

1) Find

$$\int_1^4 \frac{2}{\sqrt{x}} dx$$

- a) 15
  - b) 2
  - c) 4
  - d) 8
  - e) 16
- 2) If

$$f'(x) = -3(x-6)^2(x-9)$$

which of the following is true about  $y = f(x)$ ?

- a)  $f$  has a point of inflection at  $x = 6$  and a local maximum at  $x = 9$ .
  - b)  $f$  has a local maximum at  $x = 6$  and a local minimum at  $x = 9$ .
  - c)  $f$  has a local minimum at  $x = 6$  and a local maximum at  $x = 9$ .
  - d)  $f$  has a point of inflection at  $x = 6$  and a local minimum at  $x = 9$ .
  - e)  $f$  has a local minimum at  $x = 6$  and a point of inflection at  $x = 9$ .
- 3) A curve is described by parametric equations

$$[x = 5 \ln(t), y = t^2 - 8]$$

where  $t > 0$ . Give an expression for

$$\frac{\partial^2}{\partial x^2} y$$

- a)  $\frac{4}{5} t$
- b)  $\frac{4}{25} t^2$
- c)  $\frac{4}{25} t$
- d)  $\frac{2}{5} t^2$
- e)  $\frac{2}{5} t$

4) Give the value for

$$\lim_{x \rightarrow 0} \left( \frac{1}{8} \frac{6^x - 1}{x} \right)$$

- a)  $\frac{1}{8} \ln(6)$
- b) 0
- c) 1
- d)  $8 \ln(6)$
- e) *The limit does not exist.*

5) Which of the following series converge?

I.

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n!}$$

II.

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$$

III.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

- a) II only  
 b) I, II and III  
 c) II and III only  
 d) I and II only  
 e) I only  
 6) If  $g(f(x)) = x$ ,  $g(7) = 2$  and  $g'(7) = 12$ , then  $f'(2)$  is

- a)  $\frac{1}{12}$   
 b)  $-\frac{1}{12}$   
 c)  $\frac{1}{7}$   
 d)  $-\frac{1}{7}$   
 e)  $\frac{7}{12}$

7) If  $f$  is a differentiable function and  $f(0) = -3$  and  $f(2) = 6$ , then which of the following must be true?

- I. There exists a  $c$  in  $[0,2]$  where  $f(c) = 0$ .  
 II. There exists a  $c$  in  $[0,2]$  where  $f'(c) = 0$ .  
 III. There exists a  $c$  in  $[0,2]$  where  $f'(c) = 9/2$ .

- a) II only  
 b) I only  
 c) I and III only  
 d) II and III only  
 e) I, II and III  
 8) Evaluate

$$\int_{-1}^1 \frac{6}{x^2} dx$$

- a) 8  
 b) 0  
 c) 4

d) 2

e) *The integral diverges.*

9) Find the area enclosed by the graphs of

$$\begin{aligned}y &= e^x + 1 \\y &= 7\end{aligned}$$

and the y-axis.

a)  $7 \ln(6) + 6$

b)  $7 \ln(7) - 7$

c)  $7 \ln(7) + 7$

d)  $7 \ln(6) - 6$

e)  $6 \ln(6) - 5$

10) What is the minimum value of the function

$$f(x) = \frac{3}{\sqrt{x}} + 4\sqrt{x}$$

a)  $2\sqrt{3}$

b)  $4\sqrt{3}$

c)  $\frac{3}{4}$

d)  $\frac{1}{2}\sqrt{3}$

e)  $\sqrt{3}$

11) Give the value of

$$\int_{\pi}^{2\pi} \frac{\cos(4x)}{2 + \sin(4x)} dx$$

a) 0

b) 1

c)  $\frac{1}{4}$

d)  $\frac{1}{8}$

e)  $-\frac{1}{4}$

12) The side of a cube is expanding at a constant rate of 4 inches per second. What is the rate of change of the surface area, in  $\text{in}^2$  per second, when the volume of the cube is  $64 \text{ in}^3$ ?

a) 24

b) 96

c) 240

d) 192

e) 48

13) Give the area inside one petal of the polar graph of

$$r = 6 \sin(2\theta)$$

- a)  $9\pi$
- b)  $\frac{9}{2}\pi$
- c)  $18\pi$
- d)  $\frac{9}{4}\pi$
- e)  $3$

14) Give the solution to the initial value problem

$$[y' = 18x^2y, y(1) = 1]$$

- a)  $\frac{1}{6}\ln(x^3) + 1$
- b)  $e^{6x^3} - 6$
- c)  $\frac{1}{6}\ln(x^3) + e$
- d)  $e^{6x^3} - 6$
- e)  $e^{6x^3}$

15) The position of a particle moving along a horizontal line is given by

$$x(t) = 4(t-3)^3$$

What is the maximum speed of the particle for  $0 \leq t \leq 10$ ?

- a) 324
- b) 196
- c) 108
- d) 36
- e) 588

16)  $\int [\sec(6x)]^2 dx =$

- a)  $6[\tan(6x)]^2 + C$
- b)  $6\tan(6x) + C$
- c)  $-6\tan(6x) + C$
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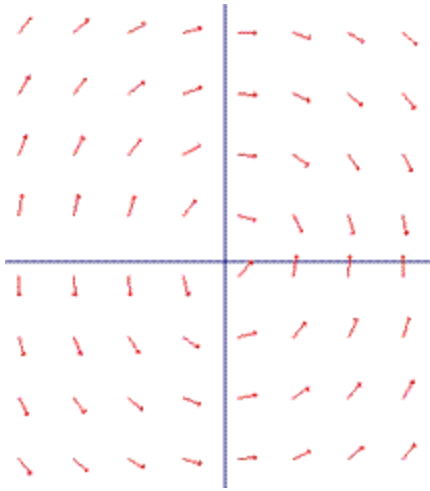
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$$f(x) = xe^{-7x}$$

for  $x > 0$ . Give the interval on which the function is increasing.

- a)  $(0, 7)$
- b)  $\left(1, \frac{1}{7}e\right)$
- c)  $\left(0, \frac{1}{7}e\right)$
- d)  $\left(0, \frac{1}{7}\right)$
- e)  $(1, 7)$

18)



Which of the following differential equations correspond to the slope field shown in the figure above?

- a)  $\frac{dy}{dx} = 2xy$
- b)  $\frac{dy}{dx} = -\frac{y}{x}$
- c)  $\frac{dy}{dx} = \frac{x}{y}$
- d)  $\frac{dy}{dx} = -\frac{x}{y}$
- e)  $\frac{dy}{dx} = \frac{y}{x}$

19) Evaluate

$$\lim_{h \rightarrow 0} \left( \frac{\cos(3x + 3h) - \cos(3x)}{h} \right)$$

- a)  $-3 \cos(3x)$
- b)  $3 \sin(3x)$
- c)  $3 \cos(3x)$
- d) *The limit does not exist.*
- e)  $-3 \sin(3x)$

20) If  $\int_0^{36} e^x dx = m$  then  $\int_0^6 x e^{x^2} dx$  is

- a)  $2m$
- b)  $m$
- c)  $\frac{1}{2}m$
- d)  $m^2$
- e)  $\frac{1}{2}m^2$

21) Find the area of the region enclosed by the graph of

$$y = 3x^2$$

and the line

$$y = 4x$$

- a)  $\frac{64}{27}$
- b)  $\frac{32}{27}$
- c)  $\frac{8}{27}$
- d)  $\frac{9}{32}$
- e)  $\frac{9}{16}$

22) Suppose

$$z = e^y$$

$$y = 3x^3 - 3$$

and

$$x = 1 + \ln(t^2)$$

What is  $dz/dt$  when  $t = 1$ ?

- a) 12
- b) 9
- c)  $\frac{9}{2}$
- d) 18
- e) 3

23) Evaluate

$$\int_0^{\frac{1}{2}\pi} \cos^2(11x) \sin(11x) dx$$

- a)  $-\frac{1}{66}$
- b)  $\frac{1}{66}$
- c)  $-\frac{1}{33}$
- d)  $\frac{1}{33}$
- e)  $\frac{1}{11}$

24) Give an equation for the tangent line to the parametric curve

$$[x = e^t, y = t^2 + 6t]$$

at  $t = 0$ .

- a)  $y - 1 = 6x - 6$
- b)  $y = 6e(x - 1)$
- c)  $y - 1 = 6x$
- d)  $y = 6x - 6$
- e)  $y = \frac{6(x - 1)}{e}$

25) Evaluate

$$\frac{\partial}{\partial x} \int_5^{7x} \ln(5t) dt$$

- a)  $7 \ln(35x) - 14 \ln(5)$
- b)  $\frac{7}{5x}$
- c)  $5 \ln(35x)$
- d)  $7 \ln(7x) - 14 \ln(5)$
- e)  $7 \ln(35x)$

26) The region bounded by

$$y = 5 \sin(x)$$

and the  $x$ -axis, for  $0 \leq x \leq \frac{1}{2} \pi$ , is rotated about the line  $y = -2$ . The volume of this solid can be represented by:

- a)  $2\pi \int_0^{\frac{1}{2}\pi} (25 \sin(x)^2 + 2) dx$
- b)  $\pi \int_0^{\frac{1}{2}\pi} ((5 \sin(x) + 2)^2 - 4) dx$
- c)  $\pi \int_0^{\frac{1}{2}\pi} (25 \sin(x)^2 - 4) dx$
- d)  $2\pi \int_0^{\frac{1}{2}\pi} 25 \sin(x + 2)^2 dx$
- e)  $2\pi \int_0^{\frac{1}{2}\pi} (5 \sin(x) + 2)^2 dx$

27) Give the third degree Taylor polynomial about  $x = 1$  of

$$f(x) = \ln(x)$$

- a)  $(x-1) + \frac{1}{3}(x-1)^2 + \frac{1}{3}(x-1)^3$
- b)  $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3$
- c)  $(x-1) - \frac{1}{3}(x-1)^2 + \frac{1}{5}(x-1)^3$
- d)  $(x-1) + (x-1)^2 + 2(x-1)^3$
- e)  $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

28) Which of the following integrals gives the length of the graph of

$$f(x) = e^{3x}$$

for  $x$  between 0 and 2?

a)  $\int_0^2 \sqrt{1 + 9e^{6x}} \, dx$

b)  $\int_0^2 \sqrt{1 + e^{6x}} \, dx$

c)  $\int_0^2 \sqrt{x + 9e^{6x}} \, dx$

d)  $\int_0^2 \sqrt{x + e^{6x}} \, dx$

e)  $\int_0^2 \sqrt{e^{3x} + 9e^{6x}} \, dx$



29) Find the average value of the function

$$f(x) = e^{5x}$$

over the interval  $[0, 2]$ .

- a)  $\frac{1}{10} e^{10}$
- b)  $\frac{1}{5} (e^{10} - 1)$
- c)  $\frac{1}{10} (e^{10} - 1)$
- d)  $\frac{1}{2} (e^{10} - 1)$
- e)  $\frac{1}{5} e^{10}$

30) What is the y-intercept of the line tangent to the curve  $y = x^2 + 6$  at  $x = 3$ ?

- a)  $(0, -3)$
- b)  $(0, 0)$
- c)  $(0, -12)$
- d)  $(0, 12)$
- e)  $(0, 3)$

31) Which of the following function(s) is continuous and differentiable?

I.  $f(x) = \frac{8}{\sqrt{x}}$

II.  $g(x) = x|x|$

III.  $h(x) = \begin{cases} 5x + 1 & x \leq 0 \\ x^2 + 1 & 0 < x \end{cases}$

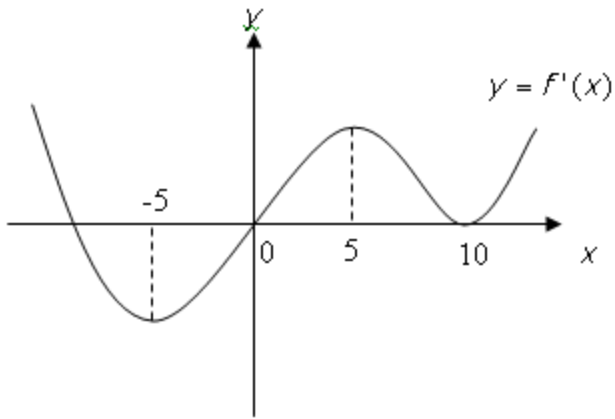
- a) II only
- b) I only
- c) III only
- d) I and II only
- e) I and III only

32) Find  $m$

$$\lim_{x \rightarrow 0} \left( \frac{e^{mx^2} - \cos(12x)}{x^2} \right) = 144$$

- a) 72
- b) 288
- c) 6
- d) 36
- e) 3

33) The graph of the derivative of  $f$  is shown below. Which of the following must be true?



- a)  $f$  is increasing on  $[-5, 5]$ .  
 b)  $f$  has a point of inflection at  $x = 10$ .  
 c)  $f$  has a local maximum at  $x = 0$ .  
 d)  $f$  is concave down on  $[0, 10]$ .  
 e)  $f$  has a local minimum at  $x = -5$ .
- 34) The sum of two positive integers  $x$  and  $y$  is 120. Find the value of  $x$  that minimizes

$$P = x^3 - 120xy$$

- a)  $x = 40$   
 b)  $x = 60$   
 c)  $x = 20$   
 d)  $x = 100$   
 e)  $x = 80$

35) A particle moves on the curve

$$[x = 5 \sin(t), y = \sin(2t)]$$

find the speed of the particle at time  $t = \pi$ .

- a) 5.1962  
 b) 4.5826  
 c) 5.3852  
 d) 3.0000  
 e) 2.6458

36) The function  $f$  is defined as

$$f(x) = \frac{(x-4)^2}{x-6}$$

$$x \neq 6$$

Which of the following is **false**?

- a)  $f$  has a local maximum at  $x = 4$ .  
 b)  $f$  has a vertical asymptote at  $x = 6$ .  
 c)  $f$  is decreasing on  $[4, 6]$ .  
 d)  $f$  is concave up for  $x > 6$ .  
 e)  $f$  has a horizontal asymptote at  $y = 1$ .
- 37) A particle is moving along the  $x$ -axis and its position at time  $t \geq 0$  is given by

$$S(t) = (t-3)^2(t-5)$$

Which of the following is (are) true?

- I. The particle changes direction at  $x = 3$  and  $x = 5$ .

II. The particle is slowing down on  $[0, 3]$ .

III. The particle is speeding up on  $[3, 5]$ .

- a) I and III only
- b) I only
- c) II and III only
- d) I, II and III
- e) II only

38)  $f(x)$  is a differentiable function and it is decreasing on  $(-\infty, \infty)$ .

If

$$g(x) = f(10x^3 - x^2)$$

then  $g$  has a local maximum at

- a)  $x = 0$
- b)  $x = \frac{1}{15}$
- c)  $x = 1$
- d)  $x = 5$
- e)  $x = \frac{1}{5}$

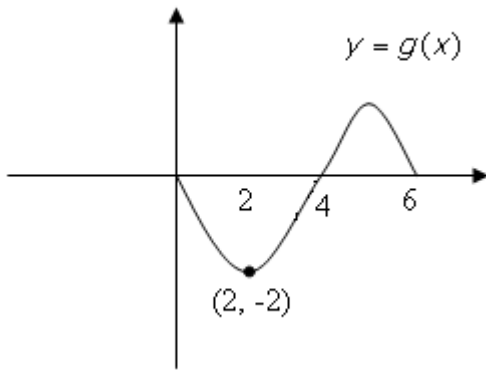
39) The rate at which a bacteria population grows is proportional to the number of bacteria present. Initially, there were 1000 bacteria present and the population doubled in 2 hours. Roughly how many hours does it take for the population to reach 10000?

- a) 6.6
- b) 8.2
- c) 10.4
- d) 4.4
- e) 5.8

40) Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{n(3x-11)^n}{2^n}$$

- a)  $\frac{11}{2}$
- b)  $\frac{2}{11}$
- c)  $\frac{11}{3}$
- d)  $\frac{2}{3}$
- e)  $\frac{3}{2}$



41)

$$g(x) = \int_0^x f(t) dt$$

for  $0 \leq x \leq 6$

The graph of  $g$  is shown above. Which of the following must be true?

- I.  $\int_0^6 f(t) dt = 0$
- II.  $\int_2^4 f(t) dt = 2$
- III.  $\int_4^0 f(t) dt = -2$

- a) I only
- b) I and II only
- c) II only
- d) I and III only
- e) II and III only

42) If the region bounded by  $y = \tan^{-1}(x)$ ,  $y = \frac{1}{4} \pi$  and the y-axis is rotated about the y-axis, the volume of

the solid formed is

- a) 0.413
- b) 1.348
- c) 0.430
- d) 0.674
- e) 0.215

43)  $f(x)$  is represented by the Maclaurin series

$$1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \dots + (-1)^n \frac{(2x)^{2n}}{(2n)!} + \dots$$

What is the slope of the line normal to the graph of  $f$  at  $x = \frac{11}{4} \pi$ ?

- a) 2
- b)  $\frac{1}{2}$
- c) 0
- d) -2

e)  $-\frac{1}{2}$

44) What are all values of  $h$  for which

$$\int_0^{\infty} \frac{4x}{(x^2 + 1)^h} dx$$

converge?

a)  $h \leq 1$

b)  $h < 1$

c)  $h \geq 1$

d)  $-1 < h < 1$

e)  $h > 1$

45) The base of a solid is the region bounded by

$$y = 6\sqrt{x}$$

the  $x$ -axis, and

the line  $x = 6$

Each cross-section of the solid perpendicular to the  $x$ -axis is a square, with one side on the  $xy$ -plane. Which of the following expressions represents the volume of the solid?

a)  $\int_0^1 6\sqrt{x} dx$

b)  $\int_0^6 6x dx$

c)  $\int_0^1 36x dx$

d)  $\int_0^6 36x dx$

e)  $\int_0^6 6\sqrt{x} dx$

1) Find

$$\int_1^4 \frac{2}{\sqrt{x}} dx$$

- a) 15
  - b) 2
  - \*c) 4
  - d) 8
  - e) 16
- 2) If

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which of the following is true about  $y = f(x)$ ?

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  - b)  $f$  has a local maximum at  $x = 6$  and a local minimum at  $x = 9$ .
  - c)  $f$  has a local minimum at  $x = 6$  and a local maximum at  $x = 9$ .
  - d)  $f$  has a point of inflection at  $x = 6$  and a local minimum at  $x = 9$ .
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4) Give the value for

$$\lim_{x \rightarrow 0} \left( \frac{1}{8} \frac{6^x - 1}{x} \right)$$

- \*a)  $\frac{1}{8} \ln(6)$
- b) 0
- c) 1
- d)  $8 \ln(6)$
- e) *The limit does not exist.*

5) Which of the following series converge?

I.

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n!}$$

II.

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$$

III.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

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I. There exists a  $c$  in  $[0,2]$  where  $f(c) = 0$ .

II. There exists a  $c$  in  $[0,2]$  where  $f'(c) = 0$ .

III. There exists a  $c$  in  $[0,2]$  where  $f'(c) = 9/2$ .

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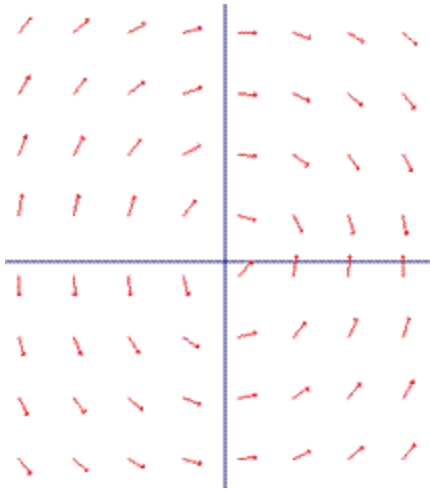
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$$\lim_{h \rightarrow 0} \left( \frac{\cos(3x + 3h) - \cos(3x)}{h} \right)$$

a)  $-3 \cos(3x)$

b)  $3 \sin(3x)$

c)  $3 \cos(3x)$

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21) Find the area of the region enclosed by the graph of

$$y = 3x^2$$

and the line

$$y = 4x$$

- a)  $\frac{64}{27}$
- \*b)  $\frac{32}{27}$
- c)  $\frac{8}{27}$
- d)  $\frac{9}{32}$
- e)  $\frac{9}{16}$

22) Suppose

$$z = e^y$$

$$y = 3x^3 - 3$$

and

$$x = 1 + \ln(t^2)$$

What is  $dz/dt$  when  $t = 1$ ?

- a) 12
- b) 9
- c)  $\frac{9}{2}$
- \*d) 18
- e) 3

23) Evaluate

$$\int_0^{\frac{1}{2}\pi} \cos^2(11x) \sin(11x) dx$$

- a)  $-\frac{1}{66}$
- b)  $\frac{1}{66}$
- c)  $-\frac{1}{33}$
- \*d)  $\frac{1}{33}$
- e)  $\frac{1}{11}$

24) Give an equation for the tangent line to the parametric curve

$$[x = e^t, y = t^2 + 6t]$$

at  $t = 0$ .

- a)  $y - 1 = 6x - 6$
- b)  $y = 6e(x - 1)$
- c)  $y - 1 = 6x$
- \*d)  $y = 6x - 6$
- e)  $y = \frac{6(x - 1)}{e}$

25) Evaluate

$$\frac{\partial}{\partial x} \int_5^{7x} \ln(5t) dt$$

a)  $7 \ln(35x) - 14 \ln(5)$

b)  $\frac{7}{5x}$

c)  $5 \ln(35x)$

d)  $7 \ln(7x) - 14 \ln(5)$

\*e)  $7 \ln(35x)$

26) The region bounded by

$$y = 5 \sin(x)$$

and the  $x$ -axis, for  $0 \leq x \leq \frac{1}{2}\pi$ , is rotated about the line  $y = -2$ . The volume of this solid can be represented

by:

a)  $2\pi \int_0^{\frac{1}{2}\pi} (25 \sin(x)^2 + 2) dx$

\*b)  $\pi \int_0^{\frac{1}{2}\pi} ((5 \sin(x) + 2)^2 - 4) dx$

c)  $\pi \int_0^{\frac{1}{2}\pi} (25 \sin(x)^2 - 4) dx$

d)  $2\pi \int_0^{\frac{1}{2}\pi} 25 \sin(x + 2)^2 dx$

e)  $2\pi \int_0^{\frac{1}{2}\pi} (5 \sin(x) + 2)^2 dx$

27) Give the third degree Taylor polynomial about  $x = 1$  of

$$f(x) = \ln(x)$$

a)  $(x-1) + \frac{1}{3}(x-1)^2 + \frac{1}{3}(x-1)^3$

b)  $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3$

c)  $(x-1) - \frac{1}{3}(x-1)^2 + \frac{1}{5}(x-1)^3$

d)  $(x-1) + (x-1)^2 + 2(x-1)^3$

\*e)  $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

28) Which of the following integrals gives the length of the graph of

$$f(x) = e^{3x}$$

for  $x$  between 0 and 2?

\*a)  $\int_0^2 \sqrt{1 + 9e^{6x}} \, dx$

b)  $\int_0^2 \sqrt{1 + e^{6x}} \, dx$

c)  $\int_0^2 \sqrt{x + 9e^{6x}} \, dx$

d)  $\int_0^2 \sqrt{x + e^{6x}} \, dx$

e)  $\int_0^2 \sqrt{e^{3x} + 9e^{6x}} \, dx$

29) Find the average value of the function

$$f(x) = e^{5x}$$

over the interval  $[0, 2]$ .

- a)  $\frac{1}{10} e^{10}$
- b)  $\frac{1}{5} (e^{10} - 1)$
- \*c)  $\frac{1}{10} (e^{10} - 1)$
- d)  $\frac{1}{2} (e^{10} - 1)$
- e)  $\frac{1}{5} e^{10}$

30) What is the y-intercept of the line tangent to the curve  $y = x^2 + 6$  at  $x = 3$ ?

- \*a)  $(0, -3)$
- b)  $(0, 0)$
- c)  $(0, -12)$
- d)  $(0, 12)$
- e)  $(0, 3)$

31) Which of the following function(s) is continuous and differentiable?

I.  $f(x) = \frac{8}{\sqrt{x}}$

II.  $g(x) = x|x|$

III.  $h(x) = \begin{cases} 5x + 1 & x \leq 0 \\ x^2 + 1 & 0 < x \end{cases}$

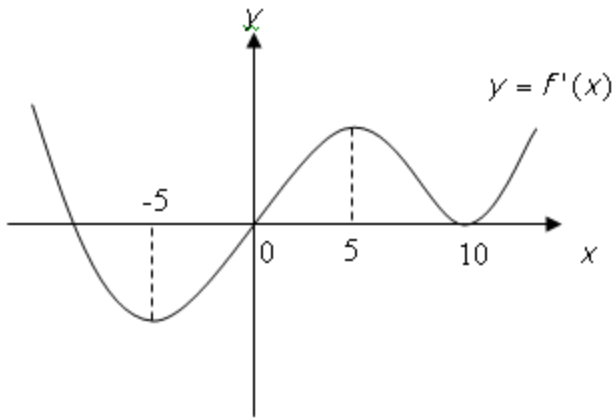
- a) II only
- \*b) I only
- c) III only
- d) I and II only
- e) I and III only

32) Find  $m$

$$\lim_{x \rightarrow 0} \left( \frac{e^{mx^2} - \cos(12x)}{x^2} \right) = 144$$

- \*a) 72
- b) 288
- c) 6
- d) 36
- e) 3

33) The graph of the derivative of  $f$  is shown below. Which of the following must be true?



a)  $f$  is increasing on  $[-5, 5]$ .

\*b)  $f$  has a point of inflection at  $x = 10$ .

c)  $f$  has a local maximum at  $x = 0$ .

d)  $f$  is concave down on  $[0, 10]$ .

e)  $f$  has a local minimum at  $x = -5$ .

34) The sum of two positive integers  $x$  and  $y$  is 120. Find the value of  $x$  that minimizes

$$P = x^3 - 120xy$$

\*a)  $x = 40$

b)  $x = 60$

c)  $x = 20$

d)  $x = 100$

e)  $x = 80$

35) A particle moves on the curve

$$[x = 5 \sin(t), y = \sin(2t)]$$

find the speed of the particle at time  $t = \pi$ .

a) 5.1962

b) 4.5826

\*c) 5.3852

d) 3.0000

e) 2.6458

36) The function  $f$  is defined as

$$f(x) = \frac{(x-4)^2}{x-6}$$

$$x \neq 6$$

Which of the following is **false**?

a)  $f$  has a local maximum at  $x = 4$ .

b)  $f$  has a vertical asymptote at  $x = 6$ .

c)  $f$  is decreasing on  $[4, 6]$ .

d)  $f$  is concave up for  $x > 6$ .

\*e)  $f$  has a horizontal asymptote at  $y = 1$ .

37) A particle is moving along the  $x$ -axis and its position at time  $t \geq 0$  is given by

$$S(t) = (t-3)^2(t-5)$$

Which of the following is (are) true?

I. The particle changes direction at  $x = 3$  and  $x = 5$ .

II. The particle is slowing down on  $[0, 3]$ .

III. The particle is speeding up on  $[3, 5]$ .

- a) I and III only
- b) I only
- c) II and III only
- d) I, II and III
- \*e) II only

38)  $f(x)$  is a differentiable function and it is decreasing on  $(-\infty, \infty)$ .

If

$$g(x) = f(10x^3 - x^2)$$

then  $g$  has a local maximum at

- a)  $x = 0$
- \*b)  $x = \frac{1}{15}$
- c)  $x = 1$
- d)  $x = 5$
- e)  $x = \frac{1}{5}$

39) The rate at which a bacteria population grows is proportional to the number of bacteria present. Initially, there were 1000 bacteria present and the population doubled in 2 hours. Roughly how many hours does it take for the population to reach 10000?

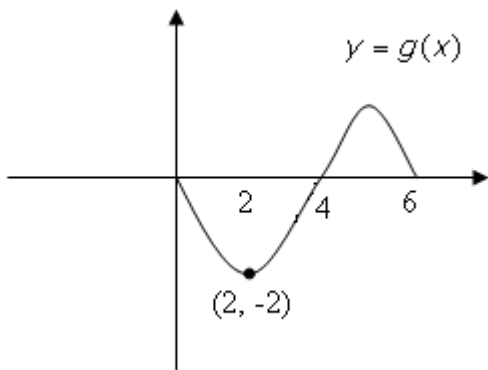
- \*a) 6.6
- b) 8.2
- c) 10.4
- d) 4.4
- e) 5.8

40) Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{n(3x-11)^n}{2^n}$$

- a)  $\frac{11}{2}$
- b)  $\frac{2}{11}$
- c)  $\frac{11}{3}$
- \*d)  $\frac{2}{3}$
- e)  $\frac{3}{2}$





41)

$$g(x) = \int_0^x f(t) dt$$

for  $0 \leq x \leq 6$

The graph of  $g$  is shown above. Which of the following must be true?

- I.  $\int_0^6 f(t) dt = 0$
- II.  $\int_2^4 f(t) dt = 2$
- III.  $\int_4^0 f(t) dt = -2$

- a) I only  
 \*b) I and II only  
 c) II only  
 d) I and III only  
 e) II and III only

42) If the region bounded by  $y = \tan^{-1}(x)$ ,  $y = \frac{1}{4}\pi$  and the  $y$ -axis is rotated about the  $y$ -axis, the volume of

the solid formed is

- a) 0.413  
 b) 1.348  
 c) 0.430  
 \*d) 0.674  
 e) 0.215

43)  $f(x)$  is represented by the Maclaurin series

$$1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \dots + (-1)^n \frac{(2x)^{2n}}{(2n)!} + \dots$$

What is the slope of the line normal to the graph of  $f$  at  $x = \frac{11}{4}\pi$ ?

- a) 2  
 b)  $\frac{1}{2}$   
 c) 0  
 d) -2

\*e)  $-\frac{1}{2}$

44) What are all values of  $h$  for which

$$\int_0^{\infty} \frac{4x}{(x^2 + 1)^h} dx$$

converge?

a)  $h \leq 1$

b)  $h < 1$

c)  $h \geq 1$

d)  $-1 < h < 1$

\*e)  $h > 1$

45) The base of a solid is the region bounded by

$$y = 6\sqrt{x}$$

the  $x$ -axis, and

the line  $x = 6$

Each cross-section of the solid perpendicular to the  $x$ -axis is a square, with one side on the  $xy$ -plane. Which of the following expressions represents the volume of the solid?

a)  $\int_0^1 6\sqrt{x} dx$

b)  $\int_0^6 6x dx$

c)  $\int_0^1 36x dx$

\*d)  $\int_0^6 36x dx$

e)  $\int_0^6 6\sqrt{x} dx$