AP CALCULUS AB STUDY SHEET

Key Definitions

limit – this is what distinguishes Calculus from other math. A limit of a function is the value that the dependent variable approaches as the independent variable approaches a given value. A limit has the general form $\lim_{x \to h} f(x)$ This gives the value that the function is tending toward as *x* approaches *h*.

derivative – this describes the slope of the graph, or the rate of change of the function. The derivative may be notated in many ways, including y', $\frac{dy}{dx}$, and $\frac{d}{dx}[y]$. Each of these examples given refers to "THE DERIVATIVE OF *Y* WITH RESPECT TO *X*." To use function notation, the derivative of $f(x)$ is $f'(x)$. The limit definition of derivative is $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

differential – a special variable used in taking derivatives and solving ***differential equations***. A differential equation is solved by ***integration***.

implicit differentiation – useful when there are multiple variables in the expression. When the derivative is taken, simply multiply by the corresponding ***differential***.

integral – this describes the area under the curve. The integral is the inverse of the derivative, and thus is also called the ***anti-derivative***. Notation is one of two ways: $\int f(x)\,dx$ or $F(x)$ In the latter case, it cannot be assumed that *F* always means the anti-derivative of *f*. The above examples are ***indefinite integrals***, meaning that they give a ***general solution***. When evaluating a ***definite integral***, the result is the area under the curve. Definite integrals have the form $\int_a^b f(x)\,dx$; this example will yield the area under $f(x)$ from *a* to *b*.

critical numbers – these are the values of the independent variable at which the ***first derivative*** is zero: $f'(x)=0$. They also occur where the first derivative is undefined, such as at a ***cusp*** or sharp corner (like absolute value). These numbers are useful when looking for changes in the trend of the graph: either ***increasing*** or ***decreasing***. Critical numbers also indicate ***relative extrema***, either ***relative maxima*** or ***relative minima***.

absolute extrema – absolute maxima and minima must be found by using CRITICAL NUMBERS ***and*** the ENDPOINTS OF THE INTERVAL. Whichever value is furthest to the desired extreme is given as the absolute extrema.

inflection – inflection is the change of the ***concavity*** of the graph. A graph is either concave up or concave down (a horizontal line would have both or none) Points of inflection occur where the ***second derivative*** is zero. Evaluation of the second derivative at any value yields the concavity;
$f''(x)>0 \Rightarrow$ concave up ; $f''(x)<0 \Rightarrow$ concave down

mean value theorem (for derivatives) – this theorem declares that there must be at least one point on a given interval for which the ***instantaneous slope*** (derivative) equals the average rate of change over that interval. The function must be continuous on the closed interval, and differentiable on the open interval. Symbolically, this is expressed $f'(c)=\frac{f(b)-f(a)}{b-a}$ where (a,b) is the open interval.

Rolle's theorem – this is a simpler version, and an application of the ***mean value theorem***. It adds the condition that, on the interval (a,b), $f(a)=f(b)$. This means that the average slope is zero, and there must be a point *c* at which $f'(c)=0$

position, velocity, and acceleration – that's the order,
$a(t)=v'(t)=s''(t)$ or $s(t)=\int v(t)\,dt=\int\left[\int a(t)\,dt\right]dt$

DERIVATIVES

*For all of the following, **x** is considered to be the independent variable, **n** is a real number, **c** is a nonzero real number, **a** is a positive integer, and **u** and **v** are differentiable functions.*

Simple Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$

Chain Rule: $\frac{d}{dx}[f(u)] = f'(u)u'$

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$ ***Product Rule***
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$ ***Quotient Rule***
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
22. $\frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1+u^2}$
23. $\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{|u|\sqrt{u^2-1}}$
24. $\frac{d}{dx}[\text{arccsc}\, u] = \frac{-u'}{|u|\sqrt{u^2-1}}$
25. $\frac{d}{dx}[\arcsin \frac{x}{a}] = \frac{1}{\sqrt{a^2-x^2}}$
26. $\frac{d}{dx}[\arccos \frac{x}{a}] = \frac{-1}{\sqrt{a^2-x^2}}$
27. $\frac{d}{dx}[\arctan \frac{x}{a}] = \frac{a}{a^2+x^2}$
28. $\frac{d}{dx}[\text{arccot} \frac{x}{a}] = \frac{-a}{a^2+x^2}$
29. $\frac{d}{dx}[\text{arcsec} \frac{x}{a}] = \frac{a}{|x|\sqrt{x^2-a^2}}$
30. $\frac{d}{dx}[\text{arccsc} \frac{x}{a}] = \frac{-a}{|x|\sqrt{x^2-a^2}}$

INTEGRALS

*For all of the following, **k**, **C**, and **n** are real numbers; **u** is a differentiable function.*

1. $\int k\,f(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$ ***General rule***
4. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C,\ n \neq -1$ ***General rule***
5. $\int \dfrac{du}{u} = \ln|u| + C$
6. $\int e^u\,du = e^u + C$
7. $\int a^u\,du = \left(\dfrac{1}{\ln a}\right)a^u + C$
8. $\int \sin u\,du = -\cos u + C$
9. $\int \cos u\,du = \sin u + C$
10. $\int \tan u\,du = -\ln|\cos u| + C$
11. $\int \cot u\,du = \ln|\sin u| + C$
12. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
13. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
14. $\int \sec^2 u\,du = \tan u + C$
15. $\int \csc^2 u\,du = -\cot u + C$
16. $\int \sec u \tan u\,du = \sec u + C$
17. $\int \csc u \cot u\,du = -\csc u + C$
18. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \arcsin \dfrac{u}{a} + C$
19. $\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a} \arctan \dfrac{u}{a} + C$
20. $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a} \operatorname{arcsec} \dfrac{|u|}{a} + C$

Fundamental Theorem of Calculus – this multi-part theorem is very important!

Part 1: $\int_a^b f(x)\,dx = F(b) - F(a)$ that is to say, that once the anti-derivative of the function is found (usually using the simple power rule for integrals), the integral on the closed interval $[a, b]$ is equal to the difference in the anti-derivatives at each endpoint.

Part 2: $\dfrac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$ or $F'(x) = f(x)$ This states that the derivative of the anti-derivative is the original function. Be careful applying this, though. The example given takes the integral over $[a, x]$, but if x is a differentiable function, you must use the ***chain rule*** eventually.

The Mean Value Theorem for Integrals – if f is continuous on the closed interval [a,b] then there exists a number c in the closed interval [a,b] such that $\int_a^b f(x)\,dx = f(c)(b-a)$ This means that there exists a rectangle such that its area is equal to the area of the region under the curve.

average value – the ***mean height*** of a function on the open interval (a, b) is $\dfrac{1}{b-a}\int_a^b f(u)\,du$

area between two curves – the basic concept here is that the area between two curves is equal to the total area from the upper curve to the zero line, minus the area from the lower curve to the zero line. For all real numbers a and b as the endpoints of the interval, for an upper function *f(x)* and lower function *g(x)*, the area between them is $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$ To use this formula, *f(x)* must be greater than *g(x)* over the entire interval. Likewise, everything could be done in terms of y: For all real numbers a and b as the endpoints of the interval, for an upper function *f(y)* and lower function *g(y)*, the area between them is $\int_a^b [f(y) - g(y)]\,dy = \int_a^b f(y)\,dy - \int_a^b g(y)\,dy$ To use this formula, *f(x)* must be greater than *g(x)* over the entire interval (This means *f(x)* is farther to the right!).

Volumes of Solids of Revolution

Representative Rectangles – Remember the Reimann sums that explained the integral concept? This is the same thing. When we say representative rectangles in solids of revolution, we mean a little strip of the region that will be revolved around the axis of revolution to become either a disk, washer, or shell.

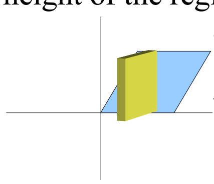
Axis of Revolution – This is a line about which a two-dimensional region is revolved to become a three-dimensional solid.

The Disk Method – This method may be used when representative rectangles are perpendicular to the axis of rotation, and when the region to be revolved is flush with the axis of revolution over the entire interval. The formula is based on the area formula for a circle, $A=\pi r^2$. Instead of a simple quantity *r*, we have an entire function represented as *R(x)*.There are two versions of the formula, one dealing with horizontal axes of revolution and one with vertical. $V=\pi\int_a^b [R(x)]^2 dx$ or $\pi\int_a^b [R(y)]^2 dy$

The Washer Method – The washer method is identical to the Disk Method, except for one condition: the region to be revolved need not be flush with the axis of revolution; in fact, it might not touch it at all! The representative rectangles still need to be perpendicular to the axis of revolution. The formula is $V=\pi\int_a^b \left([R(x)]^2-[r(x)]^2\right)dx$ or $\pi\int_a^b \left([R(y)]^2-[r(y)]^2\right)dy$ *R(x)* must be greater than *r(x)* over the entire interval, and likewise for *y*.

The Shell Method – This one is different. The region need not be flush with the axis of revolution, and the representative rectangles must be parallel to the axis of revolution. The volume formula is an application of the circumference of a circle, $C=2\pi r$. In the shell method, *p(x)* is the distance of the rectangle from the axis of revolution, and *h(x)* is the height of the rectangle. When writing the integral, make sure that if everything is in terms of *x*, and there is a *dx* at the end, that the limits of integration are also *x*-values! Everything may be applied to functions in terms of *y*. $V=2\pi\int_a^b p(x)h(x)dx$ or $2\pi\int_a^b p(y)h(y)dy$

Volumes of Known Cross-Sections – This is a strange concept. Basically, you want to make a region "pop out" of the page by placing a certain shape on it. For example, take the region shown below. Place a vertical SQUARE on each x-value. The size of the square changes at each location, and is dependent on the vertical height of the region. This dependent value is known as the base. The base is used to calculate the area of the "pop-out" for each *x* from *a* to *b*. Always use vertical rectangles. $V=\int_a^b A(x)dx$



You'll need some area formulas to do all of this right! (Notice that most have a b^2 term)

Cross-Section	***Area Formula***
Square	$A(x)=b^2$
Equilateral Triangle	$A(x)=\frac{\sqrt{3}}{4}b^2$
Isosceles Right Triangle (Hint: the right angle is opposite the base)	$A(x)=\frac{1}{4}b^2$
Semi-circle	$A(x)=\frac{1}{8}\pi b^2$
Rectangle (of given height *h*)	$A(x)=bh$