Free Response Set #1

Let $f(x)=\begin{cases} x^2-a^2x & if \;\; x<2 \\ 4-2x^2 & if \;\; x\geq 2 \end{cases}$.

(a) Find $\lim\limits_{x\to 2^-} f(x)$.

(b) Find $\lim\limits_{x\to 2^+} f(x)$.

(c) Find all values of a that make f continuous at 2. Justify your answer.

Free Response Set #2

Let $f(x) = 2x - x^2$.

(a) Find $f(4)$

(b) Find $f(4+h)$

(c) Find $\frac{f(4+h) - f(4)}{h}$

(d) Find the instantaneous rate of change of f at $x = 4$.

Free Response Set #3

Let $f(x) = x^4 - 4x^2$.

(a) Find all the points where f has horizontal tangents.

(b) Find an equation of the tangent line at $x = 1$.

(c) Find and equation of the normal line at $x = 1$.

Free Response Set #4

A particle moves along the y-axis with position given by $s(t) = -\frac{1}{2}\cos t + \frac{5}{2}$ for $t \geq 0$.

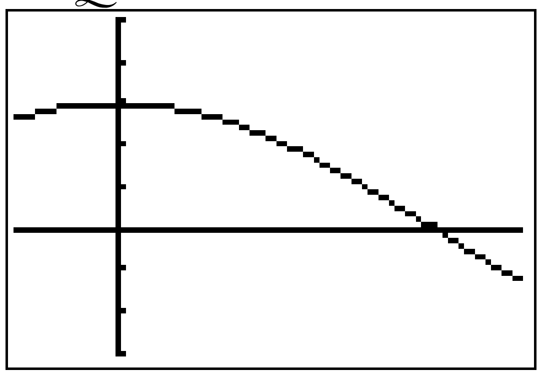
(a) In which direction (up or down) is the particle moving at $t = 1.5$? Why?

(b) Find the acceleration of the particle at $t = 1.5$. Is the velocity of the particle increasing or decreasing at
$t = 1.5$? Why or why not?

(c) Find the displacement of the particle during the first two seconds.

Free Response Set #5

Let *f* be the function given by $f(x) = 3\cos x$. As shown below, the graph of f crosses the *y*-axis at point *P* and the *x*-axis at point *Q*.



(a) Write an equation for the line passing through points P and Q.

(b) Write an equation for the line tangent to the graph of *f* at point *Q*. Show the analysis that leads to your equation.

(c) Find the *x*-coordinate of the point on the graph of *f*, between points *P* and *Q*, at which the line tangent to the graph of *f* is parallel to line *PQ*.

Free Response Set #6 – NO CALCULATOR

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all point on the curve whose x-coordinate is 1, and write and equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

Free Response Set #7 – CALCULATOR USE OK

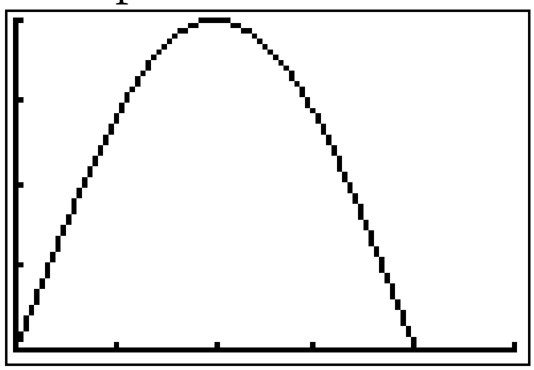
A particle moves along the x-axis so that its velocity at time t is given by $v(t) = - \;\, t + 1 \;\, \sin\left(\frac{t^2}{2}\right)$.
At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0<t<3$ when the particle changes direction. Justify your answer.

Free Response Set #8 – CALCULATOR USE OK

Let *f* be the function given by $f(x) = 4\sin x$. As shown, the graph of *f* passes through the point $M\left(\frac{\pi}{2}, 4\right)$ and crosses the *x*-axis at point $N\ \pi, 0$.



(a) Write an equation for the line passing through points *M* and *N*.

(b) Write and equation for the line tangent to the graph of *f* at point *N*. Show the analysis that leads to your equation.

(c) Find the x-coordinate of the point on the graph of, between points M and N, at which the line tangent to the graph of f is parallel to line MN.

Free Response Set #9 – CALCULATOR USE OK

Let f be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x \to -\infty} f(x)$ *and* $\lim_{x \to \infty} f(x)$.

(b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

(c) What is the range of f?

(d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

Free Response Set #10 – NO CALCULATOR

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.

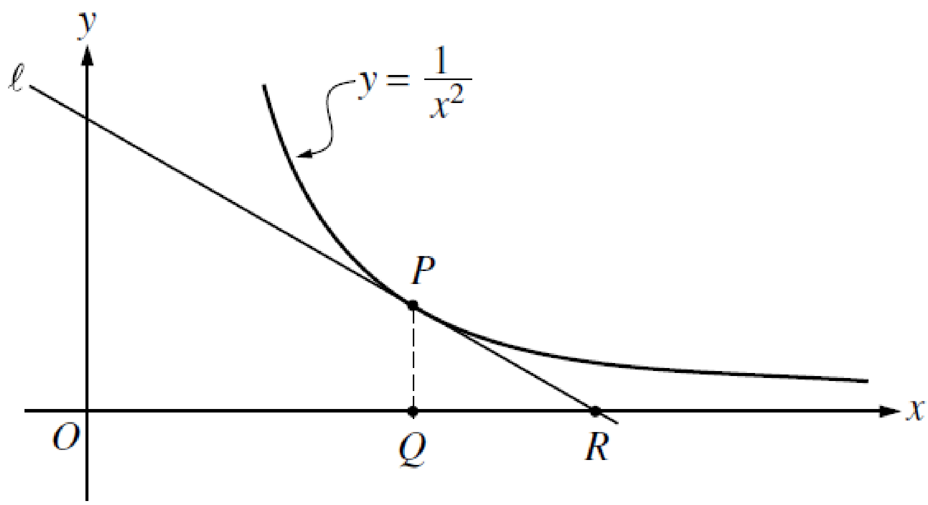
(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify that your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

Free Response Set #11 – NO CALCULATOR



In the figure above, line l is tangent to the graph of $y = \frac{1}{x^2}$ at point P, with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line l crosses the x-axis at point R, with coordinates $(k, 0)$.

(a) Find the value of k when $w = 3$.

(b) For all $w > 0$, find k in terms of w.

(c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

(d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

Free Response Set #12 CALCULATOR USE OK

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

(a) Use a midpoint Riemann sum with 4 subdivision of equal length to approximate $\int_0^{24} R(t)dt$. Using correct units, explain the meaning of your answer in terms of water flow.

(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

Free Response Set #13 Calculator Use OK

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ C°	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius C°, of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure. Using correct units, explain the meaning of your answer in terms of water flow.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire.

(c) Find $\int_0^8 T'(x)dx$, and indicate units of measure.

Explain the meaning of $\int_0^8 T'(x)dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

Free Response Set #14

The temperature outside a house during a 24-hour period is given by $F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right),\ 0 \le t \le 24$, where *F(t)* is measured in degree Fahrenheit and *t* is measure in hours.

(a) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$?

(b) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of *t* was the air conditioner cooling the house?

(c) The cost of cooling the house accumulates at the rate of $0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

Free Response Set #15 Calculator Use OK

A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1} e^{t}$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$).

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer

(c) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer

Free Response Set #16

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

Free Response Set #17

A graphing calculator is required for some problems or parts of problems.

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

Free Response Set #18

A graphing calculator is required for some problems or parts of problems.

Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

a. Find the area of R

b. The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.