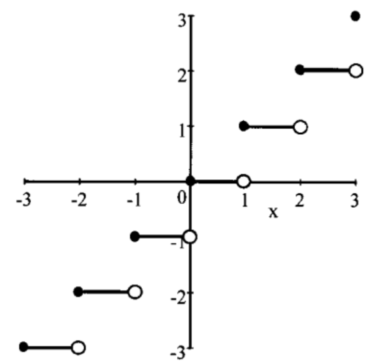
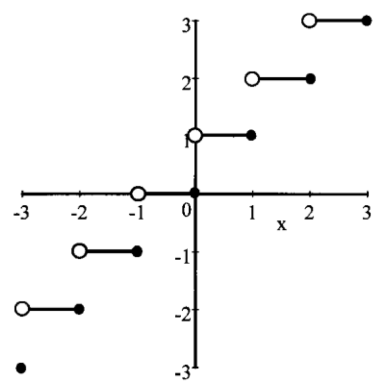
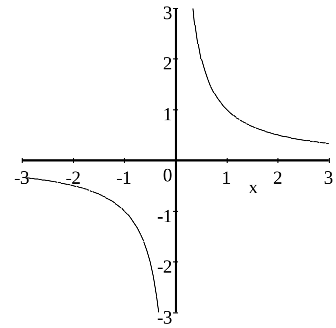
A.P. Calculus Formulas

1.	floor function (def)	Greatest integer that is less than or equal to x.
2.	$\lfloor x \rfloor$ (graph)	
3.	ceiling function (def)	Least integer that is greater than or equal to x.
4.	$\lceil x \rceil$ (graph)	
5.	$a^3+b^3=$	$(a+b)(a^2-ab+b^2)$
6.	$a^3-b^3=$	$(a-b)(a^2+ab+b^2)$

7. $f(x) = \dfrac{1}{x}$ (graph)



8. Change of base rule for logs: $\log_a x = \dfrac{\ln x}{\ln a}$

9. Circle formula: $(x-h)^2 + (y-k)^2 = r^2$

10. Parabola formula: $(x-h)^2 = 4p(y-k)$

11. Ellipse formula: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \qquad c = \sqrt{a^2 - b^2}$

12. Hyperbola formula: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1 \qquad c = \sqrt{a^2 + b^2}$

13. eccentricity: $e = \dfrac{c}{a}$

14. $\sin^2 x + \cos^2 x =$ 1

15. $1 + \tan^2 x =$ $\sec^2 x$

16. $1 + \cot^2 x =$ $\csc^2 x$

17. $\sin(u \pm v) =$ $\sin u \cdot \cos v \pm \cos u \cdot \sin v$

18. $\cos(u \pm v) =$ $\cos u \cdot \cos v \mp \sin u \cdot \sin v$

19. $\tan(u \pm v) =$ $\dfrac{\tan u \pm \tan v}{1 \mp \tan u \cdot \tan v}$

20. $\sin(2u) =$ $2\sin u \cdot \cos u$

21.	$\cos(2u) =$	$\cos^2 u - \sin^2 u$
22.	$\tan(2u) =$	$\dfrac{2\tan u}{1-\tan^2 u}$
23.	$\sin^2 u =$	$\dfrac{1-\cos 2u}{2}$
24.	$\cos^2 u =$	$\dfrac{1+\cos 2u}{2}$
25.	$\tan^2 u =$	$\dfrac{1-\cos 2u}{1+\cos 2u}$
26.	$\sin u \cdot \sin v =$	$\frac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$
27.	$\cos u \cdot \cos v =$	$\frac{1}{2}\left[\cos(u-v)+\cos(u+v)\right]$
28.	$\sin u \cdot \cos v =$	$\frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$
29.	$\cos u \cdot \sin v =$	$\frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$
30.	law of sines:	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
31.	law of cosines:	$c^2 = a^2 + b^2 - 2ab\cos C$
32.	area of triangle using trig.	Area $= \frac{1}{2} ac \sin B$
33.	parameterization of ellipse:	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ becomes $x = a\cos t$, $y = b\sin t$
34.	$\lim\limits_{x\to 0} \dfrac{\sin x}{x}$	1
35.	$\lim\limits_{x\to \infty} \dfrac{\sin x}{x}$	0

36.	Intermediate Value Theorem	If a function is continuous between a and b, then it takes on every value between $f(a)$ and $f(b)$.
37.	definition of derivative	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
38.	$\frac{d}{dx}(c)=$	0
39.	$\frac{d}{dx}(x)=$	1
40.	$\frac{d}{dx}(cu)=$	cu'
41.	$\frac{d}{dx}u^n=$	$nu^{n-1}u'$
42.	$\frac{d}{dx}(u\pm v)=$	$u'\pm v'$
43.	$\frac{d}{dx}(uv)=$	$uv'+vu'$
44.	$\frac{d}{dx}\left(\frac{u}{v}\right)=$	$\frac{vu'-uv'}{v^2}$
45.	$\frac{d}{dx}\sin u=$	$\cos u\cdot u'$
46.	$\frac{d}{dx}\cos u=$	$-\sin u\cdot u'$
47.	$\frac{d}{dx}\tan u=$	$\sec^2 u\cdot u'$
48.	$\frac{d}{dx}\cot u=$	$-\csc^2 u\cdot u'$
49.	$\frac{d}{dx}\sec u=$	$\sec u\cdot\tan u\cdot u'$
50.	$\frac{d}{dx}\csc u=$	$-\csc u\cdot\cot u\cdot u'$

51. slope of parametrized curve: $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

52. derivative formula for inverses $\left.\dfrac{df^{-1}}{dx}\right|_{x=f(a)}=\dfrac{1}{\left.\frac{df}{dx}\right|_{x=a}}$

53. $\dfrac{d}{dx}\sin^{-1}u=$ $\dfrac{u'}{\sqrt{1-u^2}}$

54. $\dfrac{d}{dx}\cos^{-1}u=$ $\dfrac{-u'}{\sqrt{1-u^2}}$

55. $\dfrac{d}{dx}\tan^{-1}u=$ $\dfrac{u'}{1+u^2}$

56. $\dfrac{d}{dx}\cot^{-1}u=$ $\dfrac{-u'}{1+u^2}$

57. $\dfrac{d}{dx}\sec^{-1}u=$ $\dfrac{u'}{|u|\sqrt{u^2-1}}$ $|u|>1$

58. $\dfrac{d}{dx}\csc^{-1}u=$ $\dfrac{-u'}{|u|\sqrt{u^2-1}}$ $|u|>1$

59. $\cot^{-1}(x)=$ $\dfrac{\pi}{2}-\tan^{-1}x$

60. $\sec^{-1}(x)=$ $\cos^{-1}\left(\dfrac{1}{x}\right)$

61. $\csc^{-1}(x)=$ $\sin^{-1}\left(\dfrac{1}{x}\right)$

62. $\dfrac{d}{dx}e^u=$ $e^u u'$

63. $\dfrac{d}{dx}\ln u=$ $\dfrac{1}{u}u'$

64.	$\frac{d}{dx}a^u =$	$a^u \ln a \cdot u'$
65.	Extreme Value Theorem	If f is continuous over a closed interval, then f has a maximum and minimum value over that interval.
66.	Mean Value Theorem (for derivatives)	If $f(x)$ is a differentiable function over $[a,b]$, then at some point between a and b: $\frac{f(b)-f(a)}{b-a} = f'(c)$
67.	linearization formula	$L(x) = f(a) + f'(a)\cdot(x-a)$
68.	Newton's Method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
69.	$\int k \cdot f\ \ u\ \ du =$	$k\int f\ \ u\ \ du$
70.	$\int [f(u) \pm g(u)]du =$	$\int f(u)du \pm \int g(u)du$
71.	Mean Value Theorem (for definite integrals)	If f is continuous on a,b , then at some point c in a,b , $f\ \ c\ = \frac{1}{b-a}\int_a^b f\ \ x\ \ dx$
72.	First fundamental theorem:	$\frac{d}{dx}\int_a^u f(t)dt = f(u)\cdot u'$
73.	Trapezoidal Rule:	$T = \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n\right)$
74.	Simpson's Rule:	$S = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + ... + 2y_{n-2} + 4y_{n-1} + y_n\right)$
75.	$\int du =$	$u + c$
76.	$\int u^n du =$	$\frac{u^{n+1}}{n+1} + c \qquad n \neq -1$

77.	$\int \sin u \ du =$	$-\cos u + c$
78.	$\int \cos u \ du =$	$\sin u + c$
79.	$\int \sec^2 u \ du =$	$\tan u + c$
80.	$\int \csc^2 u \ du$	$-\cot u + c$
81.	$\int \sec u \cdot \tan u \ du =$	$\sec u + c$
82.	$\int \csc u \cdot \cot u \ du =$	$-\csc u + c$
83.	$\int \frac{1}{u} du =$	$\ln\lvert u\rvert + c$
84.	$\int e^u du =$	$e^u + c$
85.	$\int a^u du =$	$\frac{1}{\ln a} a^u + c$
86.	$\int \tan u \ du =$	$-\ln\lvert\cos u\rvert + c$
87.	$\int \cot u \ du =$	$\ln\lvert\sin u\rvert + c$
88.	$\int \sec u \ du =$	$\ln\lvert\sec u + \tan u\rvert + c$
89.	$\int \csc u \ du =$	$-\ln\lvert\csc u + \cot u\rvert + c$
90.	$\int \frac{du}{\sqrt{a^2 - u^2}} =$	$\arcsin \frac{u}{a} + c$
91.	$\int \frac{du}{a^2 + u^2} =$	$\frac{1}{a} \arctan \frac{u}{a} + c$
92.	$\int \frac{du}{u\sqrt{u^2 - a^2}} =$	$\frac{1}{a} \operatorname{arcsec} \frac{\lvert u\rvert}{a} + c$
93.	Integration by parts (formula):	$\int u dv = uv - \int v du$

94.	order for choosing u in integration by parts:	LIPET $\Rightarrow$ logs, inverse trig., polynomial, exponential, trig.
95.	exponential change:	$y = y_0 e^{kt}$
96.	half-life	$\frac{\ln 2}{k}$
97.	continuous compound interest:	$A(t) = A_o e^{rt}$
98.	logistics differential equation:	$\frac{dP}{dt} = kP \;\; M - P$
99.	logistics growth model	$P = \frac{M}{1 + Ae^{-\,Mk\;\; t}}$
100.	surface area about x axis (Cartesian):	$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$
101.	length of curve (Cartesian):	$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

FORMULAS BELOW HERE ARE BC ONLY:

102.	$\lim_{n \to \infty} \frac{\ln n}{n} =$	0
103.	$\lim_{n \to \infty} \sqrt[n]{n} =$	1
104.	$\lim_{n \to \infty} x^{\frac{1}{n}} =$	1
105.	$\lim_{n \to \infty} x^n =$	0 $\left(\lvert x \rvert < 1\right)$
106.	$\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n =$	e^x

107. $\lim_{n\to\infty} \frac{x^n}{n!} =$ 0

108. $\sum_{k=1}^{n} k =$ $\frac{n(n+1)}{2}$

109. $\sum_{k=1}^{n} k^2 =$ $\frac{n(n+1)(2n+1)}{6}$

110. $\sum_{k=1}^{n} k^3 =$ $\left(\frac{n(n+1)}{2}\right)^2$

111. partial sum of geometric series: $S_n = \frac{a\left(1-r^n\right)}{1-r}$

112. What series? $\sum_{n=1}^{\infty} ar^{n-1}$ geometric, converges to $\frac{a}{1-r}$ if $|r|<1$

113. Maclaurin Series: $P(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots$

114. Taylor Series: $P(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$

115. Maclaurin Series for $\frac{1}{1-x}$ $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$

116. Maclaurin Series for $\frac{1}{1+x}$ $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots$

117. Maclaurin Series for e^x $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

118. Maclaurin Series for $\sin x$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

119. Maclaurin Series for $\cos x$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

120. Maclaurin Series for $\ln(1+x)$ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

121. Maclaurin Series for $\tan^{-1} x$: $\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$

122. Lagrange form of remainder $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$

123. Taylor's Inequality $\left|R_n(x)\right| \le \frac{M}{(n+1)!}\left|x-a\right|^{n+1}$

124. What series? $\sum_{n=0}^{\infty} \frac{1}{n!}$ reciprocal of factorials, converges to e

125. What series? $\sum_{n=1}^{\infty} (b_n - b_{n+1})$ telescoping series, converges to $b_1 - \lim_{n\to\infty} b_{n+1}$

126. What series? $\sum_{n=1}^{\infty} \frac{1}{n^p}$ p series, converges if $p > 1$

127. What series? $\sum_{n=1}^{\infty} \frac{1}{n}$ harmonic, diverges

128. What series? $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ alternating harmonic, converges

129. 2nd deriv. of parametrized curve: $\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}}$

130. length of curve (parametric): $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

131. surface area (parametric): $S = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

132.	position vector (standard form):	$r\ \ t\ = f\ \ t\ \mathbf{i}+g\ \ t\ \mathbf{j}+h\ \ t\ \mathbf{k}$
133.	speed from velocity vector:	speed = $\lvert v(t)\rvert$
134.	direction from velocity vector:	$\text{direction} = \dfrac{\text{velocity vector}}{\text{speed}} = \dfrac{v(t)}{\lvert v(t)\rvert}$
135.	polar to Cartesian:	$x = r\cos\theta,\ \ y = r\sin\theta$
136.	trajectory equations:	$x = x_o +\ \ v_o\cos\alpha\ \ t$ $y = y_o +\ \ v_o\sin\alpha\ \ t - \dfrac{1}{2}gt^2$
137.	slope of polar graph:	slope at $(r,\theta) = \dfrac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$
138.	slope of polar graph at origin:	slope = $\tan\theta$
139.	area inside polar curve:	$A = \int_\alpha^\beta \dfrac{1}{2}r^2 d\theta$
140.	length of curve (polar):	$L = \int_\alpha^\beta \sqrt{r^2 + \left(\dfrac{dr}{d\theta}\right)^2}\, d\theta$
141.	surface area (polar):	$S = \int_\alpha^\beta 2\pi r\sin\theta \sqrt{r^2 + \left(\dfrac{dr}{d\theta}\right)^2}\, d\theta$