01) One of the legs of a right triangle has length 4 cm. Express the length of the altitude perpendicular to the hypotenuse as a function of the length of the hypotenuse.

02) The altitude perpendicular to the hypotenuse of a right triangle is 12 cm. Express the length of the hypotenuse as a function of the perimeter.

03) Draw the graph of the equation $x + |x| = y + |y|$.

04) Sketch the region in the plane consisting of all points $(x, y)$ such that $|x| + |y| \leqslant 1$.

05) Solve the inequality $\ln(x^2 - 2x - 2) \leqslant 0$.

06) Find numbers $a$ and $b$ such that $\lim_{x \to 0} \frac{\sqrt{ax + b} - 2}{x} = 1$.

07) Suppose $f$ is a function that satisfies the equation $f(x + y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers $x$ and $y$. Suppose also that

$$\lim_{x \to 0} \frac{f(x)}{x} = 1$$

(a) Find $f(0)$. (b) Find $f'(0)$. (c) Find $f'(x)$.

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| 08) | Evaluate $\lim_{x \to 1} \dfrac{\sqrt[3]{x} - 1}{\sqrt{x} - 1}$. |
| 09) | Find all values of $a$ such that $f$ is continuous on $\mathbb{R}$: $$f(x) = \begin{cases} x + 1 & \text{if } x \leqslant a \\ x^2 & \text{if } x > a \end{cases}$$ |
| 10) | If $\lim_{x \to a} [f(x) + g(x)] = 2$ and $\lim_{x \to a} [f(x) - g(x)] = 1$, find $\lim_{x \to a} f(x)g(x)$. |
| 11) | Find the point where the curves $y = x^3 - 3x + 4$ and $y = 3(x^2 - x)$ are tangent to each other, that is, have a common tangent line. Illustrate by sketching both curves and the common tangent. |
| 12) | For what value of $k$ does the equation $e^{2x} = k\sqrt{x}$ have exactly one solution? |
| 13) | Show that, for all positive values of $x$ and $y$, $$\frac{e^{x+y}}{xy} \geqslant e^2$$ |
| 14) | Let $a$ and $b$ be positive numbers. Show that not both of the numbers $a(1 - b)$ and $b(1 - a)$ can be greater than $\frac{1}{4}$. |

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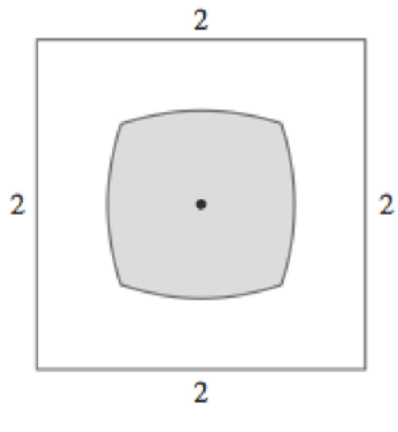
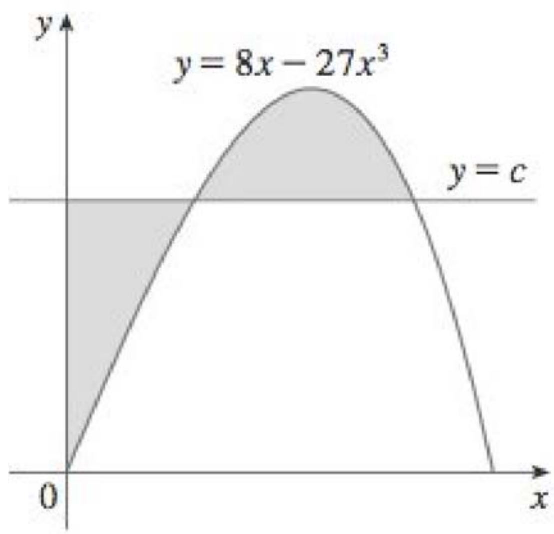
| 15) | Sketch the set of all points $(x, y)$ such that $\lvert x + y \rvert \leqslant e^x$. |
| 16) | Find a function $f$ such that $f'(-1) = \frac{1}{2}$, $f'(0) = 0$, and $f''(x) > 0$ for all $x$, or prove that such a function cannot exist. |
| 17) | For which positive numbers $a$ does the curve $y = a^x$ intersect the line $y = x$? |
| 18) | For what value of $a$ is the following equation true? $$\lim_{x \to \infty} \left( \frac{x + a}{x - a} \right)^x = e$$ |
| 19) | If $f(x) = \int_0^x x^2 \sin(t^2)\, dt$, find $f'(x)$. |
| 20) | Evaluate $\displaystyle \lim_{x \to 0} \frac{1}{x} \int_0^x (1 - \tan 2t)^{1/t}\, dt$. |
| 21) | A circular disk of radius $r$ is used in an evaporator and is rotated in a vertical plane. If it is to be partially submerged in the liquid so as to maximize the exposed wetted area of the disk, show that the center of the disk should be positioned at a height $r/\sqrt{1 + \pi^2}$ above the surface of the liquid. |

22) The figure shows a region consisting of all points inside a square that are closer to the center than to the sides of the square. Find the area of the region.




23) The figure shows a horizontal line $y = c$ intersecting the curve $y = 8x - 27x^3$. Find the number $c$ such that the areas of the shaded regions are equal.




24) A sphere of radius 1 overlaps a smaller sphere of radius $r$ in such a way that their intersection is a circle of radius $r$. (In other words, they intersect in a great circle of the small sphere.) Find $r$ so that the volume inside the small sphere and outside the large sphere is as large as possible.

25) The figure shows a curve $C$ with the property that, for every point $P$ on the middle curve $y = 2x^2$, the areas $A$ and $B$ are equal. Find an equation for $C$.