

AP Calculus Practice Exam

AB Version - Section I - Part A

Calculators ARE NOT Permitted On This Portion Of The Exam

28 Questions - 55 Minutes

1) Give $f(g(1))$, given that

$$\left[f(x) = -2x + 1, g(x) = -\frac{x}{x^2 + 1} \right]$$

a) 0

b) $\frac{1}{2}$

c) 2

d) 3

e) $\frac{-1}{2}$

2) Find the slope of the tangent line to the graph of f at $x = 4$, given that

$$f(x) = -x^2 - 4\sqrt{x}$$

a) -8

b) -7

c) -9

d) -10

e) -6

3) Determine

$$\lim_{x \rightarrow \infty} \left(\frac{2x^3 + 4x}{-2x^5 + x^2 - 2} \right)$$

a) 0

b) ∞

c) -1

d) $\frac{-3}{5}$

e) 1

4) Let

$$f(x) = x^3$$

A region is bounded between the graphs of $y = -1$ and $y = f(x)$ for x between -1 and 0 , and between the graphs of $y = 1$ and $y = f(x)$ for x between 0 and 1 . Give an integral that corresponds to the area of this region.

a) $\int_0^1 2(1 + x^3) dx$

b) $\int_{-1}^1 (1 - x^3) dx$

c) $\int_0^1 2(1 - x^3) dx$

d) $\int_{-1}^1 (1 + x^3) dx$

e) $\int_0^1 (-x^3 - 1) dx$

5) Given that

$$3x^3 - 4xy - 4y^2 = 1$$

Determine the change in y with respect to x .

a) $-\frac{6x - 4y}{-4x - 4}$

b) $-\frac{9x^2 - 4}{-4x - 8y}$

c) $-\frac{9x^2 - 4}{-4 - 8y}$

d) $-\frac{9x^2 - 4y}{-4x - 8y}$

e) $-\frac{9x^2 - 4y}{-4 - 8y}$

6) Compute the derivative of

$$4 \sec(x) - 3 \csc(x)$$

a) $4 \csc(x) + 3 \sec(x)$

b) $4 \sec(x) \tan(x) + 3 \csc(x) \cot(x)$

c) $4 (\sec(x))^2 + 3 (\csc(x))^2$

d) $4 \sec(x) \tan(x) - 3 \csc(x) \cot(x)$

e) $4 (\tan(x))^2 + 3 (\cot(x))^2$

7) Compute

$$\int_0^{\frac{1}{4}} \frac{16}{1 + 16t^2} dt$$

a) 3π

b) 2π

c) 0

d) π

e) $-\pi$

8) Determine

$$\frac{d}{dx} \left(\frac{2x^4 - 2x}{2x^4 + 2x} \right)$$

a) $\frac{6x^2}{(2x^3 + 2)^2}$

b) $\frac{12x^2 - 1}{(2x^3 + 2)^2}$

c) $\frac{12x^2}{(2x^3 + 2)^2}$

d) $\frac{6x^2}{(1 + x^3)^2}$

e) $\frac{24x^2 - 1}{(2x^3 + 2)^2}$

9) Give the equation of the normal line to the graph of

$$y = 3x\sqrt{x^2 + 6} - 3$$

at the point $(0, -3)$.

a) $x - 3\sqrt{6}y = 9\sqrt{6}$

b) $x + 3\sqrt{6}y = -9\sqrt{6}$

c) $3\sqrt{6}x + y = -3$

d) $-3\sqrt{6}x + y = -3$

e) $x + 3\sqrt{6}y = -3$

10) Determine the concavity of the graph of

$$f(x) = 2 \sin(x) + 3 (\cos(x))^2$$

at $x = \pi$.

a) **6**

b) -7

c) **3**

d) -6

e) -4

11) Compute

$$\int -3x^2 \sqrt{x^3 + 3} \, dx$$

a) $-\frac{1}{\sqrt{x^3 + 3}} + C$

b) $-\frac{4}{3} (x^3 + 3)^{(3/2)} + C$

c) $-2 (x^3 + 3)^{(3/2)} + C$

d) $-\frac{2}{\sqrt{x^3 + 3}} + C$

e) $-\frac{2}{3} (x^3 + 3)^{(3/2)} + C$

12) Give the value of x where the function

$$f(x) = x^3 + 6x^2 + 9x + 4$$

has a local maximum.

a) **1**

b) **3**

c) **-1**

d) **-2**

e) **-3**

13) The slope of the tangent line to the graph of

$$4x^2 + cx + 2e^y = 2$$

at $x = 0$ is 4. Give the value of c .

a) **4**

b) **-4**

c) **2**

d) **8**

e) **-8**

14) Compute

$$\int (7^x - 4 e^{(7 \ln(x))}) dx$$

a) $\frac{7^x}{\ln(7)} - \frac{1}{2} x^8 + C$

b) $7^x \ln(7) - \frac{4}{7} e^{(7 \ln(x))} + C$

c) $7^x \ln(7) - \frac{4}{7} \frac{e^{(7 \ln(x))}}{x} + C$

d) $\frac{7^x}{\ln(7)} - \frac{4}{7} e^{(7 \ln(x))} + C$

e) $\frac{7^x}{\ln(7)} - \frac{4}{7} x^7 + C$

15) What is the average value of the function

$$g(x) = (2x + 3)^2$$

on the interval from $x = -3$ to $x = -1$?

a) 5

b) -4

c) $\frac{7}{3}$

d) $\frac{14}{3}$

e) $\frac{4}{3}$

16) Compute

$$\lim_{t \rightarrow 0} \left(\frac{\tan\left(\frac{1}{4}\pi + t\right) - \tan\left(\frac{1}{4}\pi\right)}{t} \right)$$

a) π

b) -1

c) $\frac{1}{4} \pi$

d) 1

e) 2

17) Find the instantaneous rate of change of

$$f(t) = (2t^3 - 2t + 4) \sqrt{t^2 + 2t + 4}$$

at $t = 0$.

a) $\frac{-1}{2}$

b) -2

c) -3

d) 0

e) $\frac{1}{4}$

18) Compute

$$\frac{d}{dx} 11^{\cos(x)}$$

a) $\frac{\sin(x) 11^{\cos(x)}}{\ln(11)}$

b) $-\sin(x) 11^{\cos(x)}$

c) $-\frac{\sin(x) 11^{\cos(x)}}{\ln(11)}$

d) $-\sin(x) 11^{\cos(x)} \ln(11)$

e) $\sin(x) 11^{\cos(x)} \ln(11)$

19) A solid is generated by rotating the region enclosed by the graph of

$$y = \sqrt{x}$$

the lines $x = 1$, $x = 2$, and $y = 1$, about the x -axis. Which of the following integrals gives the volume of the

solid?

a) $\int_1^2 \pi (2 - \sqrt{x})^2 dx$

b) $\int_1^2 \pi (\sqrt{x} - 1)^2 dx$

c) $\int_1^2 \pi (2 - x)^2 dx$

d) $\int_1^2 \pi (x - 1) dx$

e) $\int_1^2 \pi (x - 1)^2 dx$

20) Compute

$$\lim_{x \rightarrow 0} \left(\frac{4x}{\sin(3x)} + \frac{x}{\cos(3x)} \right)$$

a) *undefined*

b) $\frac{4}{3}$

c) **0**

d) **1**

e) ∞

21) Given $y > 0$ and

$$\frac{dy}{dx} = \frac{3x^2 + 4x}{y}$$

If the point

$$(1, \sqrt{10})$$

is on the graph relating x and y , then what is y when $x = 0$?

a) **2**

b) **3**

- c) 1
- d) 6
- e) 10

22) Determine

$$\int_1^2 \frac{1}{\sqrt{4-t^2}} dt$$

- a) $\frac{1}{3} \pi$
- b) $\frac{1}{2} \pi$
- c) π
- d) $\frac{1}{6} \pi$
- e) $\frac{1}{4} \pi$

23) Determine

$$\int e^{(2x)} \sqrt{e^x + 1} dx$$

- a) $\frac{2}{5} (e^x + 1)^{(5/2)} - 3 (e^x + 1)^{(3/2)} + C$
- b) $e^{(2x)} (e^x + 1)^{(3/2)} + C$
- c) $\frac{2}{5} e^{\left(\frac{5}{2}x\right)} - 5 e^{\left(\frac{3}{2}x\right)} + C$
- d) $\frac{2}{5} (e^x + 1)^{(5/2)} + 3 (e^x + 1)^{(3/2)} + C$
- e) $\frac{2}{5} (e^x + 1)^{(5/2)} - \frac{2}{3} (e^x + 1)^{(3/2)} + C$

24) A particle's acceleration for $t \geq 0$ is given by

$$a(t) = 12t + 4$$

The particle's initial position is 2 and its velocity at $t = 1$ is 5. What is the position of the particle at $t = 2$?

- a) 4
- b) 12
- c) 10
- d) 20
- e) 16

25) Determine

$$\int_0^{\frac{1}{2}\pi} \sin(3x) \, dx + \int_0^{\frac{1}{6}\pi} \cos(3x) \, dx$$

- a) 1
- b) $-\frac{2}{3}$
- c) $\frac{2}{3}$
- d) 0
- e) -1

26) Determine the derivative of

$$f(x) = (\cos(2x - 2))^3$$

at $x = \pi/2$.

- a) $18 (\cos(\pi - 2))^2 \sin(\pi - 2)$
- b) $-6 (\cos(\pi - 2))^2$
- c) $-6 (\cos(\pi - 2))^2 \sin(\pi - 2)$
- d) $18 (\cos(\pi - 2))^2$
- e) $-6 \cos(\pi - 2)^2 \sin(\pi - 2)$

27) Compute the derivative of

$$f(x) = \int_0^{x^2} \ln(t^2 + 1) dt$$

a) $2x \ln(x^2 + 1)$

b) $\ln(x^4 + 1)$

c) $\frac{2x}{x^4 + 1}$

d) $\ln(x^2 + 1)$

e) $2x \ln(x^4 + 1)$

28) Determine

$$\frac{d}{dx} \ln(\ln(2 - \cos(x)))$$

a) $\frac{\cos(x)}{(2 - \cos(x)) \ln(2 - \cos(x))}$

b) $\frac{\sin(x)}{\ln(2 - \cos(x))}$

c) $\frac{\sin(x)}{(2 - \cos(x)) \ln(2 - \cos(x))}$

d) $\frac{\sin(x) (2 - \cos(x))}{\ln(2 - \cos(x))}$

e) $-\frac{\cos(x)}{\ln(2 - \cos(x))}$

AP Calculus Practice Exam

AB Version - Section I - Part B

Calculators ARE Permitted On This Portion Of The Exam

17 Questions - 50 Minutes

- 1) Give a value of c that satisfies the conclusion of the Mean Value Theorem for Derivatives for the function

$$f(x) = x^2 - x - 1$$

on the interval $[1,3]$.

- a) $\frac{3}{2}$
- b) $\frac{5}{4}$
- c) 2
- d) $\frac{1}{2}$
- e) $\frac{9}{4}$

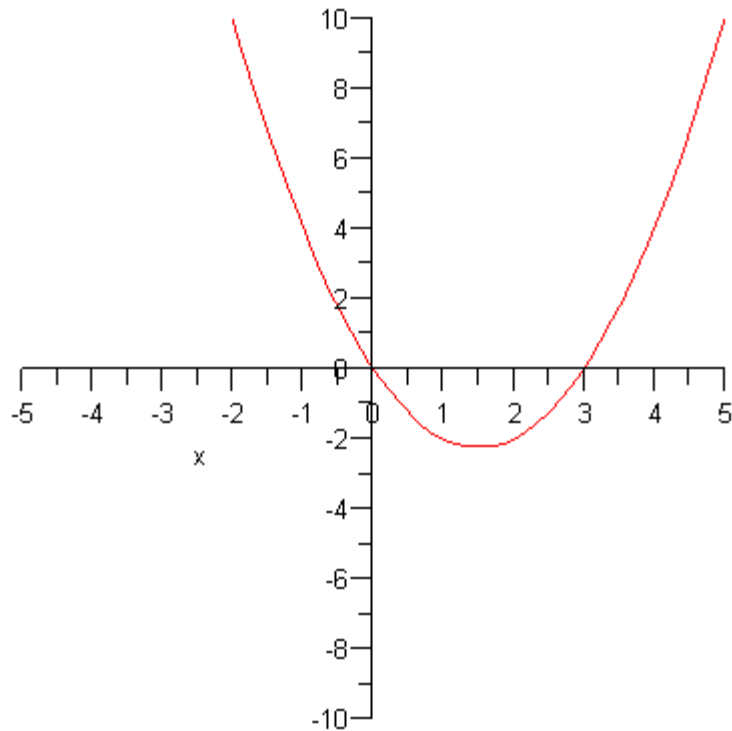
- 2) The function

$$f(x) = 4x^3 + 2e^{(2x)}$$

is invertible. Give the derivative of f^{-1} at $x = 2$.

- a) $\frac{1}{12 + 4e^2}$
- b) 4
- c) $\frac{1}{4}$
- d) $12 + 4e^2$
- e) 1

- 3) The **derivative** of f is graphed below.



Give a value of x where f has a local minimum.

- a) 3
- b) 0
- c) $\frac{3}{2}$
- d) *There is no such value of x .*
- e) 1

4) Let

$$f(x) = \begin{cases} -x + 5 & x < -2 \\ x^2 + 1 & -2 \leq x \text{ and } x \leq 1 \\ 2x^3 - 1 & 1 \leq x \end{cases}$$

Which of the following is (are) true?

- 1) f is continuous at $x = -2$.
- 2) f is differentiable at $x = 1$.
- 3) f has a local minimum at $x = 0$.
- 4) f has an absolute maximum at $x = -2$.

- a) 2 only
- b) 2 and 4
- c) 3 only
- d) 1 and 3
- e) 1 and 4
- 5) Given

$$\left[\int_0^{50} 4 f(x) \, dx = 3, \int_2^{50} f(x) \, dx = -2 \right]$$

Determine

$$\int_0^2 f(x) \, dx$$

a) $\frac{-5}{2}$

b) *There is not enough information.*

c) $\frac{11}{2}$

d) $\frac{11}{4}$

e) $\frac{-5}{4}$

6) Give the approximate location of a local maximum for the function

$$f(x) = 4x^3 + 3x^2 - 2x$$

a) (0.2287, - .2527)

b) (- .7287, 1.502)

c) (- .7287, 1.449)

d) (0.2287, - .1992)

e) (- .7287, - .2527)

7) Give the approximate average value of the function

$$f(x) = 4x \ln(3x)$$

over the interval [1,4].

a) 22.77

b) 17.77

c) 22.07647778

d) 20.77

e) 15.58

8) The region enclosed by the graphs of

$$[y = x^3 - 1, y = x - 1]$$

is rotated around the y-axis to generate a solid. What is the volume of the solid?

a) 2.356

b) 0.8380

c) 1.047

d) 1.676

e) 0.7855

9) What is the approximate instantaneous rate of change of the function

$$f(t) = \int_0^{4t} \cos(x) \, dx$$

at $t = \pi/11$?

a) 0.4158

b) 1.663

c) -3.638

d) $-.9095$

e) 0.1512

10) What is the error when the integral

$$\int_0^1 \sin(\pi x) \, dx$$

is approximated by the Trapezoidal rule with $n = 3$?

a) 0.011

b) 0.032

c) 0.109

d) 0.059

e) 0.051

11) The amount of money in a bank account is increasing at the rate of

$$R(t) = 10000 e^{(0.06 t)}$$

dollars per year, where t is measured in years. If $t = 0$ corresponds to the year 2005, then what is the approximate total amount of increase from 2005 to 2007.

a) \$32,560

b) \$4,500

c) \$18,350

d) \$16,250

e) \$21,250

12) A particle moves with acceleration

$$a(t) = 2t^2 - 4t$$

and its initial velocity is 0. For how many values of t does the particle change direction?

a) 0

b) 2

c) 3

d) 4

e) 1

13) At what approximate rate (in cubic meters per minute) is the volume of a sphere changing at the instant

when the surface area is 3 square meters and the radius is increasing at the rate of 1/4 meters per minute?

- a) 0.8105
- b) 0.7750
- c) 2.381
- d) 1.536
- e) 0.7500

14) A rectangle has one side on the x -axis and the upper two vertices on the graph of

$$y = e^{-9x^2}$$

Give a decimal approximation to the maximum possible area for this rectangle.

- a) 0.2858
- b) 0.4713
- c) $-.4713$
- d) 0.7773
- e) 0.2573

15) A rough approximation for $\ln(5)$ is 1.609. Use this approximation and differentials to approximate $\ln(521/100)$.

- a) 1.649
- b) 1.630
- c) 1.651
- d) 1.567
- e) 1.588

16) The function

$$f(x) = \begin{cases} nx^3 - x & x \leq 1 \\ mx^2 + 5 & 1 < x \end{cases}$$

is differentiable everywhere. What is n ?

- a) -17
- b) -14
- c) 13
- d) -9
- e) -11

17) Which of the following functions has a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = 2$?

- a) $f(x) = \ln(2x + 2)$
- b) $f(x) = \frac{2x^2 + 1}{x^2 - 1}$

c) $f(x) = e^{(x-1)} + 2$

d) $f(x) = \arctan(x-1) + 2 - \frac{1}{2}\pi$

e) $f(x) = \frac{x-1}{2x+2}$

- 1) c)
- 2) c)
- 3) a)
- 4) c)
- 5) d)
- 6) b)
- 7) d)
- 8) d)
- 9) b)
- 10) d)
- 11) e)
- 12) e)
- 13) e)
- 14) a)
- 15) c)
- 16) e)
- 17) b)
- 18) d)
- 19) d)
- 20) b)
- 21) a)
- 22) a)
- 23) e)
- 24) e)
- 25) c)
- 26) e)
- 27) e)
- 28) c)

- 1) c)
- 2) c)
- 3) a)
- 4) c)
- 5) d)
- 6) b)
- 7) d)
- 8) d)
- 9) b)
- 10) d)
- 11) e)
- 12) e)
- 13) e)
- 14) a)
- 15) c)
- 16) e)
- 17) b)