

Given a differential equation  $\frac{dy}{dx} = g(x, y)$ , construct the slope field (phase plane) it describes, and then sketch a particular solution.

- ✓ **Factor:** If the differential equation is not factored, factor it completely. Doing so helps study the sign of the slope measure  $\frac{dy}{dx}$  in various regions of the two-dimensional plane.
- ✓ **Zero:** Determine the conditions under which the slope is zero. Set each factor in the numerator position equal to zero, then interpret all the resulting conditions by describing the sets of points that they represent on the 2D plane. For example, if  $(y - 2x)$  is one of the factors of  $\frac{dy}{dx}$ , then all the points along the line  $y = 2x$  will have slope equal to zero. On the phase plane, we draw tiny horizontal segment lines at points with integer coordinates.
- ✓ **Undefined:** Slope will be undefined whenever factors in the denominator are set equal to zero. We mark tiny vertical segments on such points.
- ✓ **Sign:** Discuss the sign of the slope in each region of the plane. The regions are often the four quadrants, although it depends on the nature of the factors. An accurate and quick inspection of the sign may be needed especially on a multiple choice question where you are asked to match a slope field to a given differential equation, or vice-versa.
- ✓ **Magnitude:** The relative size of the slope can be illustrated on the phase plane by drawing the segment a bit steeper or flatter than the adjacent tangent marks.

**Example:** We know that solutions to  $\frac{dy}{dx} = -xy$  will have negatively sloped tangent lines in the first and third quadrants, and the slope field segments will be positively sloped in the second and fourth quadrants. Further, observe that the slope equals zero on all points along the axes and is never undefined.

