function $f(x) = x\sin x$ on the interval $[-4, 4]$. Find all the values that satisfy Rolle's Theorem. Remember to show your work.

7. Let f be a twice-differentiable function such that $f(2)=5$ and $f(5)=2$. Let g be the function given by $g(x)=f(f(x))$.

(a) Explain why there must be a value c for $2<c<5$ such that $g'(c)= \ 1$

(b) Show that $g'(2)=g'(5)$. Use this result to explain why there must be a value k for $2<k<5$ such that $g''(k)=0$. [Hint: Use the MVT]

BONUS

A particle is moving along the $x-$axis with velocity $v(t)=t\sin t$. Is the speed of the particle increasing or decreasing at $t=2$? Explain fully.