AP Calculus | Packet: More Pre-Requisites | Shubleka

Name SHUBLEKA / KEY

Topic 1: Fractional & Negative Exponents

Simplify using only positive exponents

1. $-3x^{-3} = \frac{-3}{x^3}$

2. $-5\left(\frac{3}{2}\right)(4-9x)^{\frac{-1}{2}}(-9) =$
$= 45 \cdot \frac{3}{2} \cdot \frac{1}{\sqrt{4-9x}}$
$= \frac{135}{2\sqrt{4-9x}}$

3. $2\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^2}\right] =$
$= \frac{-8}{(2-x)^3}$

4. $(16x^2y)^{\frac{3}{4}} =$
$= 16^{3/4} \cdot x^{6/4} \cdot y^{3/4}$
$= 8 \cdot x^{3/2} \cdot y^{3/4}$

5. $-\frac{x^{\frac{-1}{2}}}{2}\sin\sqrt{x} =$
$= -\frac{\sin\sqrt{x}}{2\sqrt{x}}$

6. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}} =$
$= \frac{2\sqrt{x-4}}{(x-4)^{3/4}} =$
$= \frac{2(x-4)^{1/2}}{(x-4)^{3/4}} =$
$= \frac{2}{\sqrt[4]{x-4}}$

7. $-4\left(\frac{2x-1}{2x+1}\right)^{-3}\left[\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right]$
$= \frac{-4(2x+1)^3}{(2x-1)^3} \cdot \frac{4x}{(2x+1)^2}$
$= \frac{-16x \cdot (2x+1)}{(2x-1)^3}$

8. $\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}} =$
$= \frac{1}{3\sqrt{(2x+5)^3}}$

9. $\left(\frac{1}{x^{-2}} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}}\right)^{\frac{-1}{2}} =$
$= (x^2+4xy+y^2)^{-1/2} =$
$= \frac{1}{\sqrt{x^2+4xy+y^2}}$

Topic 2: Domain

Find the domain of the following functions:

1. $y = \dfrac{3x-2}{4x+1}$

$4x+1 \neq 0$

$x \neq -1/4$

2. $y = \dfrac{x^2-4}{2x+4}$

$x \neq -2$

3. $y = \dfrac{x^2-5x-6}{x^2-3x-18}$

$x^2-3x-18$

$= (x-6)(x+3)$

$x \neq 6;\ x \neq -3.$

4. $y = \dfrac{2^{2-x}}{x}$

$x \neq 0$

5. $y = \sqrt{x-3} - \sqrt{x+3}$

$x \geq 3$ and $x \geq -3$

So: $x \geq 3.$

6. $y = \dfrac{\sqrt{2x-9}}{2x+9}$

$x \neq \frac{-9}{2}$ and

$x \geq \frac{9}{2}$

So: $x \geq \frac{9}{2}.$

7. $y = \dfrac{x^2+8x+12}{\sqrt[4]{x+5}}$

$x > -5.$

8. $y = \sqrt{x^2-5x-14}$

$x^2-5x-14 \geq 0$

$(x-7)(x+2) \geq 0$

$x \geq 7$ or $x \leq -2$

9. $y = \dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

$x^2-x-30 > 0$

$(x+5)(x-6) > 0$

$x < -5$ or $x > 6.$

10. $y = \log(2x-12)$

$2x-12 > 0$

$x > 6$

11. $y = \sqrt{\tan x}$

$\tan x \geq 0$

$\left[0, \frac{\pi}{2}\right)$

$\cup \left[\pi, \frac{3\pi}{2}\right)$

D: $\underbrace{\left[k\pi, \left(k+\frac{1}{2}\right)\pi\right)}_{\text{Union}}$

12. $y = \dfrac{x}{\cos x}$

$x \neq \frac{\pi}{2} + k\pi$

Topic 3: Solving inequalities (absolute value)

Write the following absolute value expressions as piecewise expressions

1. $y = |2x - 4|$

$$y = \begin{cases} 2x-4 & \text{if } 2x-4 \geq 0 \\ & x \geq 2 \\ 4-2x & \text{if } x < 2 \end{cases}$$

2. $y = |6 + 2x| + 1$

$$y = \begin{cases} 7+2x & \text{if } x \geq -3 \\ -5-2x & \text{if } x < -3 \end{cases}$$

3. $y = |4x + 1| + 2x - 3$

$$y = \begin{cases} 6x-2 & \text{if } x \geq -\frac{1}{4} \\ -2x-4 & \text{if } x < -\frac{1}{4}. \end{cases}$$

Solve the following absolute value inequalities

4. $|x - 3| > 12$

$x-3 > 12$ or $x-3 < -12$

$x > 15$ or $x < -9$

5. $|x - 3| \leq 4$

$-4 \leq x-3 \leq 4$

$-1 \leq x \leq 7$

6. $|10x + 8| > 2$

$10x+8 > 2$ or $10x+8 < -2$

7. $|3x - 4| > -2$

All reals.

8. $|x - 6| > -8$

All reals.

9. $|x + 1| \leq |x - 3|$

$x < -1$: Case I.

$-x-1 \leq 3-x$

$-1 \leq 3$ ✓ $(-\infty, -1)$

$-1 \leq x < 3$: Case II.

$x+1 \leq 3-x$

$2x \leq 2$

$x \leq 1$

Solution

$[-1, 1]$

$3 \leq x$: Case 3

$x+1 \leq x-3$

$1 \leq -3$

ϕ no solution.

Final Answer:

$(-\infty, 1]$

Topic 4: Solving inequalities (quadratic)

Write the following absolute value expressions as piecewise expressions

1. $|x^2-1|$

$$|x^2-1| = \begin{cases} x^2-1 & \text{if } x \geq 1 \text{ or } x \leq -1 \\ 1-x^2 & \text{if } -1 < x < 1 \end{cases}$$

2. $|x^2+x-12|$

$$= |(x-3)(x+4)|$$

$$= \begin{cases} x^2+x-12 & \text{if } x \geq 3 \text{ or } x \leq -4 \\ 12-x-x^2 & \text{if } -4 < x < 3 \end{cases}$$

3. $|x^2+4x+4| = |(x+2)^2|$

$$= x^2+4x+4$$

for all x.

Solve the following by factoring and making appropriate sign charts.

4. $x^2-16>0$

$(x-4)(x+4) > 0$

x-4: ⊖ ⊖ ⊕

x+4: ⊖ -4 ⊕ 4 ⊕

⊕ ⊖ ⊕

✓ ✓

$x<-4$ or $x>4$

5. $x^2+6x-16>0$

$(x+8)(x-2) > 0$

$x<-8$ or $x>2$

(see #4)

6. $x^2-3x \geq 10$

$(x-5)(x+2) \geq 0$

$x \leq -2$ or $x \geq 5$

(see #4)

7. $2x^2+4x \leq 3$

$2x^2+4x-3 \leq 0$

$$x = \frac{-4 \pm \sqrt{40}}{4}$$

$$\frac{-2-\sqrt{10}}{2} \leq x \leq \frac{-2+\sqrt{10}}{2}$$

Sign Chart?

8. $x^3+4x^2-x \geq 4$

$x^2(x+4)-(x+4) \geq 0$

$(x+4)(x^2-1) \geq 0$

$(x+4)(x-1)(x+1) \geq 0$

⊖ -4 ⊕ -1 ⊖ 1 ⊕

$-4 \leq x \leq -1$ or $x \geq 1$

9. $2\sin^2 x \geq \sin x \quad 0 \leq x < 2\pi$

$(2\sin x - 1)\cdot \sin x \geq 0$

$\frac{\pi}{6}, \frac{5\pi}{6} \quad 0, \pi, 2\pi$

$\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ or $[\pi, 2\pi]$

sinx: ⊕ ⊕ ⊕ ⊖

$(2\sin x - 1)$: 0 ⊖ $\pi/6$ ⊕ $\frac{5\pi}{6}$ ⊖ π ⊖ 2π

− + − +

Topic 5: Special factorization

Factor completely

1. x^3+8

$(x+2)(x^2-2x+4)$

2. $x^3-8=$

$(x-2)(x^2+2x+4)$

3. $27x^3-125y^3=$

$(3x)^3-(5y)^3=$

$(3x-5y)\left[9x^2+15xy+25y^2\right]$

4. $x^4+11x^2-80=$

$u^2+11u-80=$

$(u-5)(u+16)=$

$(x^2-5)(x^2+16)=$

$(x-\sqrt{5})(x+\sqrt{5})(x^2+16$

5. $ac+cd-ab-bd=$

$(a+d)(c-b)$

6. $2x^2+50y^2-20xy$

$=2\left[x^2-2\cdot x\cdot 5y+(5y)^2\right]$

$=2\ (x-5y)^2$

7. $x^2+12x+36-9y^2$

$=x\ (x+12)+9\,(4-y^2)$

$=x\,(x+12)+9\,(2-y)(2+y)$

?

8. $x^3-xy^2+x^2y-y^3=$

$(x+y)(x^2-y^2)$

$=(x+y)^2(x-y)$

9. $(x-3)^2(2x+1)^3+(x-3)^3(2x+1)^2$

$(x-3)^2(2x+1)^2\cdot\left[2x+1+x-3\right]$

$=(x-3)^2(2x+1)^2\,(3x-2)$

Topic 6: Function transformation

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:

1. $f(x) - 4$

vertical shift by 4 units, down.

2. $f(x-4)$

horizontal shift, to the right 4 units.

3. $-f(x+2)$

horizontal shift to the left 2 units, and a reflection about $y=0$.

4. $5f(x)+3$

vertical stretch by a factor of 5; vertical shift by +3 units.

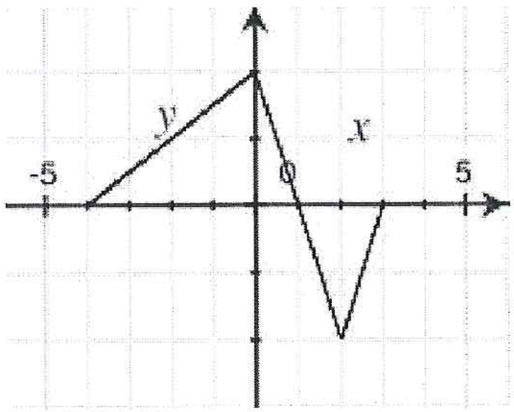
5. $f(2x)$

horizontal shrinking by a factor of $1/2$

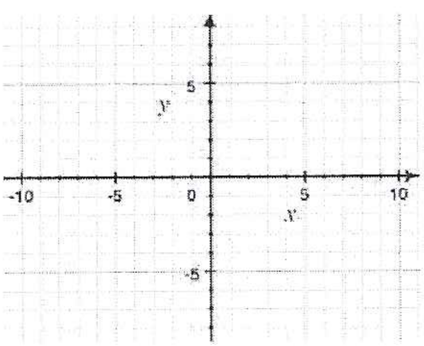
6. $|f(x)|$

Reflection about x-axis of all graph parts in QIII or QIV.

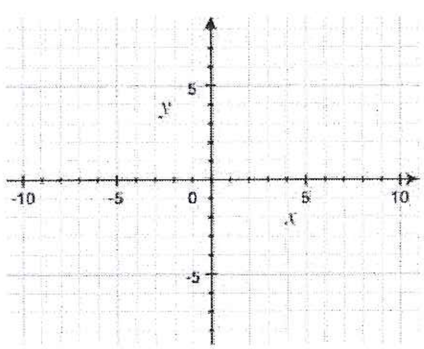
Here is a graph of $y = f(x)$. Sketch the following graphs



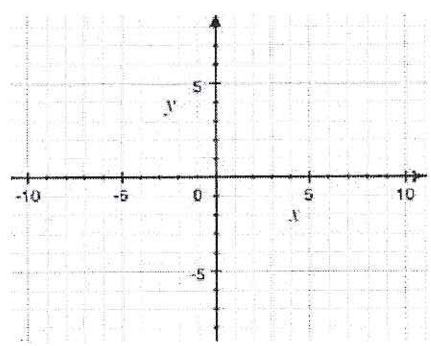
7. $y = 2f(x)$



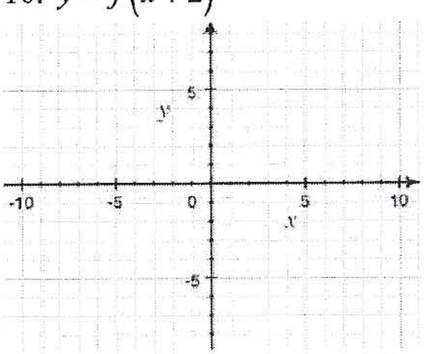
8. $y = -f(x)$



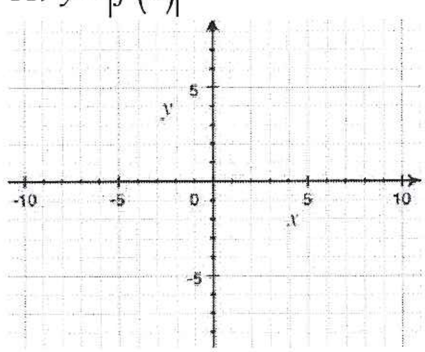
9. $y = f(x-1)$



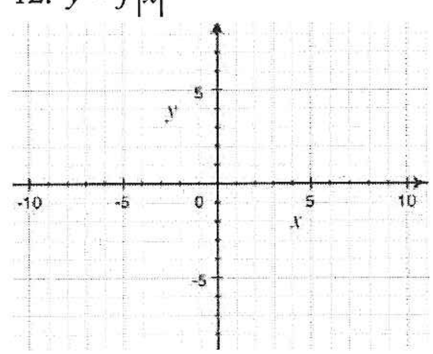
10. $y = f(x+2)$



11. $y = |f(x)|$



12. $y = f|x|$



Key only.

Topic 7: Factor theorem (*p* over *q* method/synthetic division)

Use the p over q method and synthetic division to factor the polynomial $P(x)$. Then solve $P(x) = 0$.

1. $P(x) = x^3 + 4x^2 + x - 6 =$

$= (x-1)(x+2)(x+3)$

2. $P(x) = x^3 + 5x^2 - 2x - 24 =$

$= (x-2)(x+3)(x+4)$

3. $P(x) = x^3 - 6x^2 + 3x - 10$

??

4. $P(x) = x^3 + 2x^2 - 19x - 20 =$

$= (x-4)(x+1)(x+5)$

5. $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8 =$

$= (x+2)^3(x-1)$

6. $P(x) = x^4 + 11x^3 + 41x^2 + 61x + 30 =$

$= (x+1)(x+2)(x+3)(x+5)$

EVEN: $f(x) = f(-x)$ for all x

Topic 8: Even and odd functions

ODD: $f(-x) = -f(x)$ for all x.

Show work to determine if the relation is even, odd, or neither

1. $f(x) = 2x^2 - 7$

even.

Why?

2. $f(x) = -4x^3 - 2x$

odd

Why?

3. $f(x) = 4x^2 - 4x + 4$

neither.

4. $f(x) = x - \frac{1}{x}$

odd.

Why?

5. $f(x) = |x| - x^2 + 1$

even.

6. $5x^2 - 6y = 1$

even

Why?

7. $y = e^x - \frac{1}{e^x}$

neither?

Well....

$$f(-x) = e^{-x} - \frac{1}{e^{-x}}$$

$$= \frac{1}{e^x} - e^x$$

$$= -\left[e^x - \frac{1}{e^x}\right] = -f(x)$$

Actually, odd.

8. $3y^3 = 4x^3 + 1$

neither

9. $3x = |y|$

Neither.

Topic 9: Solving quadratic equations and quadratic formula

Solve each equation

1. $7x^2 - 3x = 0$

$x(7x-3)=0$

$x=0$ or $x=\frac{3}{7}$.

2. $4x(x-2)-5x(x-1)=2$

$4x^2-8x-5x^2+5x-2=0$

$-x^2-3x-2=0$

$x^2+3x+2=0$

$(x+2)(x+1)=0$

$x=-2$; $x=-1$.

3. $x^2+6x+4=0$

$x=\frac{-6\pm\sqrt{20}}{2}$

$x=-3\pm\sqrt{5}$

4. $2x^2-3x+3=0$

$b^2-4ac=$

$9-24<0$

No solution.

5. $2x^2-(x+2)(x-3)=12$

$2x^2-x^2+6+x-12=0$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3$; $x=2$.

6. $x+\frac{1}{x}=\frac{13}{6}$ $\quad x\neq 0$

$\frac{x^2+1}{x}=\frac{13}{6}$

$6x^2+6=13x$

$6x^2-13x+6=0$

$6x^2-4x-9x+6=0$

$2x(3x-2)-3(3x-2)=0$

$(2x-3)(3x-2)=0$

$x=3/2$; $x=\frac{2}{3}$

7. $x^4-9x^2+8=0$

$u^2-9u+8=0$

with $u=x^2$

$(u-8)(u-1)=0$

$x^2=8$ $\quad$ $x^2=1$

$x=\pm 2\sqrt{2}$ $\quad$ $x=\pm 1$.

8. $x-10\sqrt{x}+9=0$

$u=\sqrt{x}$ $\quad x\geq 0$

$u^2-10u+9=0$

$(u-9)(u-1)=0$

$\sqrt{x}=9$ $\quad$ $\sqrt{x}=1$

$x=81$ $\quad$ $u=1$.

9. $\frac{1}{x^2}-\frac{1}{x}=6$

$\frac{1-x-6x^2}{x^2}=0$

$6x^2+x-1=0$

$x=\frac{-1\pm 5}{12}=-1/2,\ 1/3.$

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Topic 10: Asymptotes

KEY/SOLUTIONS (Please report any mistakes.)

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptotes (if they exist)

1. $y = \dfrac{x}{x-3}$

VA: $x=3$

HA: $y=1$

2. $y = \dfrac{x+4}{x^2-1}$

VA: $x=1$; $x=-1$

HA: $y=0$

3. $y = \dfrac{x+4}{x^2+1}$

VA: none ($x^2+1 \neq 0$ for all x)

HA: $y=0$

4. $y = \dfrac{x^2-2x+1}{x^2-3x-4}$

$y = \dfrac{(x-1)^2}{(x-4)(x+1)}$

VA: $x=4$; $x=-1$

HA: $y=1$

5. $y = \dfrac{x^2-9}{x^3+3x^2-18x}$

$y = \dfrac{(x-3)(x+3)}{x(x^2+3x-18)}$

$y = \dfrac{(x-3)(x+3)}{x(x+6)(x-3)}$

VA: $x=0$; $x=-6$

hole @ $x=3$

HA: $y=0$

6. $y = \dfrac{2x^2+6x}{x^3-3x^2-4x}$

$y = \dfrac{2x(x+3)}{x(x^2-3x-4)}$

$y = \dfrac{2x(x+3)}{x(x-4)(x+1)}$

VA: $x=-1$; $x=4$

hole @ $x=0$

HA: $y=0$

7. $y = \dfrac{x^2-x-6}{x^3-x^2+x-6}$

$y = \dfrac{(x-3)(x+2)}{(x-2)(x^2+x+3)}$

Use Factor Theorem with a guess of $x=2$ as a root.

VA: $x=2$ only

HA: $y=0$

8. $y = \dfrac{2x^3}{x^3-1}$

$y = \dfrac{2x^3}{(x-1)(x^2+x+1)}$

VA: $x=1$

HA: $y=2$

9. $y = \dfrac{\sqrt{x}}{2x^2-10}$

$y = \dfrac{\sqrt{x}}{2(x^2-5)}$

$y = \dfrac{\sqrt{x}}{2(x-\sqrt{5})(x+\sqrt{5})}$

VA: $x=-\sqrt{5}$; $x=\sqrt{5}$

HA: $y=0$

Topic 11: Complex fractions

Simplify the following

1. $\dfrac{x}{x-\frac{1}{2}} = \dfrac{x}{\frac{2x-1}{2}} = x \cdot \dfrac{2}{2x-1} = \dfrac{2x}{2x-1}$

2. $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-2} = \dfrac{\frac{1+4x}{x}}{\frac{1-2x}{x}} = \dfrac{1+4x}{x} \cdot \dfrac{x}{1-2x} = \dfrac{1+4x}{1-2x}$

3. $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}} = \dfrac{\frac{x^2-1}{x}}{\frac{x^2+1}{x}} = \dfrac{x^2-1}{x} \cdot \dfrac{x}{x^2+1} = \dfrac{x^2-1}{x^2+1}$

4. $\dfrac{\frac{3}{x}-\frac{4}{y}}{\frac{4}{x}-\frac{3}{y}} = \dfrac{\frac{3y-4x}{xy}}{\frac{4y-3x}{xy}} = \dfrac{3y-4x}{xy} \cdot \dfrac{xy}{4y-3x} = \dfrac{3y-4x}{4y-3x}$

5. $\dfrac{1-\frac{2}{3x}}{x-\frac{4}{9x}} = \dfrac{\frac{3x-2}{3x}}{\frac{9x^2-4}{9x}} = \dfrac{3x-2}{3x} \cdot \dfrac{9x}{9x^2-4} = \dfrac{9x-6}{9x^2-4}$

6. $\dfrac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}} = \dfrac{(x-y)(x+y)}{xy} \cdot \dfrac{y}{x+y} = \dfrac{x-y}{x}$ (as long as $x+y \neq 0$, $y \neq 0$)

7. $\dfrac{x^{-3}-x}{x^{-2}-1} = \dfrac{x^{-3}\left[1-x^4\right]}{x^{-2}\left[1-x^3\right]} = \dfrac{(1-x^4)}{x\,(1-x^3)}$

Can we simplify further?

8. $\dfrac{\frac{x}{1-x}+\frac{1+x}{x}}{\frac{1-x}{x}+\frac{x}{1+x}} = \dfrac{\frac{x^2+(1+x)(1-x)}{(1-x)(x)}}{\frac{(1+x)(1-x)+x^2}{x(1+x)}} = \dfrac{x^2+1-x^2}{x(1-x)} \cdot \dfrac{x(1+x)}{1-x^2+x^2} = \dfrac{1+x}{1-x}$

(as long as $x \neq 0$).

9. $\dfrac{\frac{4}{x-5}+\frac{2}{x+2}}{\frac{2x}{x^2-3x-10}+3}$; numerator $= \dfrac{4x+8+2x-10}{(x-5)(x+2)}$, denominator $= \dfrac{2x+3x^2-9x-30}{x^2-3x-10}$

$= \dfrac{6x-2}{(x-5)(x+2)} \cdot \dfrac{(x-5)(x+2)}{3x^2-7x-30} = \dfrac{2(3x-1)}{3x^2-7x-30}$.

Topic 12: Composition of functions

If $f(x) = x^2$, $g(x) = 2x - 1$, and $h(x) = 2^x$, find the following

1. $f(g(2))$

$= f(3)$

$= 9.$

2.. $f(g(2))$

$= f(3)$

$= 9$

3. $f(h(-1))$

$= f(1/2)$

$= 1/4$.

4. $h(f(-1))$

$= h(1)$

$= 2$

5. $g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$

$= g(f(\sqrt{2}))$

$= g(2)$

$= 3$

6. $f(g(x))$

$= (2x-1)^2$

7. $g(f(x))$

$= 2 \cdot x^2 - 1$

8. $g(g(x))$

$= 2(2x-1) - 1$

$= 4x - 3$

9. $f(h(x))$

$= (2^x)^2$

$= 2^{2x}$

Topic 13: Solving Rational (fractional) equations

Solve each equation for x

1. $\frac{2}{3}-\frac{5}{6}=\frac{1}{x}$

$\frac{12-15}{18}=\frac{1}{x}$

$\frac{-3}{18}=\frac{1}{x}$

$\frac{-6}{1}=\frac{x}{1}$

$\boxed{x=-6}$

Check?

2. $x+\frac{6}{x}=5$

$\frac{x^2+6}{x}=5$

$x^2+6=5x$

$x^2-5x+6=0$

$(x-2)(x-3)=0$

$\boxed{x=2 \text{ or } x=3}$

Check?

3. $\frac{x+1}{3}-\frac{x-1}{2}=1$

$6\left[\frac{x+1}{3}-\frac{x-1}{2}\right]=1\cdot 6$

$2x+2-3x-3=6$

$-x-1=6$

$-x=7$

$\boxed{x=-7}$

Check?

4. $\frac{x-5}{x+1}=\frac{3}{5}$

$5(x-5)=3(x+1)$

$5x-25=3x+3$

$2x=28$

$\boxed{x=14}$

Check?

5. $\frac{60}{x}-\frac{60}{x-5}=\frac{2}{x}$

$x(x-5)\left[\frac{60}{x}-\frac{60}{x-5}\right]=\frac{2}{x}(x-5)(x)$

$\cancel{60x}-300-\cancel{60x}=2x-10$

$-290=2x$

$\boxed{-145=x}$

6. $\frac{2}{x+5}+\frac{1}{x-5}=\frac{16}{x^2-25}$

Multiply both sides by x^2-25.

$2(x-5)+(x+5)=16$

$3x-5=16$

$3x=21$

$\boxed{x=7}$

7. $\frac{x}{x-2}+\frac{2x}{4-x^2}=\frac{5}{x+2}$

$\frac{x}{x-2}-\frac{2x}{x^2-4}=\frac{5}{x+2}$

$x(x+2)-2x=5(x-2)$

$x^2+2x-2x=5x-10$

$x^2-5x+10=0$

$x=?$

$b^2-4ac<0$

No solution.

8. $\frac{x}{2x-6}-\frac{3}{x^2-6x+9}=\frac{x-2}{3x-9}$

$\frac{x}{2(x-3)}-\frac{3}{(x-3)^2}=\frac{x-2}{3(x-3)}$

Multiply by $6(x-3)^2$ both sides.

$3x(x-3)-18=2(x-2)(x-3)$

$3x^2-9x-18=2x^2-10x+12$

$x^2+x-30=0$

$(x+6)(x-5)=0$

$x=5$ or $x=-6$

Check?

9. $\frac{2x+3}{x-1}=\frac{10}{x^2-1}+\frac{2x-3}{x+1}$

Multiply by x^2-1.

$(2x+3)(x+1)=10+(2x-3)(x-1)$

$2x^2+5x+3=2x^2-5x+13$

$10x=10$

$\boxed{x=1}$

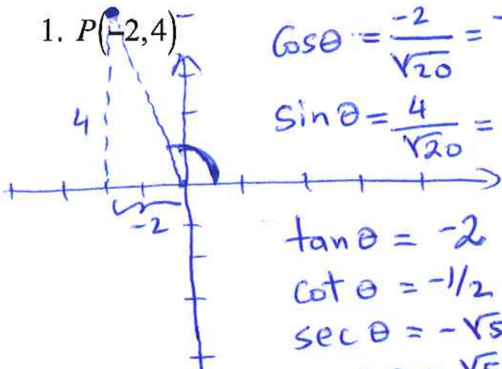
Extraneous Root, so no solution.

Topic 14: Solving Rational (fractional) equations

Solve the following problems.

If point P is on the terminal side of θ, find all 6 trig functions of θ. Draw a picture.

1. $P(-2,4)$



$\cos\theta = \frac{-2}{\sqrt{20}} = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$

$\sin\theta = \frac{4}{\sqrt{20}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

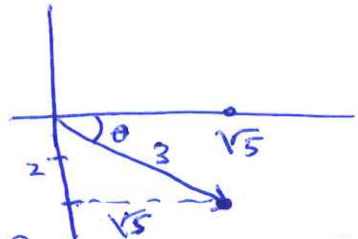
$\tan\theta = -2$

$\cot\theta = -1/2$

$\sec\theta = -\sqrt{5}$

$\csc\theta = \sqrt{5}/2$

2. $P(\sqrt{5},-2)$



$\cos\theta = \frac{\sqrt{5}}{3}$

$\sin\theta = \frac{-2}{3}$

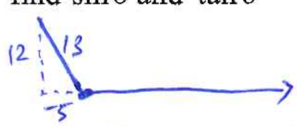
$\tan\theta = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$

$\cot\theta = -\frac{\sqrt{5}}{2}$

$\sec\theta = \frac{3\sqrt{5}}{5}$

$\csc\theta = \frac{-3}{2}$.

3. If $\cos\theta = \frac{-5}{13}, \theta$ in quadrant II,
   find $\sin\theta$ and $\tan\theta$

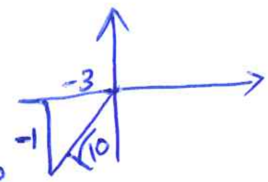


$\cos\theta = \frac{-5}{13}$ $\quad \sec\theta = \frac{-13}{5}$

$\sin\theta = \frac{12}{13}$ $\quad \csc\theta = \frac{13}{12}$

$\tan\theta = \frac{-12}{5}$ $\quad \cot\theta = -\frac{5}{12}$

4. If $\cot\theta = 3, \theta$ in quadrant III,
   find $\sin\theta$ and $\cos\theta$



$\cot\theta = \frac{3}{1}$ $\quad \tan\theta = \frac{1}{3}$

$\cos\theta = \frac{-3\sqrt{10}}{10}$ $\quad \sec\theta = \frac{-\sqrt{10}}{3}$

$\sin\theta = -\frac{\sqrt{10}}{10}$ $\quad \csc\theta = -\sqrt{10}$

Find the exact value of the following without calculators:

5. $\sin^2 225° - \cos^2 300° =$

$= \left(\frac{-\sqrt{2}}{2}\right)^2 - \left(\frac{+1}{2}\right)^2$

$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

6. $\left(6\sec 180° - 4\cot 90°\right)^2$ $\quad = 0$

$\left(6 \cdot \frac{1}{\cos 180°} - \frac{4}{\tan 90°}\right)^2$ well...

$= \left[6 \cdot (-1) - 4 \cdot 0\right]^2$

$= 36.$

7. $\left(4\cos 30° - 6\sin 120°\right)^{-2}$

$\left(4 \cdot \frac{\sqrt{3}}{2} - 6 \cdot \frac{\sqrt{3}}{2}\right)^{-2}$

$= \left(\frac{-2\sqrt{3}}{2}\right)^{-2} = \left(-\sqrt{3}\right)^{-2}$

$= \left(-\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$

Solve the following triangles (3 decimal place accuracy)

8. $A =$ $\quad a = 21.7$
   $B = 16°$ $\quad b =$
   $C = 90°$ $\quad c =$

9. $A =$ $\quad a = 6$ feet
   $B =$ $\quad b =$
   $C = 90°$ $\quad c = 95$ inches

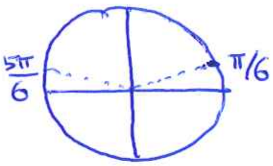
Review Law of Sines / Law of Cosines

Topic 15: Solving Trigonometric equations

Solve each equation on the interval $[0,2\pi)$

1. $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}\ ;\ \frac{5\pi}{6}$



2. $\cos^2 x = \cos x$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0$ or $\cos x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $x = 0$

3. $2\cos x + \sqrt{3} = 0$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}.$

4. $4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

5. $2\sin^2 x + \sin x = 1$

$2\sin^2 x + \sin x - 1 = 0$

$u = \sin x$

$2u^2 + u - 1 = 0$

$(2u-1)(u+1) = 0$

$u = \frac{1}{2}$ or $u = -1$

$\sin x = \frac{1}{2}$ $\sin x = -1$

$x = \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

6. $\cos^2 x + 2\cos x = 3$

$\cos^2 x + 2\cos x - 3 = 0$

$u = \cos x$

$u^2 + 2u - 3 = 0$

$(u+3)(u-1) = 0$

$u = -3$ or $u = 1$

$\cos x = -3$ $\cos x = 1$

impossible $x = 0.$

7. $2\sin x\cos x + \sin x = 0$

$\sin x\,[2\cos x + 1] = 0$

$\sin x = 0$ $2\cos x + 1 = 0$

$x = 0, \pi$ $\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

8. $8\cos^2 x - 2\cos x = 1$

$8u^2 - 2u - 1 = 0$

with $u = \cos x$

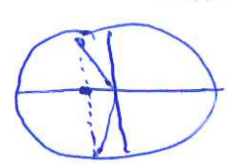
$(4u+1)(2u-1) = 0$

$u = \frac{-1}{4}$ $u = \frac{1}{2}$

$\cos x = \frac{-1}{4}$ $\cos x = \frac{1}{2}$

$x = \arccos\left(\frac{-1}{4}\right)$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$

2 angles



9. $\sin^2 x - \cos^2 x = 0$

$(\sin x - \cos x)(\sin x + \cos x) = 0$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$