UH.EDU 2006 AP Calculus BC Exam 2 | Free Response Solutions

Problem 1

a)

$$\frac{dh}{dt} = \frac{1}{2} ft/s$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{h}{r} = \frac{50}{15} \rightarrow r = \frac{3h}{10}$$

$$V(r) = \frac{1}{3}\pi\left(\frac{3h}{10}\right)^2 h = \frac{3\pi h^3}{100}$$

b)

$$\frac{dV}{dt} = \frac{3\pi}{100}3h^2\frac{dh}{dt} = \frac{9\pi}{100}2^2\frac{1}{2} = \frac{9\pi}{50} ft^3/s$$

c)

$$A = \pi r^2 = \pi\left(\frac{3h}{10}\right)^2 = \frac{9\pi}{100}h^2$$

$$\frac{dA}{dt} = \frac{9\pi}{50}h\frac{dh}{dt} = \frac{9\pi}{50}2\frac{1}{2} = \frac{9\pi}{50} ft^2/s$$

Problem 2

a)

$$y_{avg[-2,2]} = \frac{1}{2-(-2)}\int_{-2}^{2} f(x)dx = \frac{1}{4}\int_{-2}^{2} f(x)dx = \frac{1}{4}*2\int_{0}^{2} f(x)dx = \frac{1}{2}\int_{0}^{2} f(x)dx = 1$$

$$\rightarrow \int_0^2 f(x)dx = 2$$

$$\int_0^2 \left(cx^2\right)^{1/3} dx = 2$$

$$\int_0^2 \left(c^{1/3}x^{2/3}\right)dx = c^{1/3}\int_0^2 \left(x^{2/3}\right)dx = 2$$

$$\rightarrow c^{1/3} = \frac{2}{1.90488}$$

$$c = 1.157$$

$$f(x) = 1.157^{1/3} * x^{2/3}$$

b)

SHELL:

$$y = 1.157^{1/3} x^{2/3} \rightarrow x^2 = \frac{y^3}{1.157} \rightarrow x = \sqrt{\frac{y^3}{1.157}}$$

$$V_{shell} = \int_{y=0}^{y=4} d(y)h(y)\,dy = \int_{y=0}^{y=4} \left(2\pi(4-y)\right)\left(2 * \sqrt{\frac{y^3}{1.157}}\right)dy = 170.901$$

DISK :

$$y = 1.157^{1/3} x^{2/3} = 4 \rightarrow x = 7.44066$$

$$V = 2\pi \int_0^{7.44066} \left(4 - 1.157^{1/3} x^{2/3}\right)^2 dx = 170.901$$

Problem 3

a)

$$I : \left(x'(t), y'(t)\right) = \left(1, 2t\right) \rightarrow \left(x'(3), y'(3)\right) = \left(1, 6\right)$$

$$II : \left(x'(t), y'(t)\right) = \left(4, -2\right) \rightarrow \left(x'(3), y'(3)\right) = \left(4, -2\right)$$

b)

$$\int_1^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_1^3 \sqrt{(4)^2 + (-2)^2}\,dt$$

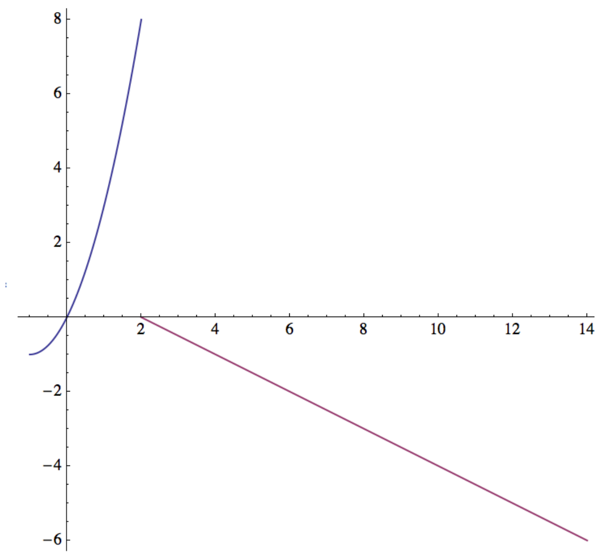
c)

Collision :

$$\begin{cases} x_1(t) = x_2(t) \\ y_1(t) = y_2(t) \end{cases} \rightarrow \begin{cases} t - 1 = 4t + 2 \\ t^2 - 1 = -2t \end{cases} \rightarrow \begin{cases} t = -1 \\ 0 = 2 \end{cases} \rightarrow impossible.$$

The particles do not collide.

d) Use the Parametric Mode in the graphing calculator to help sketch the paths of the two particles. The following was created in Mathematica:



Problem 4

a)

$f(x)=xe^{2x}$

$f(x)=x(1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+...+\frac{(2x)^n}{n!}+...)=\sum_{n=0}^{\infty}\frac{2^n x^{n+1}}{n!}$

b)

$f(x)=xe^{2x}$

$L.S.=f'(x)=\frac{d}{dx}\left(x+2x^2+\frac{2^2x^3}{2!}+\frac{2^3x^4}{3!}+...+\frac{2^n x^{n+1}}{n!}+...\right)=1+4x+6x^2+...$

$R.S.=e^{2x}+2xe^{2x}=\left(1+2x+\frac{4x^2}{2!}+\frac{8x^3}{3!}+...\right)+2x\left(1+2x+\frac{4x^2}{2!}+\frac{8x^3}{3!}+...\right)=$

$\left(1+2x+\frac{4x^2}{2!}+\frac{8x^3}{3!}+...\right)+\left(2x+4x^2+\frac{8x^3}{2!}+\frac{16x^4}{3!}+...\right)=$

$=1+4x+6x^2+...$

$L.S.=R.S.$

c)

$$\int_0^1 xe^{2x}\,dx = \int_0^1 x(1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\ldots+\frac{(2x)^n}{n!}+\ldots)\,dx =$$

$$= \int_0^1 x+2x^2+\frac{(2)^2x^3}{2!}+\frac{(2)^3x^4}{3!}+\ldots+\frac{(2)^nx^{n+1}}{n!}+\ldots dx =$$

$$= \int_0^1 \frac{x^2}{2}+\frac{2x^3}{3}+\frac{(2)^2x^4}{2!}+\frac{(2)^3x^4}{4*3!}+\ldots+\frac{(2)^nx^{n+2}}{(n+2)*n!}+\ldots dx$$

Problem 5

a)

$$A = \int_1^2 3\ln x - \frac{x-1}{2}\,dx = \left(3x\ln x - 3x - \frac{x^2}{4}+\frac{x}{2}\right)_1^2 = 6\ln 2 - \frac{13}{4}$$

b)

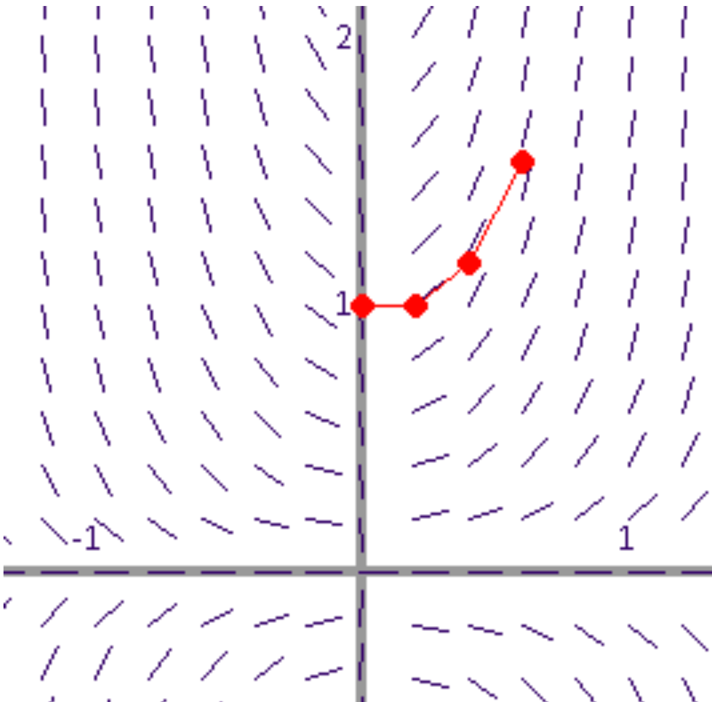
$$V_{WASHER} = \pi\int_1^2 (3\ln x)^2 - \left(\frac{x-1}{2}\right)^2 dx$$

c)

$$V_{SHELL} = \int_{x=1}^{x=2} 2\pi(1+x)\left(3\ln x - \frac{x-1}{2}\right)dx$$

Problem 6

a), b), and c)



Segment 1: $y-1=0(x-0) \to y(0.2)=1$

Segment 2: $y-1=0.8(x-0.2) \to y(0.4)=1.160$

Segment 3: $y-1.16=4*0.4*1.16(x-0.4) \to y(0.6)=1.5312$

Euler's Method Results:

1) (0.0000, 1.0000)
2) (0.2000, 1.0000)
3) (0.4000, 1.1600)
4) (0.6000, 1.5312)

d)

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(4xy\right)=4y+4x\frac{dy}{dx}=4y+4x*4xy=4y(1+4x^2)$$