I.

1. $\dfrac{3x^2+10x+8}{6x^2+17x+10}=\dfrac{3x^2+6x+4x+8}{6x^2+12x+5x+10}=\dfrac{3x(x+2)+4(x+2)}{6x(x+2)+5(x+2)}=\dfrac{(3x+4)(x+2)}{(6x+5)(x+2)}=\dfrac{(3x+4)}{(6x+5)}$

2. $\dfrac{x^3-8}{x-2}=\dfrac{x^3-2^3}{x-2}=\dfrac{(x-2)(x^2+2x+4)}{x-2}=x^2+2x+4$

3. $\dfrac{5-x}{x^2-25}=\dfrac{5-x}{x^2-5^2}=\dfrac{5-x}{(x-5)(x+5)}=\dfrac{-(x-5)}{(x-5)(x+5)}=\dfrac{-1}{x+5}$

4. $\dfrac{2x^2+5x-12}{x^2-16}=\dfrac{2x^2+8x-3x-12}{x^2-16}=\dfrac{2x(x+4)-3(x+4)}{(x-4)(x+4)}=\dfrac{(2x-3)(x+4)}{(x-4)(x+4)}=\dfrac{2x-3}{x-4}$

II.

1. Pythagorean: $\cos^2 x+\sin^2 x=1 \quad 1+\tan^2 x=\sec^2 x \quad \cot^2 x+1=\csc^2 x$
2. Double Angles

$\cos 2x=\cos^2 x-\sin^2 x$

$\cos 2x=2\cos^2 x-1$

$\cos 2x=1-2\sin^2 x$

$\sin 2x=2\sin x\cos x$

$\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$

III.

1. $\dfrac{1}{x+h}-\dfrac{1}{x}=\dfrac{x-(x+h)}{x(x+h)}=\dfrac{-h}{x(x+h)}$

2. $\dfrac{\frac{2}{x^2}}{\frac{10}{x^5}}=\frac{2}{x^2}\cdot\frac{x^5}{10}=\frac{x^3}{5}$

3. $\dfrac{\frac{1}{3+x}-\frac{1}{3}}{x}=\dfrac{\frac{3-(3+x)}{(3+x)(3)}}{x}=\dfrac{-x}{(3+x)(3)}\cdot\dfrac{1}{x}=\dfrac{-1}{3(3+x)}$

4.

$$\frac{2x}{x^2-6x+9}-\frac{1}{x+1}-\frac{1}{x^2-2x-3}=$$

$$=\frac{2x}{(x-3)^2}-\frac{1}{x+1}-\frac{1}{(x-3)(x+1)}=\frac{2x(x+1)-(x-3)^2-(x-3)}{(x-3)^2(x+1)}=$$

$$=\frac{2x^2+2x-x^2+6x-9-x+3}{(x-3)^2(x+1)}=\frac{x^2+7x-6}{(x-3)^2(x+1)}$$

IV.

1.

$$4x+10yz=0$$
$$10yz=-4x$$
$$z=\frac{-4x}{10y}=\frac{-2x}{5y}$$

2.

$$y^2+3yz-8z-4x=0$$
$$3yz-8z=4x-y^2$$
$$z(3y-8)=4x-y^2$$
$$z=\frac{4x-y^2}{3y-8}$$

V.

1. $(f+h)(1)=f(1)+h(1)=7+6=13$

2. $(k-g)(5)=k(5)-g(5)=30-\sqrt{2}$

3. $(f\circ h)(3)=f(h(3))=f(2)=4$

4. $(g\circ k)(7)=g(k(7))=g(54)=\sqrt{51}$

5. $f^{-1}(x)=\{(5,3),(4,2),(7,1)\}$

6.

$k^{-1}(x) = ?$
$y = x^2 + 5$
$x = y^2 + 5$
$x - 5 = y^2$
$y = \pm\sqrt{x-5}$ (not a function!)

7. $\dfrac{1}{f(x)} = \{(3, \tfrac{1}{5}), (2, \tfrac{1}{4}), (1, \tfrac{1}{7})\}$

8. $(kg)(x) = k(x) \cdot g(x) = (x^2 + 5)\sqrt{x-3}$

VI.

1.

$$\frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^2 - 2(x+h)] - [x^2 - 2x]}{h} =$$
$$= \frac{[x^2 + 2xh + h^2 - 2x - 2h] - x^2 + 2x}{h} =$$
$$= \frac{2xh + h^2 - 2h}{h} = 2x + h - 2$$

2. $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

3. $x^{\frac{3}{2}}(x + x^{\frac{5}{2}} - x^2) = x^{\frac{3+2}{2}} + x^{\frac{3+5}{2}} - x^{\frac{3+4}{2}} = x^{\frac{5}{2}} + x^4 - x^{\frac{7}{2}}$ [this was a bit strange]

4. $\begin{cases} x = t^3 + 3 \\ y = 2t \end{cases} \Rightarrow \begin{cases} x = \left(\dfrac{y}{2}\right)^3 + 3 \\ \dfrac{1}{2}y = t \end{cases} \Rightarrow x = \dfrac{y^3}{8} + 3$

VII.

1. $\displaystyle\sum_{n=0}^{4} \frac{n^2}{2} = \frac{0^2}{2} + \frac{1^2}{2} + \frac{2^2}{2} + \frac{3^2}{2} + \frac{4^2}{2} = \frac{1}{2}(1 + 4 + 9 + 16) = 15$

2. $\sum_{n=1}^{3} \frac{1}{n!} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} = 1 + \frac{1}{2} + \frac{1}{6} = \frac{10}{6}$

VIII.

1. $\frac{\sqrt{x}}{x} = \frac{\sqrt{x}}{\sqrt{x}\sqrt{x}} = \frac{1}{\sqrt{x}}$
2. $e^{\ln 3} = 3$
3. $e^{1+\ln x} = e^{\ln x} e = xe$
4. $\ln 1 = 0$
5. $\ln e^7 = 7 \ln e = 7$
6. $\log_3 \frac{1}{3} = \log_3 3^{-1} = -1$
7. $\log_{\frac{1}{2}} 8 = \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-3} = -3$
8. $\ln\left(\frac{1}{2}\right) = \ln\left(2^{-1}\right) = -\ln 2$
9. $e^{3\ln x} = \left(e^{\ln x}\right)^3 = x^3$
10. $\frac{4xy^{-2}}{12x^{\frac{-1}{3}} y^{-5}} = \frac{x^{1-\left(\frac{-1}{3}\right)} y^{-2-(-5)}}{3} = \frac{x^{\frac{4}{3}} y^3}{3}$
11. $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 9$
12. $\left(5a^{\frac{2}{3}}\right)\left(4a^{\frac{2}{3}}\right) = 20a^{\frac{4}{3}}$
13. $\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}} = 4^{\frac{3}{2}} a^{\frac{5}{3}\cdot\frac{3}{2}} = 8a^{\frac{5}{2}}$
14. $\frac{3(n+1)!}{5n!} = \frac{3(n+1)\cdot n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot 2\cdot 1}{5n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot 2\cdot 1} = \frac{3(n+1)}{5}$