Increasing/Decreasing Test

If $f'(x) > 0$ in (a,b), then $f(x)$ is increasing in (a,b).

If $f'(x) < 0$ in (a,b), then $f(x)$ is decreasing in (a,b).

First Derivative Test

Suppose $x = c$ is a critical number of $f(x)$.

I. If $f'(x)$ changes sign from + to – at $x = a$, then $f(a)$ is a local maximum.
II. If $f'(x)$ changes sign from – to + at $x = a$, then $f(a)$ is a local minimum.
III. If $f'(x)$ does not change sign at $x = a$, then $f(a)$ is not a local extremum.

Concavity Test

If $f''(x) > 0$ in (a,b), then $f(x)$ is concave up in (a,b).

If $f''(x) < 0$ in (a,b), then $f(x)$ is concave down in (a,b).

Second Derivative Test

Suppose $f(x)$ is twice differentiable near a critical number $x = c$.

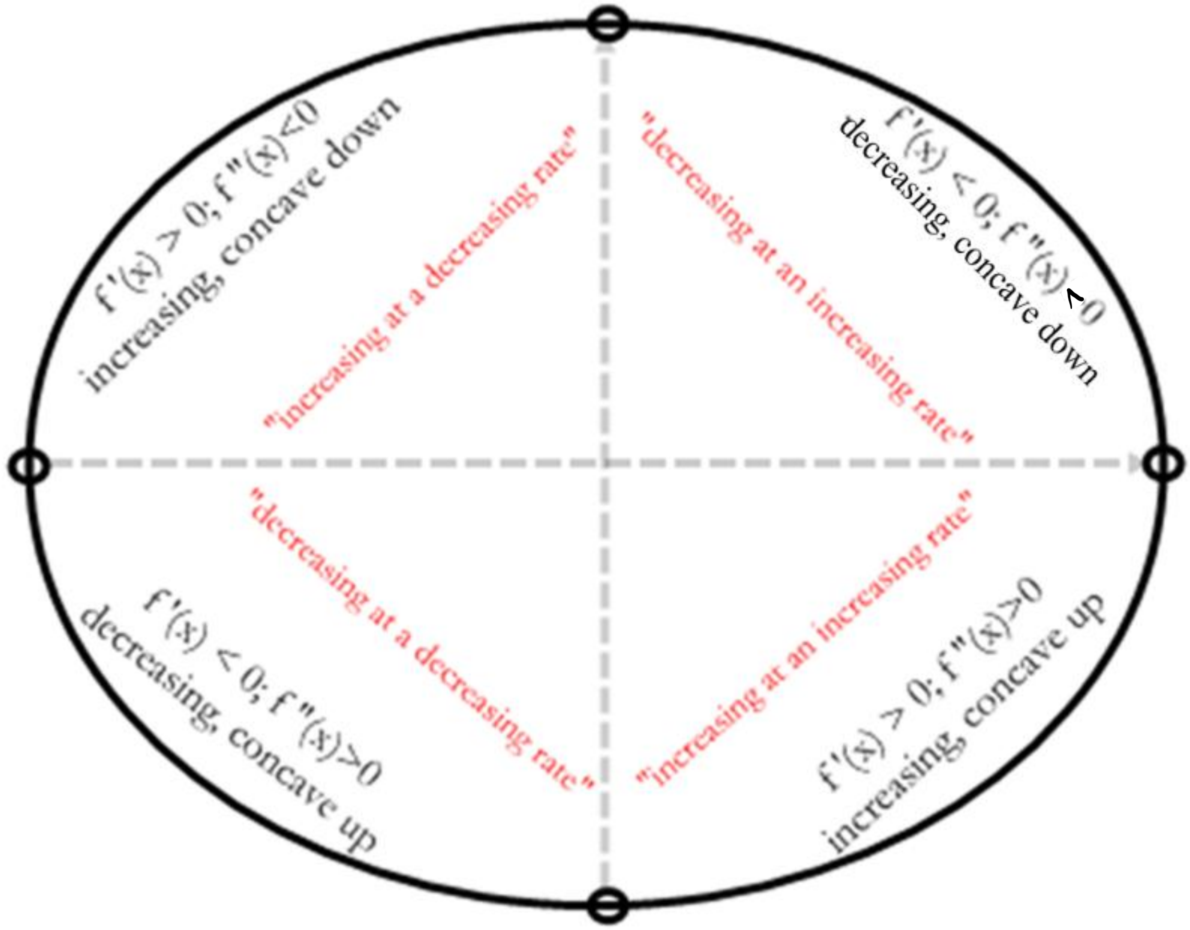
I. If $f'(c) = 0$ and $f''(c) > 0$, then $f(c)$ is a local minimum.
II. If $f'(c) = 0$ and $f''(c) < 0$, then $f(c)$ is a local maximum.
IV. If $f'(c) = f''(c) = 0$, then the second derivative test is inconclusive.

Inflection Point

$f(x)$ has an inflection point at $x = c$ if and only if it satisfies the following conditions:

I. $f(x)$ is continuous at $x = c$
II. $f''(x)$ changes sign at $x = c$ (concavity changes)
III. $f'(x)$ does not change sign at $x = c$.

Summary: What do f' and f'' say about f?



Sketching Guidelines: Discussing a Function.

- Find the domain, the $x-$intercepts and the $y-$intercepts. If possible, comment on range.
- Find asymptotes: vertical, horizontal, slant.
- Find any discontinuities
- Describe the end behavior: what happens to $y-$values as x approaches infinity or negative infinity?
- If possible, factor $f(x)$ completely, and then study its sign using a table.
- Find $f'(x)$, and then factor it completely to determine the critical numbers of $f(x)$..

- Study the sign of $f'(x)$ to determine the increasing/decreasing intervals for the original function $f(x)$.
- Find $f''(x)$, and then factor it completely to determine the critical numbers of $f'(x)$..
- Study the sign of $f''(x)$ to determine concavity for the original function $f(x)$.
- Combine all the information (increasing/decreasing, concavity, intercepts, sign of $f(x)$ etc. to draw a sketch of the original function; verify your sketch using a graphing device.