AP Calculus Practice Exam
AB Version - Section I - Part A

Calculators ARE NOT Permitted On This Portion Of The Exam
28 Questions - 55 Minutes

1) Give $f(g(1))$, given that

$$\left[f(x) = 2x + 2,\ g(x) = -\frac{x}{2 + x^2}\right]$$

a) $\frac{-8}{9}$

b) $\frac{7}{3}$

c) 2

d) $\frac{4}{3}$

e) $\frac{-2}{9}$

2) Find the slope of the tangent line to the graph of f at $x = 4$, given that

$$f(x) = -x^2 + 4\sqrt{x}$$

a) -8

b) -10

c) -9

d) -5

e) -7

3) Determine

$$\lim_{x \to \infty} \left(\frac{-2x^3 + x}{-4x^5 + 2x^2 + 2}\right)$$

a) ∞

b) $\mathbf{0}$

c) $\dfrac{1}{2}$

d) $\dfrac{3}{10}$

e) $\mathbf{1}$

4) Let

$$f(x) = x^3$$

A region is bounded between the graphs of y = -1 and y = f(x) for x between -1 and 0, and between the graphs of y = 1 and y = f(x) for x between 0 and 1. Give an integral that corresponds to the area of this region.

a) $\displaystyle\int_{-1}^{1} \left(1 - x^3\right)\, dx$

b) $\displaystyle\int_{0}^{1} 2\left(1 - x^3\right)\, dx$

c) $\displaystyle\int_{0}^{1} 2\left(1 + x^3\right)\, dx$

d) $\displaystyle\int_{-1}^{1} \left(1 + x^3\right)\, dx$

e) $\displaystyle\int_{0}^{1} \left(-x^3 - 1\right)\, dx$

5) Given that

$$5\,x^3 - 4\,x\,y - 2\,y^2 = 1$$

Determine the change in y with respect to x.

a) $-\dfrac{15\,x^2 - 4}{-4 - 4\,y}$

b) $-\dfrac{15\,x^2 - 4\,y}{-4 - 4\,y}$

c) $-\dfrac{15x^2-4}{-4x-4y}$

d) $-\dfrac{10x-4y}{-4x-2}$

e) $-\dfrac{15x^2-4y}{-4x-4y}$

6) Compute the derivative of

$$-4\sec(x)+2\csc(x)$$

a) $-4\sec(x)\tan(x)-2\csc(x)\cot(x)$

b) $-4\csc(x)-2\sec(x)$

c) $-4\,(\sec(x))^2-2\,(\csc(x))^2$

d) $-4\sec(x)\tan(x)+2\csc(x)\cot(x)$

e) $-4\,(\tan(x))^2-2\,(\cot(x))^2$

7) Compute

$$\int_0^{\frac{1}{2}}\frac{4}{1+4t^2}\,dt$$

a) $-\pi$

b) $\dfrac{3}{2}\pi$

c) $\dfrac{1}{2}\pi$

d) π

e) 0

8) Determine

$$\frac{d}{dx}\left(\frac{4x^4 - 2x}{4x^4 + 2x}\right)$$

a) $\dfrac{24x^2 - 1}{\left(4x^3 + 2\right)^2}$

b) $\dfrac{48x^2 - 1}{\left(4x^3 + 2\right)^2}$

c) $\dfrac{12x^2}{\left(2x^3 + 1\right)^2}$

d) $\dfrac{24x^2}{\left(4x^3 + 2\right)^2}$

e) $\dfrac{6x^2}{\left(4x^3 + 2\right)^2}$

9) Give the equation of the normal line to the graph of

$$y = 2x\sqrt{x^2 + 8} + 2$$

at the point (0 , 2).

a) $x - 4\sqrt{2}\,y = -8\sqrt{2}$

b) $x + 4\sqrt{2}\,y = 8\sqrt{2}$

c) $4\sqrt{2}\,x + y = 2$

d) $-4\sqrt{2}\,x + y = 2$

e) $x + 4\sqrt{2}\,y = 2$

10) Determine the concavity of the graph of

$$f(x) = 3\sin(x) + 4\,(\cos(x))^2$$

at $x = \pi$.

a) 8

b) -10

c) 4

d) -8

e) -6

11) Compute

$$\int 4\,x^2\,\sqrt{x^3+4}\;dx$$

a) $\frac{8}{3}\left(x^3+4\right)^{(3/2)}+C$

b) $\frac{16}{9}\left(x^3+4\right)^{(3/2)}+C$

c) $\frac{8}{9}\left(x^3+4\right)^{(3/2)}+C$

d) $\frac{4}{3}\,\frac{1}{\sqrt{x^3+4}}+C$

e) $\frac{8}{3}\,\frac{1}{\sqrt{x^3+4}}+C$

12) Give the value of *x* where the function

$$f(x)=x^3-9\,x^2+24\,x+4$$

has a local maximum.

a) 4

b) -2

c) 2

d) -4

e) 3

13) The slope of the tangent line to the graph of

$$4\,x^2+c\,x-2\,e^y=-2$$

at $x = 0$ is 4. Give the value of c.

a) -2

b) 4

c) 8

d) -4

e) -8

14) Compute

$$\int \left(5^x + 2\, e^{(5 \ln(x))}\right) dx$$

a) $\dfrac{5^x}{\ln(5)} + \dfrac{2}{5}\, e^{(5\ln(x))} + C$

b) $5^x \ln(5) + \dfrac{2}{5}\, e^{(5\ln(x))} + C$

c) $5^x \ln(5) + \dfrac{2}{5}\, \dfrac{e^{(5\ln(x))}}{x} + C$

d) $\dfrac{5^x}{\ln(5)} + \dfrac{2}{5}\, x^5 + C$

e) $\dfrac{5^x}{\ln(5)} + \dfrac{1}{3}\, x^6 + C$

15) What is the average value of the function

$$g(x) = (2x + 3)^2$$

on the interval from *x* = -3 to *x* = -1?

a) $\dfrac{7}{3}$

b) -4

c) 5

d) $\dfrac{14}{3}$

e) 3

16) Compute

$$\lim_{t \to 0} \left(\frac{\tan\left(\frac{1}{4}\pi + t\right) - \tan\left(\frac{1}{4}\pi\right)}{t} \right)$$

a) 1

b) $\frac{1}{4}\pi$

c) π

d) 2

e) -1

17) Find the instantaneous rate of change of

$$f(t) = \left(2t^3 - 3t + 4\right)\sqrt{t^2 + 3t + 4}$$

at $t = 0$.

a) -3

b) $\frac{-3}{4}$

c) 0

d) -4

e) $\frac{-5}{4}$

18) Compute

$$\frac{d}{dx} 2^{\cos(x)}$$

a) $\sin(x)\ 2^{\cos(x)}\ \ln(2)$

b) $-\sin(x)\ 2^{\cos(x)}\ \ln(2)$

c) $-\sin(x)\ 2^{\cos(x)}$

d) $-\frac{\sin(x)\ 2^{\cos(x)}}{\ln(2)}$

e) $\frac{\sin(x)\ 2^{\cos(x)}}{\ln(2)}$

19) A solid is generated by rotating the region enclosed by the graph of

$$y = \sqrt{x}$$

the lines $x = 1$, $x = 2$, and $y = 1$, about the x-axis. Which of the following integrals gives the volume of the solid?

a) $\int_1^2 \pi\ (x - 1)\ dx$

b) $\int_1^2 \pi\ (x - 1)^2\ dx$

c) $\int_1^2 \pi\ \left(\sqrt{x} - 1\right)^2\ dx$

d) $\int_1^2 \pi\ (2 - x)^2\ dx$

e) $\int_1^2 \pi\ \left(2 - \sqrt{x}\right)^2\ dx$

20) Compute

$$\lim_{x \to 0} \left(-\frac{4x}{\sin(2x)} + \frac{x}{\cos(2x)} \right)$$

a) ∞

b) 0

c) $\frac{-5}{2}$

d) -2

e) *undefined*

21) Given $y > 0$ and

$$\frac{dy}{dx} = \frac{3x^2 + 4x}{y}$$

If the point

$$\left(1, \sqrt{10}\right)$$

is on the graph relating x and y, then what is y when $x = 0$?

a) 3

b) 2

c) 1

d) 6

e) 10

22) Determine

$$\int_{1}^{2} \frac{1}{\sqrt{4-t^2}}\, dt$$

a) $\frac{1}{2}\pi$

b) $\frac{1}{3}\pi$

c) π

d) $\frac{1}{6}\pi$

e) $\frac{1}{4}\pi$

23) Determine

$$\int e^{(2x)} \sqrt{e^x + 1}\, dx$$

a) $\frac{2}{5}\left(e^x + 1\right)^{(5/2)} - \frac{2}{3}\left(e^x + 1\right)^{(3/2)} + C$

b) $e^{(2x)}\left(e^x + 1\right)^{(3/2)} + C$

c) $\frac{2}{5}\,e^{\left(\frac{5}{2}x\right)} - 5\,e^{\left(\frac{3}{2}x\right)} + C$

d) $\frac{2}{5}\left(e^x + 1\right)^{(5/2)} - 3\left(e^x + 1\right)^{(3/2)} + C$

e) $\frac{2}{5}\left(e^{x}+1\right)^{(5/2)}+3\left(e^{x}+1\right)^{(3/2)}+C$

24) A particle's acceleration for $t \geq 0$ is given by

$$a(t) = 12t + 4$$

The particle's initial position is 2 and its velocity at $t = 1$ is 5. What is the position of the particle at $t = 2$?

a) 10

b) 12

c) 16

d) 4

e) 20

25) Determine

$$\int_{0}^{\frac{1}{2}\pi} \sin(3x)\,dx + \int_{0}^{\frac{1}{6}\pi} \cos(3x)\,dx$$

a) -1

b) 1

c) 0

d) $\frac{2}{3}$

e) $\frac{-2}{3}$

26) Determine the derivative of

$$f(x) = (\cos(2x-4))^{3}$$

at $x = \pi/2$.

a) $-6\,(\cos(\pi-4))^{2}$

b) $-6\cos(\pi-4)^{2}\sin(\pi-4)$

c) $-6\,(\cos(\pi-4))^{2}\sin(\pi-4)$

d) $18\,(\cos(\pi-4))^{2}\sin(\pi-4)$

e) $18\ (\cos(\pi - 4))^2$

27) Compute the derivative of

$$f(x) = \int_0^{x^2} \ln\left(t^2 + 1\right)\, dt$$

a) $\ln\left(x^4 + 1\right)$

b) $2\,x\,\ln\left(x^4 + 1\right)$

c) $\dfrac{2\,x}{x^4 + 1}$

d) $2\,x\,\ln\left(x^2 + 1\right)$

e) $\ln\left(x^2 + 1\right)$

28) Determine

$$\frac{d}{dx} \ln(\ln(2 - \cos(x)))$$

a) $\dfrac{\cos(x)}{(2 - \cos(x))\ \ln(2 - \cos(x))}$

b) $\dfrac{\sin(x)}{\ln(2 - \cos(x))}$

c) $\dfrac{\sin(x)}{(2 - \cos(x))\ \ln(2 - \cos(x))}$

d) $\dfrac{\sin(x)\ \ (2 - \cos(x))}{\ln(2 - \cos(x))}$

e) $-\dfrac{\cos(x)}{\ln(2 - \cos(x))}$

AP Calculus Practice Exam
AB Version - Section I - Part B

Calculators ARE Permitted On This Portion Of The Exam
17 Questions - 50 Minutes

1) Give a value of *c* that satisfies the conclusion of the Mean Value Theorem for Derivatives for the function

$$f(x) = -2x^2 - x + 2$$

on the interval [1,3].

a) $\frac{9}{4}$

b) $\frac{3}{2}$

c) $\frac{1}{2}$

d) 2

e) $\frac{5}{4}$

2) The function

$$f(x) = 3x^3 + 2e^{(2x)}$$

is invertible. Give the derivative of f^{-1} at $x = 2$.

a) $9 + 4e^2$

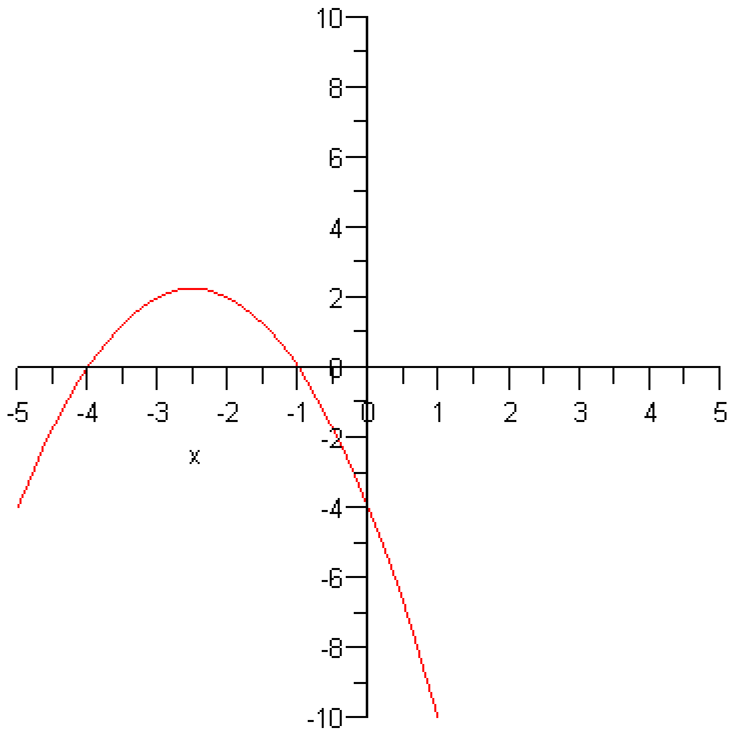
b) 4

c) $\frac{1}{9 + 4e^2}$

d) 1

e) $\frac{1}{4}$

3) The **derivative** of *f* is graphed below.



Give a value of *x* where *f* has a local maximum.

a) -4

b) -1

c) $\frac{-5}{2}$

d) *There is no such value of* x.

e) 1

4) Let

$$f(x) = \begin{cases} -x+5 & x < -2 \\ x^2+1 & -2 \le x \textbf{ and } x \le 1 \\ 2x^3-1 & 1 \le x \end{cases}$$

Which of the following is (are) true?

1) *f* is continuous at *x* = -2.
2) *f* is differentiable at *x* = 1.
3) *f* has a local minimum at *x* = 0.
4) *f* has an absolute maximum at *x* = -2.

a) 2 and 4
b) 3 only
c) 2 only
d) 1 and 3
e) 1 and 4
5) Given

$$\left[\int_0^{50} 3\, f(x)\, dx = 3,\ \int_2^{50} f(x)\, dx = -4\right]$$

Determine

$$\int_0^2 f(x)\, dx$$

a) 10

b) -3

c) *There is not enough information.*

d) -6

e) 5

6) Give the approximate location of a local maximum for the function

$$f(x) = 3\,x^3 + 5\,x^2 - 3\,x$$

a) $(-1.357,\ 5.779)$

b) $(0.2457,\ -.3908)$

c) $(-1.357,\ 5.713)$

d) $(0.2457,\ -.3216)$

e) $(-1.357,\ -.3908)$

7) Give the approximate average value of the function

$$f(x) = 4\,x\,\ln(2\,x)$$

over the interval [1,4].

a) 19.71

b) 12.54

c) 16.71

d) 18.02182670

e) 18.71

8) The region enclosed by the graphs of

$$[\,y = x^3 - 1,\ y = x - 1\,]$$

is rotated around the *y*-axis to generate a solid. What is the volume of the solid?

a) 0.8380

b) 0.7855

c) 1.676

d) 1.047

e) 2.356

9) What is the approximate instantaneous rate of change of the function

$$f(t) = \int_0^{3t} \cos(x)\, dx$$

at $t = \pi/7$?
a) $-.9009$

b) -7.207

c) 3.473

d) 0.4341

e) -1.030

10) What is the error when the integral

$$\int_0^1 \sin(\pi x)\, dx$$

is approximated by the Trapezoidal rule with $n = 3$?
a) 0.011

b) 0.032

c) 0.109

d) 0.059

e) 0.051

11) The amount of money in a bank account is increasing at the rate of

$$R(t) = 10000\, e^{(0.06\, t)}$$

dollars per year, where t is measured in years. If $t = 0$ corresponds to the year 2005, then what is the approximate total amount of increase from 2005 to 2007.
a) $18,350
b) $4,500
c) $21,250
d) $32,560
e) $16,250
12) A particle moves with acceleration

$$a(t) = 3\, t^2 - 2\, t$$

and its initial velocity is 0. For how many values of t does the particle change direction?
a) 3

b) 2

c) 1

d) 0

e) 4

13) At what approximate rate (in cubic meters per minute) is the volume of a sphere changing at the instant when the surface area is 5 square meters and the radius is increasing at the rate of 1/3 meters per minute?
a) 5.271

b) 1.700

c) 1.667

d) 1.080

e) 2.714

14) A rectangle has one side on the *x*-axis and the upper two vertices on the graph of

$$y = e^{(-2x^2)}$$

Give a decimal approximation to the maximum possible area for this rectangle.

a) 1.649

b) 1.

c) $-1.$

d) 0.5458

e) 0.6065

15) A rough approximation for ln(5) is 1.609. Use this approximation and differentials to approximate ln(128/25).

a) 1.633

b) 1.621

c) 1.632

d) 1.585

e) 1.597

16) The function

$$f(x) = \begin{cases} n x^3 - x & x \leq 1 \\ m x^2 + 5 & 1 < x \end{cases}$$

is differentiable everywhere. What is *n*?

a) -9

b) 13

c) -17

d) -11

e) -14

17) Which of the following functions has a vertical asymptote at *x* = -1 and a horizontal asymptote at *y* = 2?

a) $f(x) = \dfrac{2x^2 + 1}{x^2 - 1}$

b) $f(x) = \ln(2x + 2)$

c) $f(x) = e^{(x-1)} + 2$

d) $f(x) = \arctan(x - 1) + 2 - \dfrac{1}{2}\pi$

e) $f(x) = \dfrac{x-1}{2x+2}$

1) d)
2) e)
3) b)
4) b)
5) e)
6) a)
7) c)
8) c)
9) b)
10) d)
11) c)
12) c)
13) c)
14) e)
15) a)
16) d)
17) a)
18) b)
19) a)
20) d)
21) b)
22) b)
23) a)
24) c)
25) d)
26) c)
27) b)
28) c)

1) d)
2) e)
3) b)
4) b)
5) e)
6) a)
7) c)
8) c)
9) b)
10) d)
11) c)
12) c)
13) c)
14) e)
15) a)
16) d)
17) a)

SECTION II

Time: 1 hour and 30 minutes
Percent of total grade: 50

Part A: 45 minutes, 3 problems
(A graphing calculator is required for some problems or parts of problems.)

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

Part B: 45 minutes, 3 problems
(No calculator is allowed for these problems.)

During the timed portion for Part B, you may keep Part A, and continue to work on the problems in Part A without the use of any calculator.

GENERAL INSTRUCTIONS FOR SECTION II PART A AND PART B
For each part of Section II, you may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

- YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM WITH A PENCIL OR PEN IN THE SPACE PROVIDED FOR THAT PART IN THE PINK EXAM BOOKLET. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. You will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS AB
SECTION II, Part A
Time --- 45 minutes
Number of Problems --- 3

A graphing calculator is required for some problems or parts of the problems.

1. Let *S* be the region in the first quadrant bounded by the graphs of $y = e^{-x^2}$ and $y = 2x^2$, and the *y*-axis.

a) Find the area of the region *S*.

b) Find the volume of the solid generated when the region *S* is rotated about the x-axis.

c) The region *S* is the base of a solid for which each cross section perpendicular to the *x*-axis is a semi-circle with diameter in the xy plane. Find the volume of this solid.

2. Let $F(x) = \int_0^x \sin^2(t)\,dt$ on the interval $[0,\pi]$.

a) Approximate $F(\pi)$ using the trapezoid rule with *n* = 4.

b) Find $F'(x)$.

c) Find the average value of $F'(x)$ on the interval $[0,\pi]$.

3. A particle moves along the *x*-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t - 4$. At time $t = 1$, the velocity of the particle is $v(1) = 7$ and its position is $x(1) = 4$.

a) Write an expression for the velocity of the particle $v(t)$.

b) At what values of *t* does the particle change direction?

c) Write an expression for the position $x(t)$ of the particle.

d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

CALCULUS AB
SECTION II, Part B
Time --- 45 minutes
Number of Problems --- 3

No Calculator is allowed for these problems.

4. Consider the graph of $f(x) = x^4 - 6x^2$.

a) Find the relative maxima and minima (both *x* and *y* coordinates).
b) Find the coordinates of the point(s) of inflection.
c) Determine the interval(s) on which the function is increasing.
d) Determine the interval(s) on which the function is concave up.

5. Consider the equation $x^2 + 3y^2 + xy = 3$.

a) Write an expression for the slope of the curve at any point (x, y).

b) Find the equation of the normal line to the curve at the point $(0,1)$.

c) What is $\frac{d^2y}{dx^2}$ at $(0,1)$?

d) If $(a, -2a)$ is a point on the curve, determine the nature of the line tangent to the curve at that point.

6. Water is poured into a tank in the shape of a right circular cone, standing on its vertex. The height of the cone is 40 feet and its radius is 8 feet. The water level in the tank is increasing at a constant rate of 2 feet per second.

a) Find an expression for the volume of the water (in ft^3) in the tank in terms of its height.

b) How fast (in ft/sec) is the water being poured into the tank at the instant the depth of the water is 10 feet?

c) How fast is the area of the surface of the water increasing at the instant the depth of the water is 10 feet?

END OF EXAM