AP Calculus Practice Exam
BC Version - Section I - Part A

Calculators ARE NOT Permitted On This Portion Of The Exam
28 Questions - 55 Minutes

1) Given

$$3y^2 - 4e^{(-2x)} - xy = 4$$

Find dy/dx.

a) $\frac{-4e^{(-2x)} - y}{6y - x}$

b) $-\frac{8e^{(-2x)}}{6y + x}$

c) $\frac{8e^{(-2x)}}{6y + x}$

d) $-\frac{8e^{(-2x)} - y}{6y - x}$

e) $\frac{8e^{(-2x)} - y}{6y - x}$

2) Give the volume of the solid generated by revolving the region bounded by the graph of *y* = ln(*x*), the *x*-axis, the lines *x* = 1 and *x* = *e*, about the *y*-axis.

a) $\frac{1}{2}\pi(e^4 - 1)$

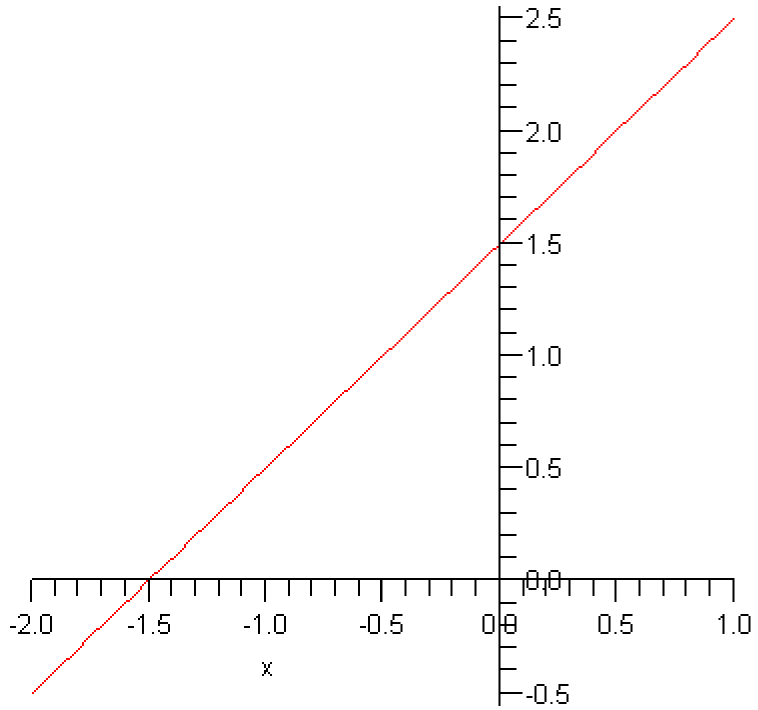
b) $\frac{1}{4}\pi(e^2 + e)$

c) $\frac{1}{4}\pi(e^2 - e)$

d) $\frac{1}{2}\pi(e^4 + 1)$

e) $\frac{1}{2}\pi(e^2 + 1)$

3) The graph of the **derivative** of *f* is shown below.



Find the area bounded between the graph of *f* and the *x*-axis over the interval [-2,1], given that *f*(0) = 1.

a) $\frac{13}{4}$

b) $\frac{29}{12}$

c) $\frac{8}{3}$

d) $\frac{31}{12}$

e) $\frac{11}{4}$

4) Determine dy/dt, given that

$$y = x^2 + 4x$$

and

$$x = \cos(3t)$$

a) $-6\cos(3t)\sin(3t)$

b) $-3(2\cos(3t)+4)\sin(3t)$

c) $6\cos(3t)+12$

d) $-18\sin(3t)$

e) $3(2\cos(3t)+4)\cos(3t)$

5) The function

$$f(x) = 5x^2 + 3e^{(2x)}$$

is invertible. Give the slope of the normal line to the graph of f^{-1} at $x = 3$.

a) $-\frac{1}{30 + 6e^6}$

b) $\frac{-2}{3}$

c) $\frac{1}{6}$

d) $30 + 6e^6$

e) -6

6) Determine

$$\int (\sin(6x))^2 (\cos(6x))^2 \, dx$$

a) $\frac{1}{8}x - \frac{1}{192}\sin(24x) + C$

b) $\frac{1}{8}x + \frac{1}{96}\sin(12x) + C$

c) $\frac{1}{8}x - \frac{1}{192}\cos(24x) + C$

d) $\frac{1}{8}x + \frac{1}{192}\sin(24x) + C$

e) $\frac{1}{8}x - \frac{1}{96}\sin(12x) + C$

7) Give the polar representation for the circle of radius 2 centered at (0 , 2).

a) $r = 2\sin(\theta) + 2\cos(\theta)$

b) $r = 4\cos(\theta)$

c) $r = 4\sin(\theta)$

d) $r\sin(\theta) = 2$

e) $r = 4\sin(\theta) - \cos(\theta)$

8) Determine

$$\lim_{t \to \infty} \left(4t^2 \left(\sin\left(\frac{2}{t}\right)\right)^2\right)$$

a) 2

b) 1

c) 16

d) $\frac{1}{2}$

e) 32

9) Determine

$$\int_{1}^{2} \frac{1}{\sqrt{4 - x^2}}\, dx$$

a) $\frac{2}{3}\pi$

b) $\frac{1}{3}\pi$

c) $-\frac{1}{3}\pi$

d) $\frac{1}{6}\pi$

e) $-\frac{1}{6}\pi$

10) Give the radius of convergence for the series

$$\sum_{k=1}^{\infty} \frac{3^{(k+2)}\, x^k}{k+1}$$

a) *The series diverges for all x.*

b) 1

c) 0

d) $\frac{1}{3}$

e) 3

11) Determine

$$\lim_{t \to \infty} \left(t \left(\ln\left(2 + \frac{1}{t}\right) - \ln(2)\right)\right)$$

a) $\frac{1}{2}\ln(2)$

b) 2

c) $\frac{1}{2}$

d) 0

e) ∞

12) The position of a particle moving along the *x*-axis at time *t* is given by

$$x(t) = (\sin(4\pi t))^2$$

At which of the following values of *t* will the particle change direction?

I) *t* = 1/8
II) *t* = 1/6
III) *t* = 1
IV) *t* = 2

a) I, II and III
b) I and II
c) I, III and IV
d) II, III and IV
e) III and IV

13) Determine

$$\int_0^{\pi} x \cos(x)\, dx$$

a) $-1 + \frac{1}{2}\pi$

b) π

c) -2

d) $-\frac{4}{3} + \frac{1}{3}\pi$

e) $-\frac{3}{2} + \frac{1}{4}\pi$

14) Determine the *y*-intercept of the tangent line to the curve

$$y = \sqrt{x^2 + 33}$$

at *x* = 4.

a) $\frac{45}{7}$

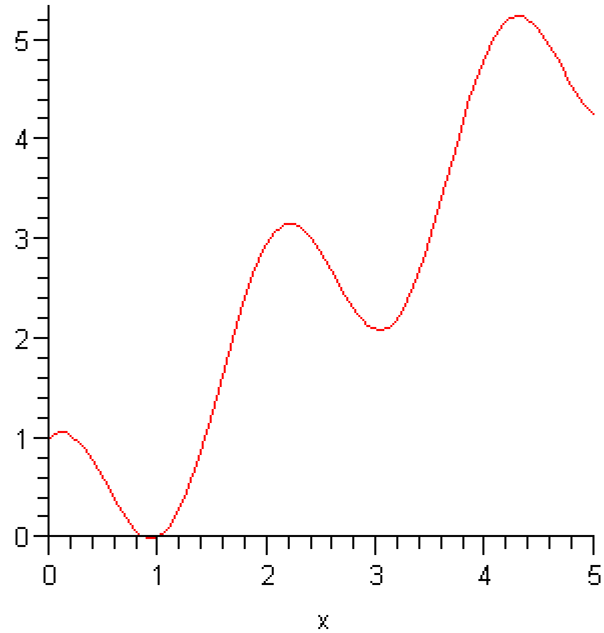
b) $\frac{66}{49}$

c) $\frac{-33}{49}$

d) $\frac{135}{49}$

e) $\frac{33}{7}$

15) The function *f* is graphed below.



Give the number of values of *c* that satisfy the conclusion of the Mean Value Theorem for derivatives on the interval [2,5].

a) 3

b) 2

c) 1

d) 4

e) 3.2

16) Give the average value of the function

$$f(x) = 2\,e^{(x-4)}$$

on the interval [1,3].

a) $e^{(-1)} + e^{(-2)}$

b) $-\frac{2}{3}e^{(-3)} + \frac{2}{3}e^{(-1)}$

c) $\frac{2}{3}e^{(-1)}$

d) $-e^{(-3)} + e^{(-1)}$

e) $-2\,e^{(-3)} + 2\,e^{(-1)}$

17) A rectangle has both a changing height and a changing width, but the height and width change so that the area of the rectangle is always 200 square feet. Give the rate of change of the width (in ft/sec) when the height is 5 feet, if the height is decreasing at that moment at the rate of 1/2 ft/sec.

a) $\frac{2}{5}$

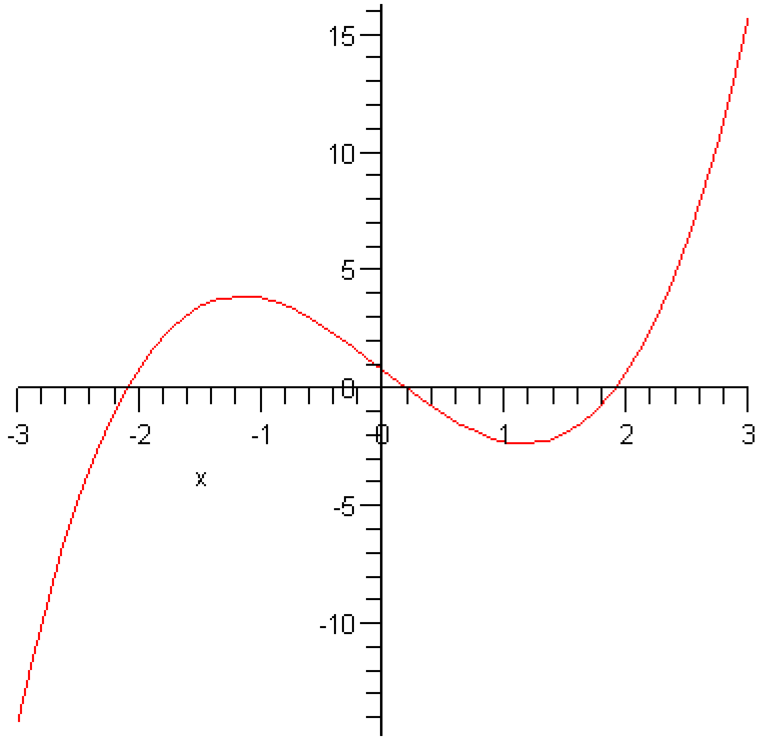
b) $\frac{-2}{5}$

c) $\frac{1}{40}$

d) $\frac{1}{60}$

e) 205

18) The graph of the **derivative** of f is shown below.



Give the number of values of x in the interval [-3,3] where the graph of f has inflection.

a) 1

b) 2

c) 0

d) 3

e) *There is not enough information.*

19) A rectangle has its base on the x-axis and its vertices on the positive portion of the parabola

$$y = 2 - 3x^2$$

What is the maximum possible area of this rectangle?

a) $\frac{8}{27}\sqrt{3}\sqrt{6}$

b) $\frac{2}{9}\sqrt{3}\sqrt{6}$

c) $\frac{4}{15}\sqrt{5}\sqrt{6}$

d) $\frac{2}{15}\sqrt{5}\sqrt{6}$

e) $\frac{1}{3}\sqrt{2}\sqrt{6}$

20) Compute

$$\int e^{(2x)} (\tan(e^{(2x)}))^2 \, dx$$

a) $\frac{1}{2}(\sec(e^{(2x)}))^2 + C$

b) $\frac{1}{2}\tan(e^{(2x)})\, e^{(2x)} + C$

c) $4\tan(e^{(2x)})\,(\sec(e^{(2x)}))^2\, e^{(2x)} + C$

d) $\frac{1}{2}\tan(e^{(2x)}) - \frac{1}{2}e^{(2x)} + C$

e) $\frac{1}{2}\tan(e^{(2x)}) + \frac{1}{2}e^{(2x)} + C$

21) Determine

$$\int_0^\infty \frac{1}{36+x^2}\, dx$$

a) $\frac{1}{2}\pi$

b) $\frac{1}{12}\pi$

c) 3π

d) ∞

e) 6π

22) Determine

$$\lim_{x\to\infty} (4^x + 7^x)^{\left(\frac{1}{x}\right)}$$

a) $\frac{11}{2}$

b) 7

c) ∞

d) e^7

e) 11

23) Give the exact value of

$$\sum_{n=0}^{\infty} \frac{\cos(n\pi)\, 5^n}{n!}$$

a) $e^{(-5)}$

b) $\sin(5)$

c) $\cos(5)$

d) e^5

e) $-\sin(5)$

24) Determine

$$\lim_{x \to 0} \left(\frac{e^x + e^{(-x)} - 2}{1 - \cos(x)} \right)$$

a) 0

b) 1

c) 2

d) *undefined*

e) $\frac{3}{2}$

25) Give the derivative of

$$f(x) = x^{(-2x)}$$

a) $-2\, x\, x^{(-2x)} - 2\, x^{(-2x)} \ln(x)$

b) $-2\, x\, x^{(-2x-1)} + x^{(-2x)} \ln(x)$

c) $x\, x^{(-2x-1)} - 2\, x^{(-2x)} \ln(x)$

d) $-2\, x\, x^{(-2x-1)} - 2\, x^{(-2x)} \ln(x)$

e) $-2\, x\, x^{(-2x-1)}$

26) Give the first 3 nonzero terms in the Taylor series expansion about $x = 0$ for the function

$$f(x) = \cos(2\, x)$$

a) $1 - 2x^2 + 4x^4$

b) $1 - 2x^2 + \frac{2}{3}x^4$

c) $x - \frac{4}{3}x^3 + \frac{4}{15}x^5$

d) $1 + 2x + 2x^2$

e) $1 - 2x^2$

27) Determine

$$\int \frac{x}{x^2 + 2x - 8}\,dx$$

a) $\frac{1}{2}\ln((x-2)(x+4)) + C$

b) $\frac{1}{3}\ln(|x-2|) + \frac{2}{3}\ln(|x+4|) + C$

c) $-\frac{2}{3}\ln(|x-2|) - \frac{1}{3}\ln(|x+4|) + C$

d) $-\frac{2}{3}\ln(|x+4|) - \frac{1}{3}\ln(|x-2|) + C$

e) $\frac{1}{2}\ln(|(x-2)(x+4)|) + C$

28) Which of the following series converge(s)?

$$\left[A = \sum_{n=1}^{\infty} \frac{1}{n^{(3/4)}},\ B = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^5}},\ C = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}\right]$$

a) B only
b) A, B and C
c) B and C
d) A and B
e) A and C

AP Calculus Practice Exam
BC Version - Section I - Part B

Calculators ARE Permitted On This Portion Of The Exam
17 Questions - 50 Minutes

1) The limit of the sequence

$$u_n = \frac{1 + c\,n^2}{(2\,n + 3 + 2\sin(n))^2}$$

as *n* approaches ∞ is -3. What is the value of *c*?

a) $\frac{-3}{4}$

b) -9

c) $\frac{-3}{2}$

d) -12

e) -27

2) If

$$\frac{dy}{dx} = 4\,y\,x^2$$

and *y* = 3 when *x* = -2, then what is *y*?

a) $\dfrac{3\,e^{\left(-\frac{4}{3}x^3\right)}}{e^{\left(\frac{32}{3}\right)}}$

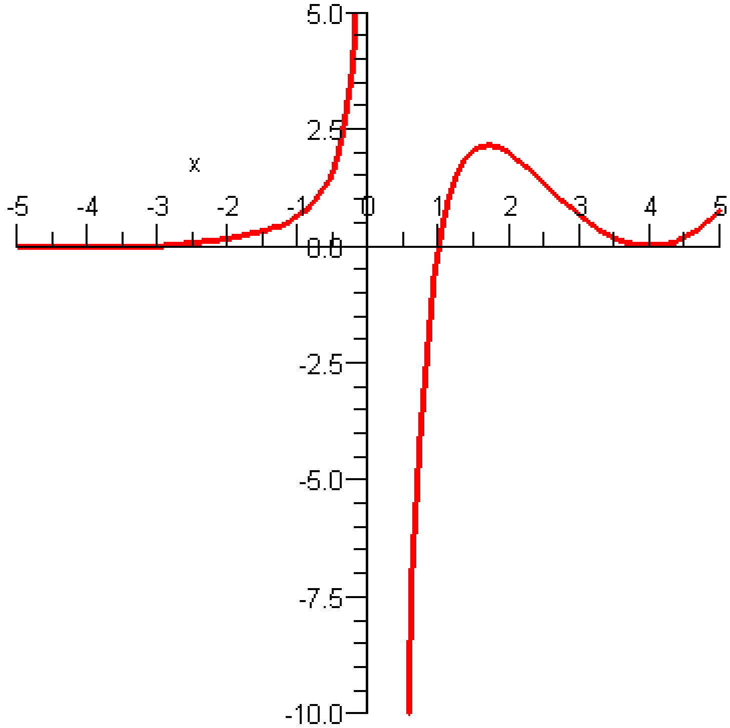
b) $\dfrac{3\,e^{\left(\frac{4}{3}x^2\right)}}{e^{\left(\frac{16}{3}\right)}}$

c) $\dfrac{3\,e^{\left(\frac{4}{3}x\right)}}{e^{\left(\frac{-8}{3}\right)}}$

d) $\dfrac{3\,e^{\left(-\frac{4}{3}x^2\right)}}{e^{\left(\frac{16}{3}\right)}}$

e) $\dfrac{3\,e^{\left(\frac{4}{3}x^3\right)}}{e^{\left(\frac{-32}{3}\right)}}$

3) The graph of the **derivative** of *f* is given below.



Which of the following is FALSE about the function *f*?

a) *f* is increasing on [1,4].

b) *f* is concave down on [1,5/2].

c) *f* is concave down on [-3,0).

d) *f* is not differentiable at 0.

e) The funciton is constant on (-∞,-3].

4) Determine

$$\int_{-8}^{1} \frac{1}{x^{(5/3)}}\, dx$$

a) $\frac{-3}{2}$

b) *The integral does not exist.*

c) $\frac{3}{2}$

d) $\frac{3}{4}$

e) $\frac{1}{12}$

5) Give the area that lies below the *x*-axis and is contained within the region bounded by the polar curve

$$r = 1 + 2\sin(\theta)$$

a) $\pi - 4 + \frac{1}{2}\sqrt{3}$

b) $\frac{1}{4}\pi - 2 + \frac{1}{4}\sqrt{3}$

c) $\frac{1}{2}\pi - 2 + \frac{3}{2}\sqrt{3}$

d) $\pi - 2 + \frac{3}{2}\sqrt{3}$

e) $\frac{1}{2}\pi - 4 + \frac{3}{2}\sqrt{3}$

6) Give the error that occurs when the area between the curve

$$y = x^3 + 1$$

and the x-axis over the interval [0,1] is approximated by the trapezoid rule with $n = 4$.

a) 0.016

b) 0.046

c) 0.025

d) 0.128

e) 0.008

7) Let

$$f(x) = \sum_{k=0}^{\infty} (1 - (\sin(x))^2)^k$$

Determine $f(2\pi/3)$.

a) ∞

b) 2

c) $\frac{4}{3}$

d) $\frac{1}{2}$

e) $\frac{3}{2}$

8) Give the length of the curve determined by

$$[x = 4t^2, y = t^3 + 2t]$$

for t from 0 to 2.

a) 20.0849

b) 20.1390

c) 20.1084

d) 20.0735

e) 20.0886

9) Particles A and B leave the origin at the same time and move along the *y*-axis. Their positions are determined by the functions

$$[\, y_A = 2\sin(2t),\ y_B = 4\cos(t)\,]$$

for *t* between 0 and 8. What is the velocity of particle B when particle A stops for the first time?

a) 0

b) $-2\sqrt{2}$

c) -4

d) 4

e) $2\sqrt{2}$

10) The base of a solid is the region in the *xy* plane enclosed by the curves

$$[\ f(x) = \sin(x),\ g(x) = \cos(x)\ ]$$

over the interval $[0, \pi/4]$. Cross sections of the solid perpendicular to the *x*-axis are squares. Determine the volume of the solid.

a) 0.3061

b) 0.2564

c) 0.3146

d) 0.2855

e) 0.2572

11) Give the minimum value of the function

$$f(x) = 2x^3 - 9x + 5$$

for $x \geq 0$.

a) -2.258

b) -2.368

c) -2.349

d) -2.213

e) -2.175

12) Select the TRUE statement associated with the function

$$f(x) = \frac{\sin(x)}{x^2}$$

a) The graph of the function passes through the origin.
b) The function does not have a horizontal asymptote.
c) The function has a vertical asymptote at $x = 0$.
d) The graph of the function is symmetric about the *x*-axis.
e) The graph is always concave up.

13) The function *g* is the derivative of

$$\int_0^x (t^3 - 5)\, dt$$

What is the derivative of the inverse of *g* at *x* = 3?

a) 12

b) $\frac{1}{4}$

c) $\frac{1}{12}$

d) 4

e) $\frac{1}{3}$

14) The half-life of radium-226 is 1625 years. What percentage of a given amount of the radium will remain after 1000 years?

a) 65.34%
b) 65.20%
c) 65.25%
d) 65.35%
e) 65.30%

15) The function *f* satisfies the equation

$$\int_0^{2x} f(t)\, dt = 4\sin(x) + x$$

Evaluate $f(\pi/3)$.

a) 2.232

b) 1.500

c) 4.464

d) 3.

e) 2.432

16) A rectangular box with square base and top is to be made to contain 2160 cubic feet. The material for the base costs 30 cents per square foot, the material for the top costs 50 cents per square foot, and the material for the sides costs 20 cents per square foot. Give the length of one side of the base (in feet) so that the cost is minimized.

a) 9.16

b) 9.78

c) 8.32

d) 10.26

e) 12.38

17) Which expression represents the volume of the solid generated when the region between the curves

$$\left[y = 6 - x^2,\ y = \frac{1}{2}x^2 \right]$$

over the interval [0,2] is rotated around the *x*-axis?

a) $2\pi \int_{2}^{6} y\sqrt{6-y}\, dy + 2\pi \int_{0}^{2} y^{(3/2)}\sqrt{2}\, dy$

b) $2\pi \int_{0}^{2} y\sqrt{6-y}\, dy - 2\pi \int_{0}^{2} y^{(3/2)}\sqrt{2}\, dy$

c) $2\pi \int_{2}^{6} y\sqrt{6-y}\, dy - 2\pi \int_{0}^{2} y^{(3/2)}\sqrt{2}\, dy$

d) $2\pi \int_{0}^{6} y\sqrt{6-y}\, dy + 2\pi \int_{0}^{2} y^{(3/2)}\sqrt{2}\, dy$

e) $2\pi \int_{0}^{6} y\sqrt{6-y}\, dy - 2\pi \int_{0}^{2} y^{(3/2)}\sqrt{2}\, dy$

1) d)
2) e)
3) b)
4) b)
5) e)
6) a)
7) c)
8) c)
9) b)
10) d)
11) c)
12) c)
13) c)
14) e)
15) a)
16) d)
17) a)
18) b)
19) a)
20) d)
21) b)
22) b)
23) a)
24) c)
25) d)
26) b)
27) b)
28) c)

1) d)
2) e)
3) b)
4) b)
5) e)
6) a)
7) c)
8) c)
9) b)
10) d)
11) c)
12) c)
13) c)
14) e)
15) a)
16) d)
17) a)

SECTION II

Time: 1 hour and 30 minutes
Percent of total grade: 50

Part A: 45 minutes, 3 problems
(A graphing calculator is required for some problems or parts of problems.)

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

Part B: 45 minutes, 3 problems
(No calculator is allowed for these problems.)

During the timed portion for Part B, you may keep Part A, and continue to work on the problems in Part A without the use of any calculator.

GENERAL INSTRUCTIONS FOR SECTION II PART A AND PART B

For each part of Section II, you may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

- YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM WITH A PENCIL OR PEN IN THE SPACE PROVIDED FOR THAT PART IN THE PINK EXAM BOOKLET. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. You will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS BC
SECTION II, Part A
Time --- 45 minutes
Number of Problems --- 3

A graphing calculator is required for some problems or parts of the problems.

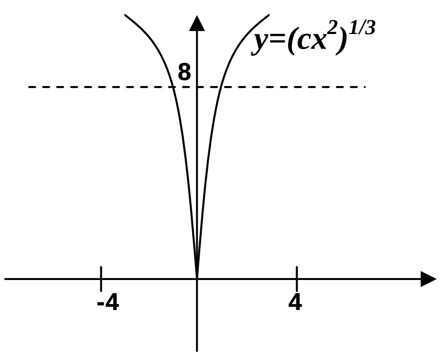
1. Water is poured into a tank in the shape of a right cone, standing on its vertex. The height of the cone is 30 feet and its radius is 12 feet. The water level in the tank is increasing at a constant rate of 0.2 feet per second.

a) Find an expression for the volume of the water (in ft^3) in the tank in terms of its height.

b) How fast (in ft/sec) is the water being poured at the instant the depth of the water is 5 feet?

c) How fast is the area of the surface of the water increasing at the instant the depth of the water is 5 feet?

2)



The graph of $y=(cx^2)^{1/3}$ is given above.

a) Find the value of *c* so that the average value of *y* on the interval [-4,4] is 12.

b) Using the value of *c* determined in (a), set up and evaluate the volume of the solid generated by revolving the region enclosed by the curve $y = (cx^2)^{1/3}$ and the line $y=8$, about the line $y=8$.

3. Two particles move in the *xy* – plane. For time $t \geq 0$, the position of particle I is given by $x = t + 1$ and $y = t^2 - 1$, and the position of particle II is given by $x = 6t + 3$ and $y = -2t$.

a) Find the velocity vector for each particle at time $t = 4$.

b) Give the integral expression (but do not evaluate) for the distance traveled by particle II from time $t = 1$ to $t = 2$.

c) Do the particles collide? If yes, at what time? Justify your answer.

d) Sketch the paths of both particles from time $t = 0$ to $t = 4$. Indicate the direction of each particle along its path.

CALCULUS BC
SECTION II, Part B
Time --- 45 minutes
Number of Problems --- 3

No Calculator is allowed for these problems.

4. Let f be the function $f(x) = xe^x$.

a) Find a power series representation of $f(x)$.

b) Use term-by-term differentiation to show that $f\,'(x) = e^x + xe^x$.

c) Write down (but do not evaluate) $\int_0^1 xe^x dx$ as a series by integrating the power series from part (a).

5. Let *S* be the region enclosed by the graphs of $y = 3e^x$ and $y = 2x$, and the lines $x = 0$ and $x = 1$.

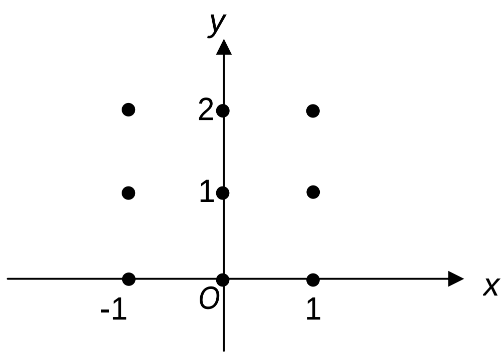
a) Find the area of *S*.

b) State (but do not evaluate) an integral expression, in terms of a single variable, for the volume of the solid generated when *S* is revolved about the *x* – axis.

c) State (but do not evaluate) an integral expression, in terms of a single variable, for the volume of the solid generated when *S* is revolved about the line $x = -1$.

6. Consider the differential equation $\frac{dy}{dx} = 2xy$

a) On the axes provided, sketch a slope field for the given differential equation at the 9 points indicated.



b) Sketch the solution curve through the point (0,1).

c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 2$. Use Euler's Method, starting at x=0 with 3 steps of equal size, to approximate $f(0.6)$. Show your work clearly.

d) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

END OF EXAM