Firm Theory with Differential Calculus

Function	Description	
x	Quantity	Quantity supplied or demanded
$p(x)$	Price Function	Price at which x units are demanded
$R(x)=xp(x)$	Revenue	The product of price and quantity
F	Fixed Cost	Cost that does not vary with output
$V(x)$	Variable Cost	Cost that varies with the output level
$C(x)=F+V(x)$	Total Cost	Direct sum of fixed and variable costs
$A(x)=\frac{C(x)}{x}$	Average Cost	Total Cost divided by quantity
$R'(x)=MR(x)$	Marginal Revenue	The rate of revenue with respect to output
$C'(x)=MC(x)$	Marginal Cost	The rate of total cost with respect to output
$\Pi(x)=R(x)-C(x)$	Profit Function	The difference between revenue and cost.

Interpretations:

$R'(100)=$ the revenue brought in by the 101^{st} unit.

$R'(100)=$ the cost of producing the 101^{st} unit

F = the $y-$intercept of the total cost function

Two types of firm behavior are discussed here: average-cost-minimization and profit-maximization.

I. Minimizing Average Cost

Objective Function: $A(x) = \frac{C(x)}{x}$

$\Rightarrow A'(x) = \frac{xC'(x) - C(x)}{x^2} \to x = 0 \ \vee \ xC'(x) - C(x) = 0$

$\Rightarrow xC'(x) - C(x) = 0$

$\Rightarrow C'(x) = \frac{C(x)}{x}$

$\Rightarrow C'(x) = A(x)$

$\Rightarrow MC = AC$

Remark: One must justify a minimum average cost using one of the derivative tests.

II. Maximizing Profit

Objective Function: $\prod(x) = R(x) - C(x)$

$\Rightarrow \prod'(x) = R'(x) - C'(x) = 0$

$\Rightarrow R'(x) = C'(x)$

$\Rightarrow MR = MC$

Remark: One must justify a maximum profit using one of the derivative tests.