AP CALCULUS
REVIEW WORKSHEETS

ACCELERATION, VELOCITY AND POSITION

In problems 1-4, determine the position function, x(t), from the given information.

1. $v(t) = t^3 + 4t$, x(0) = 5
2. $v(t) = 5 - \dfrac{1}{t^2}$, x(1) = 0 $(t \geq 1)$
3. $a(t) = t$, v(0) = 3, x(0) = -2
4. $a(t) = 2 - t^2$, v(0) = 15, x(0) = 3

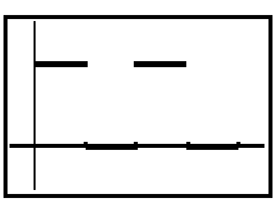
5. An object moving along an x-coordinate axis with its scale measured in meters has a velocity of $6t^{2/3}$ m/sec in the positive direction at time t. At t = 1 the object is at x = 0. Where is the object at t = 8?
6. A store takes in money at the rate of $10t - t^2$ dollars/hour t hours after it has opened. How much does it take in during its ten-hour day?
7. Water flows into a tank at the rate of $10t^4 + 100t + 10$ ft^3/day at time t (days) and leaks out at the constant rate of 6 ft^3/day. At t = 0 the tank contains 25 ft^3 of water. How much does the tank contain one day later?
8. An object's velocity in the positive direction on the x-axis is $v(t) = t^2 - t$ yards/sec at time t (seconds). At t = 0 it is at x = 10 yards.
   a) When is the velocity zero and where is it at those times?
   b) At what time(s) is it again at x = 10 yards?
9. An object's acceleration on the x-axis is $12t^2$ m/sec^2 at time t (seconds). At t = 0 it is at x = 0 meters and its velocity is 0 m/sec^2. How long does it take to reach x = 10 meters and what is its velocity at that time?

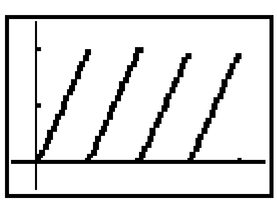
NET DISTANCE/TOTAL DISTANCE

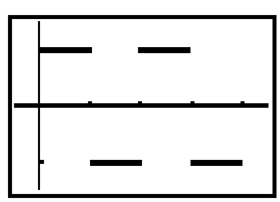
The function, v(t) is the velocity in meters per second of a body moving along a coordinate line. Find :
a) the total distance traveled by the body during the given time interval
b) the shift in the body's position

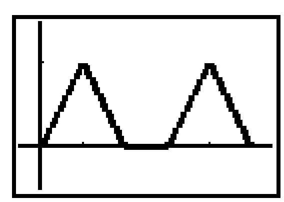
1. $v(t) = 5\cos t \quad 0 \leq t \leq 2\pi$
2. $v(t) = 6\sin 3t \quad 0 \leq t \leq \pi/2$
3. $v(t) = 49 - 9.8t \quad 0 \leq t \leq 10$
4. $v(t) = 6t^2 - 18t + 12 \quad 0 \leq t \leq 3$
5. $v(t) = t/2 - 1 \quad 0 \leq t \leq 6$

The following graphs are velocity graphs of four bodies moving on coordinate lines. Find the distance traveled by each body and the position shift for the given time interval.

6 

7. 

8. 

9. 

SEPARABLE DIFFERENTIAL EQUATIONS

Solve the differential equations and initial value problems in problems 1-7.

1. $\frac{dy}{dt} = y\sin t$, y(0)=1 2. $y^2dx + \left(x^2+1\right)dy = 0$ 3. $\sqrt{1-y^2}\,dx - dy = 0$ 4. $\frac{dy}{dx} = \cos^2 y$

5. $xdy + y^2dx = 0$, y(1)=2 6. $(x+1)ydx + x(y-1)dy = 0$ 7. $\left(y^2+1\right)dx - y\left(x^2+1\right)dy = 0$

8. The thickness x(t) (in inches) of ice forming on a lake satisfies the differential equation $\frac{dx}{dt} = \frac{3}{x}$. At time t = 0 days the ice is one inch thick. When is the ice two inches thick?

9. A newly created lake is stocked with 400 fish. Thereafter, the population increases at the rate of $\sqrt{P}$ fish per month. Write an expression P(t) to model the population of the fish at any time t.

L'HOPITAL'S RULE

Find the following limits. Support all answers.

1. $\lim\limits_{x\to 0} \frac{2^x-1}{x}$ 2. $\lim\limits_{x\to\infty} \frac{e^x}{x^2}$ 3. $\lim\limits_{x\to 0} \frac{\tan x - x}{x^3}$ 4. $\lim\limits_{x\to\pi} \frac{\sin x}{1-\cos x}$

5. $\lim\limits_{x\to 0^+} x^2\ln x$ 6. $\lim\limits_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x^2}\right)$ 7. $\lim\limits_{x\to 2^+}\left(\frac{1}{x^2-4}-\frac{x}{x-2}\right)$ 8. $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x$

9. $\lim\limits_{x\to 0^+}(\sin x)^x$ 10. $\lim\limits_{x\to 0}(1-4x)^{1/x}$ 11. $\lim\limits_{x\to\infty}(x+2)^x$ 12. $\lim\limits_{x\to 0}\frac{\sin 2x}{\sin 3x}$

SOLUTIONS BY EULER'S METHOD

1. Let $\frac{dy}{dx} = 2x$ and suppose y = .5 when x = 1.
   a) Use Euler's method to estimate the solution of the differential equation when x = 1.5. Use 5 steps, i.e., let n = 5. Calculate your answer by hand and list all intermediate answers.
   b) Use 10 steps and calculate the same answer.
   c) Did you overestimate or underestimate the exact answer?
2. Let $\frac{dy}{dx} = y$ and suppose y = 1 when x = 0.
   a) Use Euler's method to estimate the solution when x = 1 by using 5 steps. Show all intermediate answers.
   b) Use a calculator program and 100 steps to estimate the solution when x = 1.
   c) Have you overestimated or underestimated? Explain your reasoning.
   d) Construct a slope field for $\frac{dy}{dx}$. Do you think your reasoning for part c) was correct?
3. If P dollars is invested at 8% per year compounded continuously and A dollars is the amount of the investment after t years, then $\frac{dA}{dt} = .08A$. Suppose \$500 is invested initially. Use Euler's method to estimate the amount of the investment after 5 years using steps of : a) one year. b) six months. c) one month.
4. The velocity of a particle moving along a horizontal line at any time t is given by $v(t) = e^{-t^2}$, where v is measured in ft/sec and t is measured in seconds. If the particle is on the one foot point when t = 0, estimate the position of the particle using Euler's method at the end of 4 seconds. Use 16 steps.
5. Suppose the rate of growth of a predator population is given by $\frac{dP}{dt} = .001P(200-P)$, where P is the population of the predator in a confined area at time t years. If the population of the predator numbers 50 in 2001, how many will be in the predator population at the same time in 2003?

INTEGRAL AS AN ACCUMULATOR

1. Suppose a car is moving with non-decreasing speed according to the table below.

TIME (sec)	0	2	4	6	8	10
SPEED (ft/sec)	30	36	40	48	54	60

a) What is an upper estimate for the distance traveled in the first two seconds?
b) Determine upper and lower estimates for the change in position for the first 10 seconds.

2. Suppose the density of cars, in cars/mile, for the first 30 miles along the Mass Pike from Boston during certain hours of the day can be modeled by: $p(x) = 100\left(2 - \sqrt[3]{0.1x + 0.2}\right)$
where x represents the number of miles from Boston.
a) Write a function that gives the number of cars on the Mass Pike from Boston to a point x miles from Boston.
b) Use this function to determine the number of cars on this 30 mile stretch of road.

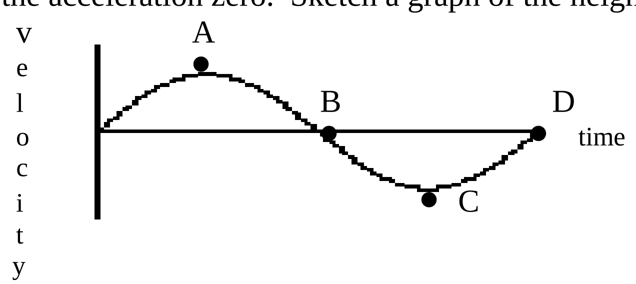
3. If $y' = \ln(x + \sin x)$ and y(2) = 3, what is y(3)?

4. Oil is leaking from a tanker at the rate of $R(t) = 2000e^{-0.2t}$ gallons/hour, where t is measured in hours. How much oil has leaked out of the tanker after 10 hours?

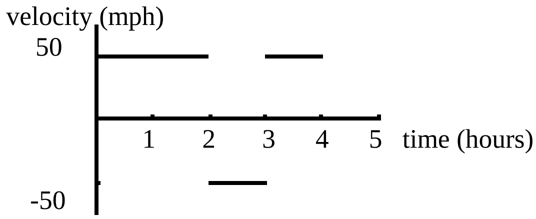
5. The EPA was recently asked to investigate a spill of radio active iodine. Measurements showed the ambient radiation levels at the site to be four times the maximim acceptable limit, so the EPA ordered an evacuation of the surrounding neighborhood. It is known that the level of radiation from an iodine source decreases acccording to the formula $R(t) = R_0 e^{-0.004t}$
where R is the radiation level in millirems/hour at time t, R_0 is the intial radiation level, and t is the time in hours.
a) How long will it take for the site to reach an acceptable level of radiation?
b) How much total radiation (in millirems) will have been emitted by that time, assuming the maximum acceptable limit is 0.6 millirems/hour.

6. Suppose the density of a circular oil slick on the surface of the water is $p(r) = \frac{100}{1+r^2}\ kg/m^2$.
a) Given that the slick extends from r = 0 to r = 1000 m, write a Reimann sum that approximates the total mass of the oil.
b) Determine the exact value of the mass.
c) Within what radius is 75% of the mass?

7. The velocity of a cork bobbing up and down on the waves in the sea is given by the graph below. Upward is considered positive. Describe the motion of the cork at each of the labeled points. At which point(s), if any, is the acceleration zero. Sketch a graph of the height of the cork above the sea floor as a function of time.



8. Suppose the velocity, v, of a car traveling along a straight road is approximated by the graph below. Find a formula for the distance of the car from its starting point as a function of time in hours. Draw a graph of this function.



MORE INTEGRALS AND LIMITS

Evaluate the integrals.

1. $\int_{-\infty}^{0} xe^{-x^2}\,dx$ 2. $\int_{0}^{2} \frac{1}{1-x}dx$ 3. $\int_{-\infty}^{\infty} \frac{e^x}{e^{2x}+1}dx$

4. Compute the area bounded by y = 0 and $y = \frac{1}{x^2+16}$.

Evaluate the limits.

5. $\lim_{x\to 1} \frac{x^3+x^2-2}{\ln x}$ 6. $\lim_{x\to 0^+} x^2 \ln x$ 7. $\lim_{x\to\infty} x^2 e^x$ 8. $\lim_{x\to 0^+} (1-3x)^{1/x}$

9. Using the following information

a(t) = 2t – 7 m/sec^2, v(1) = 4 m/sec and x(0) = 5 m

find: a) x(t) b) the total distance and the net distance traveled on the interval $0 \le t \le 3$.

Solve the following differential equations and initial value problems. Solve each for y.

10. $\frac{dy}{dx} = 5 - y$; y(0) = 10 11. $(y^2+1)dx + (2x^2y + 2y)dy = 0$; y(0) = 0

12. Use Newton's method to find all of the solutions to the nearest thousandth for $\sin x = x^5$.

13. A table of values for a continuous function, f(x) is given below. Use that table to approximate the integral. State which method used.

$$\int_{1}^{3} f(x)dx$$

X	1	1.5	2	2.5	3
f(x)	2	3	5	6	8

CRW answers

Tuesday, April 07, 2009
12:08 PM

A/V/T: 1. $x(t)=\frac{t^4}{4}+2t^2+5$ 2. $x(t)=5t+\frac{1}{t}-6$ 3. $x(t)=\frac{t^3}{6}+3t-2$ 4. $x(t)=-\frac{t^4}{12}+t^2+15t+3$

5. 111.6 m 6. \$166.67 7. 81 ft^3 8. a) t = 0 and x = 10; t = 1 and x = 59/6 b) t = 1.5

9. $\sqrt[4]{10}$ sec and $4\sqrt[4]{10^3}$ m/sec

Net/Total Dist.: 1. a) 20 m b) 0 m 2. a) 6 m b) 2 m 3. a) 245 m b) 0 m 4. a) 11 m b) 9m 5. a) 5m b) 3 m
6. d=2, s=2 7. d=4, s=4 8. d=4, s=0 9. d=2, s=2

DEQs: 1. $y=e^{1-\cos t}$ 2. $y=\frac{1}{\tan^{-1}x+C}$ 3. $y=\sin(x+C)$ 4. $y=\tan^{-1}(x+C)+(k\pi)$

5. $y=\frac{1}{\ln x+\frac{1}{2}}$ 6. $x+\ln|x|+y-\ln|y|=C$ 7. $\tan^{-1}x-\frac{1}{2}\ln\left(y^2+1\right)=C$ 8. $\frac{1}{2}$ day 9. $P(t)=\left(\frac{t}{2}+20\right)^2$

L'Hopital's: 1. $\ln 2$ 2. ∞ 3. $\frac{1}{3}$ 4. 0 5. 0 6. $-\infty$ 7. $-\infty$ 8. e^2 9. 1 10. $\frac{1}{e^4}$ 11. ∞ 12. $\frac{2}{3}$

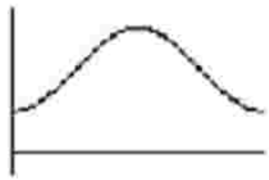
Euler's: 1. a) 1.7 b) 1.725 c) underestimated 2. a) 2.4883 b) 2.704813829 c) underestimated—slope field is concave up 3. a) \$734.66 b) \$740.12 c) \$744.92 4. 2.011 5. ~66 (66.2754)

Integral as accumulator: 1. a) 72 ft b) lower: 416 feet upper: 476 ft 2. a) cars = $\sum_{i=1}^{n} p(x_i) \bullet \Delta x_i$ or

$C(x) = \int_0^x p(t)dt$

b) 2551 cars 3. 4.123 4. 8647 gallons 5. a) 346.574 hours b) 450 millirems

6. a) $\sum_{i=1}^{n}(2\pi r_i \Delta r_i) \cdot \frac{100}{1+r^2}$ b) 4340.3 kg c) 177.8 meters 7. rising at A and max height at B, falling at C and minimum height at D, acceleration = 0 at A and C



8. $s(t) = \begin{cases} 50t & , 0 \le t < 2 \\ 200 - 50t & , 2 \le t < 3 \\ -100 + 50t & , 3 \le t \le 4 \end{cases}$

Limits, Integrals: 1. $-\frac{1}{2}$ 2. diverges with no limit 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{4}$ 5. 5 6. 0 7. ∞ 8. $\frac{1}{e^3}$

9. a) $x(t) = \frac{t^3}{3} - \frac{7}{2}t^2 + 10t + 5$ b) net distance: $7\frac{1}{2}$ m total distance: $9\frac{5}{6}$ m 10. $y = 5e^{-x} + 5$

11. $y = \sqrt{e^{-\tan^{-1} x} - 1}$ 12. 0, ± 0.961 13. LRAM = 8, RRAM = 11 or Trapezoid Rule = 9.5