Disk Method

We use the disk method whenever the region borders the rotational axis.

Horizontal Rotational Axis – Disk Method

- ✓ The equation of the curve is of the form $y = f(x)$.
- ✓ The interval of $x-$values is $[a,b]$.
- ✓ The rotational axis has equation $y = m$.

$$Volume = \int_a^b A(x)\,dx = \int_a^b \pi\left(R(x)\right)^2 dx = \int_a^b \pi\left|f(x) - m\right|^2 dx$$

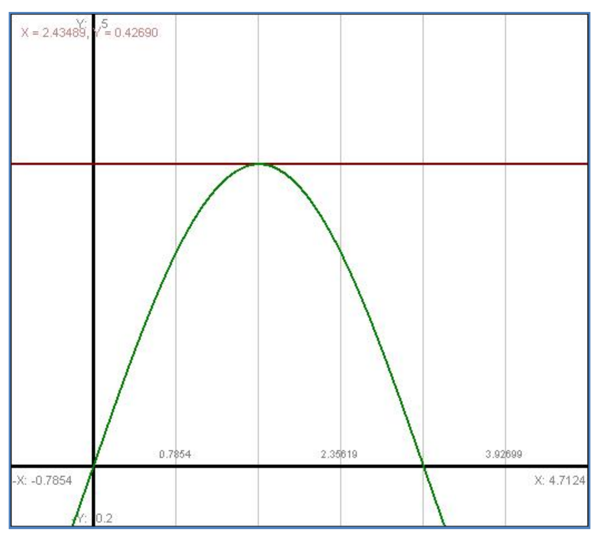
- ✓ Special case: the rotational axis is $y = 0$.

$$Volume = \int_a^b \pi\left[f(x)\right]^2 dx$$

Example: Set up, but do not evaluate the volume of the solid generated when the region enclosed by the graph of $y = \sin x$, and the lines $y = 1$, $x = 0$, and $x = \pi$ is rotated about $y = 1$.

$$V = \int_0^\pi \pi\left(f(x) - 1\right)^2 dx = \int_0^\pi \pi\left(1 - f(x)\right)^2 dx =$$

$$V = \int_0^\pi \pi\left(1 - \sin x\right)^2 dx$$



Vertical Rotational Axis – Disk Method

- ✓ The equation of the curve is of the form $x = g(y)$.
- ✓ The interval of $y-$values is $[c,d]$.
- ✓ The rotational axis has equation $x = n$.

$$Volume = \int_c^d A(y)\,dy = \int_c^d \pi\left(R(y)\right)^2 dy = \int_c^d \pi\left|g(y)-n\right|^2 dy$$

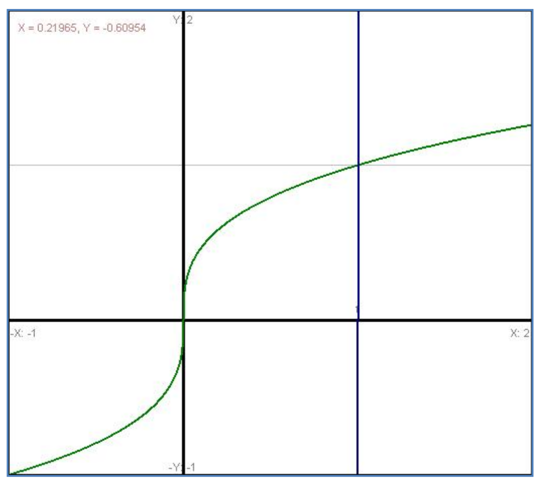
- ✓ Special case: the rotational axis is $x = 0$.

$$Volume = \int_c^d \pi\left[g(y)\right]^2 dy$$

Example: Set up, but do not evaluate the volume of the solid generated when the region enclosed by the graph of $x = y^3$, and the lines $y = 0$ and $x = 1$ is rotated about $x = 1$.

$$V = \int_{y=0}^{y=1} \pi\left(f(y)-1\right)^2 dy = \int_0^1 \pi\left(1-f(y)\right)^2 dy =$$

$$V = \int_0^1 \pi\left(1-y^3\right)^2 dy =$$



Washer Method

We use the washer method whenever the region is enclosed by two functions. The distance from the farthest function to the rotational axis is called outer radius. The distance from the nearest function to the rotational axis is called inner radius.

Horizontal Rotational Axis – Washer Method

- ✓ The equation of the outer function is of the form $y = f(x)$.
- ✓ The equation of the inner function is of the form $y = g(x)$.
- ✓ The interval of x – values is $[a,b]$.
- ✓ The rotational axis has equation $y = m$, located at a distance above or below the enclosed region by $f(x)$ and $g(x)$.

$$Volume = DISK_{OUTER} - DISK_{INNER} = \int_a^b \pi\left(f(x)-m\right)^2 dx - \int_a^b \pi\left(g(x)-m\right)^2 dx$$

$$Volume = \pi\int_a^b \left(f(x)-m\right)^2 - \left(g(x)-m\right)^2 dx$$

- ✓ Special case: the rotational axis is $y = 0$.

$$Volume = \pi\int_a^b [f(x)]^2 - [g(x)]^2 dx = \pi\int_a^b [OUTER(x)]^2 - [INNER(x)]^2 dx$$

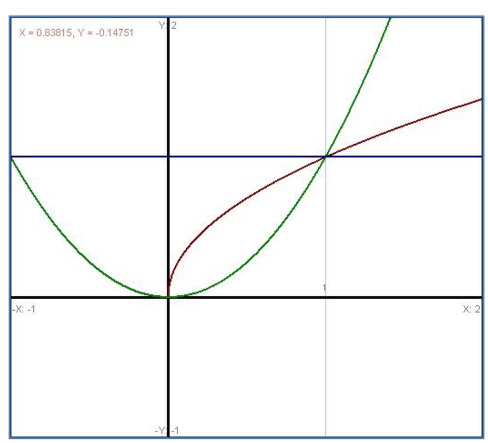
Example: Set up, but do not evaluate the volume of the solid generated when the region enclosed by the graph of $y = \sqrt{x}$ and $y = x^2$ is rotated about $y = 1$.

$$OUTER(x) = 1 - x^2 = \left|x^2 - 1\right|$$

$$INNER(x) = 1 - \sqrt{x} = \left|\sqrt{x} - 1\right|$$

$$V = \pi\int_{x=0}^{x=1} \left(OUTER(x)\right)^2 - \left(INNER(x)\right)^2 dx$$

$$V = \pi\int_{x=0}^{x=1} \left(1-x^2\right)^2 - \left(1-\sqrt{x}\right)^2 dx$$



Vertical Rotational Axis – Washer Method

- ✓ The equation of the outer function is of the form $x = f(y)$.
- ✓ The equation of the inner function is of the form $x = g(y)$.
- ✓ The interval of $y-$values is $[c,d]$.
- ✓ The rotational axis has equation $x = n$, located at a distance to the left or to the right of the enclosed region by $f(y)$ and $g(y)$.

$$Volume = DISK_{OUTER} - DISK_{INNER} = \int_c^d \pi\left(f(y)-n\right)^2 dy - \int_c^d \pi\left(g(y)-n\right)^2 dy$$

$$Volume = \pi\int_c^d \left(f(y)-n\right)^2 - \left(g(y)-n\right)^2 dy$$

- ✓ Special case: the rotational axis is $x = 0$.

$$Volume = \pi\int_c^d [f(y)]^2 - [g(y)]^2 dy = \pi\int_c^d [OUTER(y)]^2 - [INNER(y)]^2 dy$$

Example: Set up, but do not evaluate the volume of the solid generated when the region enclosed by the graph of $x = y$ and $x = y^2$ in the first quadrant is rotated about $x = 1$.

$$OUTER(y) = 1 - y^2 = \left|y^2 - 1\right|$$

$$INNER(y) = 1 - y = \left|y - 1\right|$$

$$V = \pi\int_{y=0}^{y=1} \left(OUTER(y)\right)^2 - \left(INNER(y)\right)^2 dy$$

$$V = \pi\int_0^1 \left(1-y^2\right)^2 - \left(1-y\right)^2 dy$$