AP Calculus Worksheet #1
End of Chapter P
(No Calculator)
Due: ________________

Grading:

100% = All 5 correct	65% = 3 correct	Work must support your answers. No exceptions.
90% = 4 ½ correct	55% = 2 ½ correct	
85% = 4 correct	45% = 2 correct	
75% = 3 ½ correct	35% = 1 correct	This is an answer sheet only!

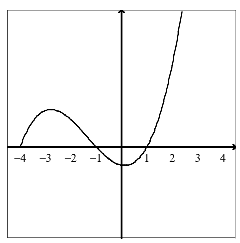
1. Find the domain and range of the function $f(x)=\sqrt{4-x^2}$. D: __________ R: __________

2. If h is the function given by $h(x)=f(g(x))$, where $f(x)=2x^4-x^2+5$ and $g(x)=|x-1|$, then $h(x)=$ ______________________.

3. The fundamental period of $3\tan(6x)$ is ______________________.

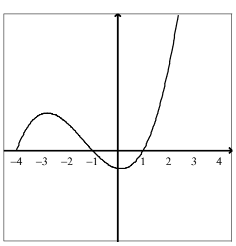
4. If the graph of $y=\dfrac{ax+9}{x+c}$ has a x-intercept at $x = 3$ and a vertical asymptote $x = -3$, then $a + c =$ ______________________.

5.

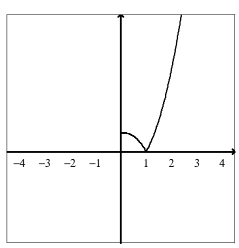


The graph of $y=f(x)$ is shown in the figure above. Which of the following could be the graph of $y=f(|x|)$? Justify your answer using complete sentences.

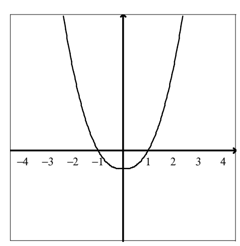
(A)



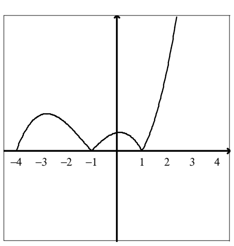
(B)



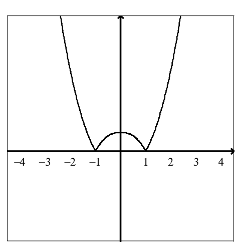
(C)



(D)



(E)



Justify: ______________________________
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____
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Name: ______________________

AP Worksheet #2
End of Chapter 1
(No Calculator)
Due: _______________

Work must support your answers. No exceptions. This is just an answer sheet!

Grading:

100%	= All 11 correct	75%	= 7.5 correct	40%	= 4 correct
95%	= 10.5 correct	70%	= 7 correct	35%	= 3.5 correct
90%	= 10 correct	65%	= 6.5 correct	30%	= 3 correct
88%	= 9.5 correct	60%	= 6 correct	25%	= 2.5 correct
85%	= 9 correct	55%	= 5.5 correct	20%	= 2 correct
82%	= 8.5 correct	50%	= 5 correct	15%	= 1.5 correct
80%	= 8 correct	45%	= 4.5 correct	10%	= 1 correct

1. If the function *f* is continuous for all real numbers and if $f(x)=\frac{x^2-7x+12}{x-4}$ where $x\neq 4$, then $f(4)=$________.

2. If $f(x)=\frac{5}{x^2+1}$ and $g(x)=3x$, then $g(f(2))=$_______.

3. The equation $y=2-3\sin\frac{\pi}{4}(x-1)$ has a fundamental period of _________.

4. Find: $\lim\limits_{x\to 0}\frac{\sin x\cos x-\sin x}{x^2}=$_________.

5. The domain of the function $f(x)=\sqrt{4-x^2}$ is _________.

6. Find: $\lim\limits_{x\to 5}\frac{x^2-25}{x-5}=$_________.

7. Find *k* so that $f(x)=\begin{cases}\frac{x^2-16}{x-4}; & x\neq 4\\ k & ; x=4\end{cases}$ is continuous for all *x*. *k* = _________.

8. Find: $\lim\limits_{x\to 0}\frac{\tan^3(2x)}{x^3}=$___________

9. Find: $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos x}{x-\frac{\pi}{4}}=$_________ .

10. If the function $f(x)$ is continuous $=\begin{cases}ax^2-6x; & \text{if } x\leq 1\\ -24x^2+4; & \text{if } x>1\end{cases}$ then $a=$________.

11. Which of the following functions is not everywhere continuous? Justify your decision for the one that is not. Hint: It must involve 3 pieces.

(a) $y = |x|$

(b) $y = \dfrac{x}{x^2 + 1}$

(c) $y = \sqrt{x^2 + 8}$

(d) $y = x^{2/3}$

(e) $y = \dfrac{4}{(x+1)^2}$

Show justification here:

AP Worksheet #4

End of Chapter 2

All work must be shown and done on another sheet of paper!

This is just your answer sheet!

Grading:

100%	= 19 correct	78%	= 14 correct
95%	= 18 correct	75%	= 13 correct
90%	= 17.5 correct	70%	= 12 correct
88%	= 17 correct	68%	= 11 correct
85%	= 16 correct	65%	= 10.5 correct
80%	= 15 correct	60%	= 10 correct

Work must support your answers.
No exceptions.

Due Date: Friday, October 24th

Score:___________ Name:_______________________

Do <u>not</u> use calculator (even for basic math) unless ** is by the problem.
Problems marked with @ are problems that will not be able to be done until the end of chapter 2.

1. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$ 1.____________________

2. @If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\frac{dy}{dx} =$ 2.____________________

3. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \neq -2$, the $f(-2) =$ 3.____________________

4. An equation of the line (in standard form) tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1,5)$ is 4.____________________

5. If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$ 5.____________________

6. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$ 6.____________________

7. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$ 7.____________________

8. The fundamental period of $2\cos(3x)$ is 8.____________________

9. The slope of the line normal (perpendicular) to the graph of $y = 2\sec x$ at $x = \frac{\pi}{4}$ is 9.____________________

10. @**Boats A and B leave the same place at the same time. Boat A heads due North at 12 km/hr. Boat B heads due east at 18km/hr. After 2.5 hours, how fast is the distance between the boats increasing? 10.____________________

11. If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0) =$ 11.____________________

12. A particle moves along the y-axis so that at time *t*, where $0 \le t \le \pi$, its position is given by $s(t) = -2\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

12.____________________

13. $\lim_{\theta \to 0} \frac{1-\cos\theta}{2\sin^2\theta}$ is

13.____________________

14. @**The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

14.____________________

15. @Consider the equation $x^2 - 2xy + 4y^2 = 52$. Find the equation of the tangent line(s) to the curve at the point $x = 2$.

15.____________________

16. If *f* is a differentiable function, then $f'(a)$ is given by which of the following? Justify.

16.____________________

I. $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

II. $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

III. $\lim_{x \to h} \frac{f(x+h) - f(x)}{h}$

17. @The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in the circumference. At this instant, the radius of the circle is

17.____________________

18. If $f(x) = \sqrt{1+\sqrt{x}}$, find $f'(x)$.

18.____________________

19. If $f(x) = \sin^2 x$, find $f'''(x)$.

19.____________________

20. If $y = \left(\frac{x^3 - 2}{2x^5 - 1}\right)^4$, find $\frac{dy}{dx}$ at $x = 1$.

20.____________________

I did not use my calculator (even for basic math) on these problems unless the problem was marked with a **.
Signature: _________________________

AP Worksheet #5
End of Chapter 3

Due: November 25^{th}

Score: __________ Name: ________________________

Record all answers on answer sheet.

<u>Section I – No calculators</u> (Please show all work on separate paper)

<u>Grading Scale</u>
(37 questions total)

100% = 34	78% = 28
95% = 33	75% = 26
90% = 32	70% = 24
88% = 31	68% = 23
85% = 30	65% = 22
80% = 29	60% = 20
	55% = 19
	50% = 17
	45% = 15

1. If $f(x) = 5x^{1/3}$, then $f'(8) =$ ________

2. $\lim\limits_{x\to\infty} \dfrac{5x^2 - 3x + 1}{4x^2 + 2x + 5} =$ __________

3. If $f(x) = \dfrac{3x^2 + x}{3x^2 - x}$ then $f'(x)$ is ____________ (write as a single fraction)

4. If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$ _________

5. If $x^2 - 2xy + 3y^2 = 8$, then $\dfrac{dy}{dx} =$ ________________ (write as a single fraction)

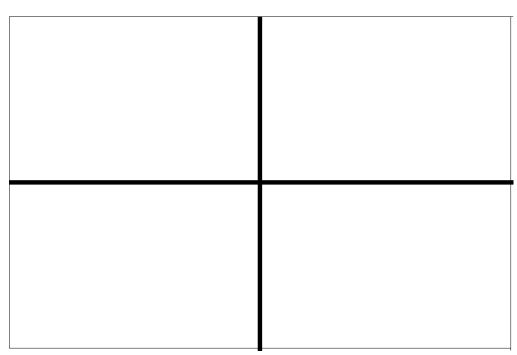
6. If $f(x) = \sin x + \csc x$, then $f'(x) =$ ____________________

7. An equation of a line normal to the graph of $y = \sqrt{(3x^2 + 2x)}$ at (2,4) is ________________

8. If $f(x) = \cos^2 x$, then $f''(\pi) =$ _____________

9. If $f(x) = \dfrac{5x}{x^2 + 1}$ and $g(x) = 3x - 2$, then $g(f(2)) =$ ___________

10. Let f be the function given by $f(x) = 2x^4 - 4x^2 + 1$.
    - Find an equation of the line tangent to the graph at (2,17). _______________
    - Find the x- and y-coordinates of the relative maxima and relative minima. ____________
    - Find the x- and y-coordinates of the points of inflection. _________________
    - Sketch the graph of $f(x)$. Graph:



11. The equation $y = 2 - 3\sin\frac{\pi}{4}(x-1)$ has a fundamental period of ____________

12. For what value of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum? ___

13. If $f(x) = \begin{cases} x^2 + 5 \text{ if } x < 2 \\ 7x - 5 \text{ if } x \geq 2 \end{cases}$, for all real numbers x, which of the following must be true? Justify

I. $f(x)$ is continuous everywhere.
II. $f(x)$ is differentiable everywhere.
III. $f(x)$ has a local minimum at $x = 2$.

(A) I only (B) I and II only (C) II and III only
(D) I and III only (E) I, II, and III

Justify:____
____
____
____.

14. If $f(x) = \sqrt{(x^3 + 5x + 121)}(x^2 + x + 11)$ then $f'(0) =$_________

15. $\lim_{x \to 0} 4\frac{\sin x \cos x - \sin x}{x^2} =$__________

16. If $f(x) = \cos^3(x+1)$, then $f'(\pi) =$__________

•

Section II (calculators may be used)

17. Consider the equation $x^2 - 2xy + 4y^2 = 52$. Exact values only.

- Write an expression for the slope of the curve at the point (x, y). ____________________
- Find the equation(s) of the tangent lines to the curve at the point $x = 2$. _______________
- Find $\frac{d^2y}{dx^2}$ at (2, 4). ________________ _______________

18. Water is draining at the rate of $48\pi\ ft^3/\text{min}$ from a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.

- Find an expression for the volume of water in the tank in terms of its radius. _________
- At what rate is the radius of the water in the tank shrinking when the radius is 16 ft? ____
- How fast is the height of the water in the tank dropping at the instant the radius is 16 ft?

____

19. Cars A and B leave the same place at the same time. Car A heads due North at 10 km/hr. Car B heads due East at 12 km/hr. After 2.5 hours, how fast is the distance between the cars increasing (in km/hr)?__________

20. $\lim_{h\to 0}\dfrac{\sin\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}\right)}{h}=$ __________

21. Find two nonnegative numbers x and y whose sum is 100 and for which xy^2 is a maximum.

____

22. A 20-foot ladder slides down a wall at 5 ft/sec. At what speed is the bottom sliding out when the top is 10 feet from the floor? (in ft/sec) ________________

23. The graph of $y=x^3-5x^2+4x+2$ has a local minimum point(s) at __________

24. Find the value(s) of $\dfrac{dy}{dx}$ of $x^2y+y^2=5$ at $y=2$

25. The graph of $y=5x^4-x^5$ has an inflection point (or points) at ________________

26. The graph of $y=x^3-2x^2-5x+2$ has local maximum at ________________

27. If $f(x)=\left(1+\dfrac{x}{20}\right)^5$, find $f''(40)$.______________

28. If the function $f(x)$ is continuous and differentiable $f(x)=\begin{cases} ax^3-6x; & \text{if } x\le 2 \\ bx^2+4; & x>2 \end{cases}$ then $b=$ ___

29. Two particles leave the origin at the same time and move along the y-axis and their respective position determined by the functions $y_1=\cos 3t$ and $y_2=5\sin t$ for $0<t<2$. For what values of t, if any, do the particles have the same acceleration? ________________

30. The temperature on New Years Day in Hinterland was given by $T(H)=-A-B\cos\left(\dfrac{\pi H}{12}\right)$, where T is the temperature in degrees Fahrenheit and H is the numbers of hours from midnight $(0\le H\le 24)$. If the initial temperature at midnight was $-15°F$, and at Noon of New Year's Day was $5°F$. Find an <u>expression</u> for the rate the temperature is changing with respect to H. ________________

AP Worksheet #6 (Due: ______________________)

End of Chapter 4

Score: __________ Name: ________________________________

Section I – No calculators (Please show all work)

Grading Scale
(60 Points Possible)

100%	= 56 correct	68%	= 38 correct
95%	= 54 correct	65%	= 36 correct
90%	= 52 correct	60%	= 34 correct
88%	= 50 correct	58%	= 32 correct
85%	= 48 correct	55%	= 30 correct
80%	= 46 correct	50%	= 28 correct
78%	= 44 correct	48%	= 26 correct
75%	= 42 correct	45%	= 24 correct
70%	= 40 correct	40%	= 22 correct

1. If $f(x) = 2x^{1/4}$, then $f^{-1}(8) =$ _________
2. $\lim_{x\to\infty} \dfrac{2x^2 - 3x + 1}{3x^3 + 2x + 5} =$ ___________
3. If $f(x) = \dfrac{2x+1}{x-1}$ then $f'(x)$ is ______________ (write as a single fraction)
4. If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 + x - 12}{x+4}$ when $x \neq -4$, then $f(-4) =$ __________
5. If $x^3 + 4x^2y - 3y^2 = 8$, then $\dfrac{dy}{dx} =$ _________________ (write as a single fraction)
6. If $f(x) = \tan x + \sec^2 x$, then $f'(x) =$ ______________________
7. An equation of a line normal to the graph of $y = 3x^2 + 2x - 1$ at (2,15) is __________________
8. $\int_{-1}^{1} \dfrac{4x}{(1+x^2)^2}\, dx =$ ____________
9. If $f(x) = \tan^2 x$, then $f''(\pi) =$ ______________
10. If $f(x) = \dfrac{5}{x^2+1}$ and $g(x) = 3x$, then $f(g(2)) =$ ____________
11. $\int x\sqrt{5x^2 - 4}\, dx =$ _____________ (write using rational exponents)
12. The slope of the line tangent to the graph $3x^2 + 5y^2 = 17$ at (2,1) is ____________
13. The equation $y = 1 + 5\sin\dfrac{\pi}{6}(x+5)$ has a fundamental period of ______________
14. For what value of x does the function $f(x) = 2x^3 - 18x^2 - 240x$ have a local minimum? ___
15. If $f(x) = \begin{cases} x^2 + 5 & \text{if } x < 2 \\ 4x - 5 & \text{if } x \geq 2 \end{cases}$, for all real numbers x, which of the following must be true? Justify.

    I. $f(x)$ is continuous everywhere.
    II. $f(x)$ is differentiable everywhere.
    III. $f(x)$ has a local minimum at $x = 2$.

    (A) I only (B) II only (C) II and III only
    (D) III only (E) I, II, and III

    ____
    ____
    ____
    ____.

16. The acceleration of a particle moving along the y-axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle is changing direction at $t =$ ___________

17. The average value of a function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is ______

18. If $F(x) = \int \sqrt{(x^3 + 3x + 121)}(x^2 + 1)dx$ then $F(x) =$ ______________

19. $\lim_{x \to 0} \frac{\sin 2x \cos x - \sin 2x}{x^2} =$ __________

20. If $f(x) = \tan^3(x + \pi)$, then $f'(\pi) =$ __________

21. $\int x\sqrt{x+3}\; dx =$ ___________

22. $\frac{d}{dx}\left[\int_2^{x^2} \ln(3t-5)dt\right] =$ _________

23. If a particle moves on a line according to the law $s = t^5 + 2t^3$, then how many times does it reverse directions? __________

24. A rectangular pigpen is to be built against a wall so that only three sides will require fencing. If p feet of fencing are to be used, the area of the largest possible pen is ___________.

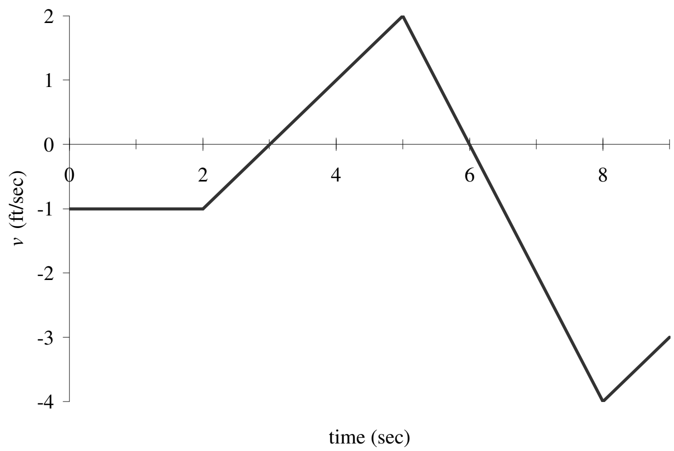
25. A smooth curve with equation $y = f(x)$ is such that its slope at each x equals x^2. If the curve goes through the point (-1,2), then its equation is ___________________.

26. If $G(2) = 5$ and $G'(x) = \frac{10x}{9 - x^2}$, then an estimate of G (2.2) using local linearization is approximately _____________. (Refer to page 228 in your text book.)

27. The average value of $f(x) = 3 + |x|$ on the interval [-2,4] is _______________.

28. Suppose $f(x) = \frac{x^2 + x}{x}$, if $x \neq 0$ and $f(0) = 1$. Prove below that f is continuous at $x = 0$.

Section II (calculators may be used)



The graph shown is for questions 29 and 30. It shows the velocity of an object during the interval $0 \le t \le 9$.

29. The object obtains the greatest speed at $t =$ _____________.

30. The object's position was at the origin at $t = 3$. It returned to the origin at ____________________.

31. $\int_0^{\pi/4} \sin x \, dx + \int_{-\pi/4}^{0} \cos x \, dx =$ ___________

32. $\lim_{h \to 0} \dfrac{\sec\left(\frac{\pi}{6} + h\right) - \sec\left(\frac{\pi}{6}\right)}{h} =$ __________

33. If $\int_{30}^{100} f(x)dx = A$ and $\int_{50}^{100} f(x)dx = B$, then $\int_{30}^{50} f(x)dx =$ ______

34. If $f(x) = 3x^2 - x$, and $g(x) = x^2$, then $\int g(f(x))dx =$ __________

35. The graph of $y = 2x^3 - 5x^2 + x + 2$ has a local minimum at ___________

36. The average value of the function $f(x) = \dfrac{2x^2 - 3x + 1}{x - 1}$ on the interval [2,4] is __________

37. $\dfrac{d}{dx}\left(\int_0^{3x} \cos(t)dt\right) =$ ___________

38. If the definite integral $\int_1^3 (x^2 + 1)dx$ is approximated by using the Trapezoid Rule when $n = 4$, the error from the actual is ___________.

39. $\int\left(\cot^2 x\right)dx =$__________

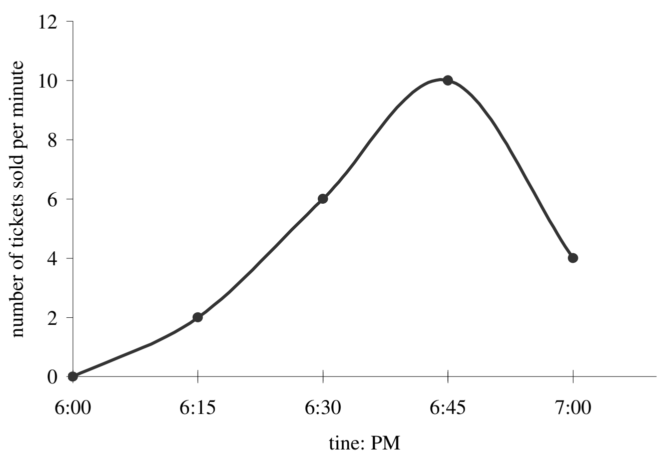
40. Find the distance traveled (to three decimal places) in the first 4 seconds, for a particle whose velocity is given by $v(t) = 7\sin^2 t$; where t stands for time. __________

41. $\int \tan^6 x \sec^2 x\ dx =$___________

42. The intervals on which the function $f(x) = x^4 - 4x^3 + 4x^2 + 6$ increases are (is) __________.

43. If we replace $\sqrt{x-2}$ by u, then $\int_3^6 \frac{\sqrt{x-2}}{x}dx$ is equivalent to the integral _______________ (make sure the integral is in terms of u)

44. How many point of inflection does the function f have on the interval $0 \le x \le 6$ if $f''(x) = 2 - 3\sqrt{x}\left(\cos^3 x\right)$? ________________



45. The graph shows the rate at which tickets were sold at a movie theater during the last hour before show time. Using right-rectangle method, and estimate of the size of the audience is ____________.

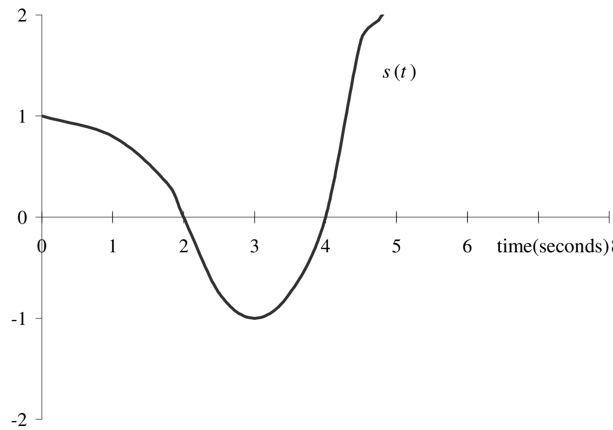
Section III Free Response Questions (No calculator) – Work is to be shown on this page.
Note: On the free response sections I will be grading your written reasons as well as organization and neatness.

1) Let *f* be the function given by $f(x) = 1 + \frac{2}{x} + \frac{1}{x^2}$.

   a) Find the *x* and *y* intercepts. Justify.
   b) Write an equation for each vertical and horizontal asymptote for the graph of *f*. Justify.
   c) Find the intervals on which *f* is increasing and decreasing. Justify.
   d) Find the maximum and minimum values of *f*. Justify.

No Calculator – Work is to be shown on this page.

2) Let the graph of $s(t)$, the position function (in feet) of a moving particle, be given below. Let t be measured in seconds. The concavity changes at $t = 2$ and $t = 4$



a) Find the values of t for which the particle is moving to the right and when it is moving to left (i.e., when velocity is positive or negative, respectively). Justify.
b) Find the values of t for which the acceleration is positive and for which it is negative. Justify.
c) Find the values of t for which the particle is speeding up (i.e., when $|v|$ is increasing). Justify.

Section IV Free Response (calculator may be used) – Work is to be shown on this page.
Note: On the free response sections I will be grading your written reasons as well as organization and neatness.

3) A particle moves along the *x*-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position $x(1) = 9$.

a) Write an expression for the velocity of the particle $v(t)$.

b) At what values of *t* does the particle change direction? Justify.

c) Write an expression for the position function, $x(t)$, of the particle.

d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 4$.

Calculator Allowed – Work is to be shown on this page.

4) A floodlight is on the ground 45 meters from a building. A thief 2 meters tall runs from the floodlight towards the building at 6 meters/second.
   a) Using a triangle(s) draw a picture of the situation.
   b) What is the relationship (equation) between the shadow on the building and the distance the thief is from the floodlight?
   c) How rapidly is the length of the shadow on the building changing when he is 15 meters from the building?

AP Worksheet #7
End of Chapter 5
Due Date: ________________________

Score: ____________ Name:________________________

Grading Scale
(61 Points Possible)

100%	= 56 correct	68%	= 38 correct
95%	= 54 correct	65%	= 36 correct
90%	= 52 correct	60%	= 34 correct
88%	= 50 correct	58%	= 32 correct
85%	= 48 correct	55%	= 30 correct
80%	= 46 correct	50%	= 28 correct
78%	= 44 correct	48%	= 26 correct
75%	= 42 correct	45%	= 24 correct
70%	= 40 correct	40%	= 22 correct

Section I – No calculators (Please show all work)

1. If $f(x)=5x^{3/4}$, then $\left(f^{-1}\right)'(5)=$ ________

2. $\lim\limits_{x\to\infty}\dfrac{5x^2-3x+1}{4x^2+2x+5}=$ __________

3. If $f(x)=\dfrac{3x^2+x}{3x^2-x}$ then $f'(x)$ is ____________ (write as a single fraction)

4. If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2-7x+12}{x-4}$ when $x\neq 4$, then $f(4)=$ _________

5. If $x^2-2xy+3y^2=8$, then $\dfrac{dy}{dx}=$ ________________ (write as a single fraction)

6. If $f(x)=\sec x+\csc x$, then $f'(x)=$ ____________________

7. An equation of a line normal to the graph of $y=\sqrt{(3x^2+2x)}$ at (2,4) is ________________

8. $\displaystyle\int_{-1}^{1}\frac{4}{1+x^2}\,dx=$ ___________

9. If $f(x)=\cos^2 x$, then $f''(\pi)=$ _____________

10. If $f(x)=\dfrac{5}{x^2+1}$ and $g(x)=3x$, then $g(f(2))=$ ___________

11. $\displaystyle\int x\sqrt{5x^2-4}\,dx=$ ____________ (write using rational exponents)

12. The slope of the line tangent to the graph $3x^2+5\ln y=12$ at (2,1) is ___________

13. The equation $y = 2 - 3\sin\frac{\pi}{4}(x-1)$ has a fundamental period of ______________

14. For what value of *x* does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum? ___

15. If $f(x) = \begin{cases} x^2 + 5 \text{ if } x < 2 \\ 7x - 5 \text{ if } x \geq 2 \end{cases}$, for all real numbers *x*, which of the following must be true? Justify

I. $f(x)$ is continuous everywhere.
II. $f(x)$ is differentiable everywhere.
III. $f(x)$ has a local minimum at $x = 2$.

(A) I only (B) I and II only (C) II and III only
(D) I and III only (E) I, II, and III

16. The acceleration of a particle moving along the *y*-axis at time *t* is given by $a(t) = 4t - 12$. I the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle is changing direction at *t* = ____________

17. The average value of a function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is ______

18. If $f(x) = \sqrt{(x^3 + 5x + 121)(x^2 + x + 11)}$ then $f'(0) =$__________

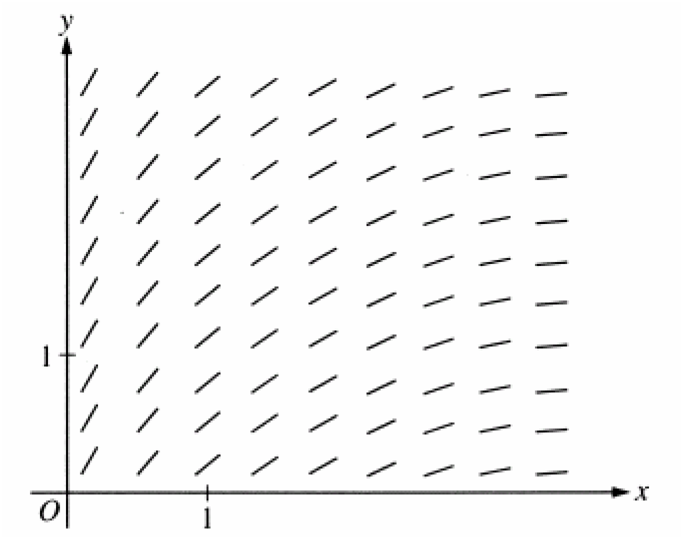
19. $\lim_{x \to 0} 4\frac{\sin x \cos x - \sin x}{x^2} =$___________

20. If $f(x) = \cos^3(x+1)$, then $f'(\pi) =$___________

21. $\int x\sqrt{x+3}\ dx =$___________

22. If $f(x)=\ln\left(\ln(1-x)\right)$, then $f'(x)=$_________

23.



The slope field for a certain differential equation is shown to the right.

Which of the following could be a specific solution to that differential equation? Explain your reasoning.

A. $y=x^2$
B. $y=e^x$
C. $y=e^{-x}$
D. $y=\ln x$

24. If F' is a continuous function for all real x, then $\lim_{h\to 0}\frac{1}{h}\int_a^{a+h} F'(x)\,dx =$ _______________

Section II (calculators may be used)

25. $\int_0^{\pi/4}\sin x\ dx+\int_{-\pi/4}^{0}\cos x\ dx=$___________

26. Boats A and B leave the same place at the same time. Boat A heads due North at 12 km/hr. Boat B heads due East at 18 km/hr. After 2.5 hours, how fast is the distance between the boats increasing (in km/hr)?__________

27. $\lim_{h\to 0}\dfrac{\tan\left(\dfrac{\pi}{6}+h\right)-\tan\left(\dfrac{\pi}{6}\right)}{h}=$ __________

28. If $\int_{30}^{100} f(x)dx = A$ and $\int_{50}^{100} f(x)dx = B$, then $\int_{30}^{50} f(x)dx =$ ______

29. if $f(x)=3x^2-x$, and $g(x)=f^{-1}(x)$, then $g'(10)$ could be __________

30. The graph of $y=x^3-5x^2+4x+2$ has a local minimum at ___________

31. The average value of the function $f(x)=\ln^2 x$ on the interval [2,4] is __________

32. $\dfrac{d}{dx}\int_0^{3x}\cos(t)dt =$ ___________

33. If the definite integral, $\int_1^3 (x^2+1)dx$, is approximated by using the Trapezoid Rule when $n=4$, the error is ___________.

34. $\dfrac{d}{dx}\left[\int_1^{x^2-3}\ln(2t)\,dt\right]=$ __________

35. If the function $f(x)$ is continuous and differentiable $f(x)=\begin{cases} ax^3-6x; & \text{if } x\le 1 \\ bx^2+4; & x>1 \end{cases}$ then $a =$ ____

36. Two particles leave the origin at the same time and move along the y-axis and their respective position determined by the functions $y_1=\cos 2t$ and $y_2=4\sin t$ for $0<t<6$. For how many values of t do the particles have the same acceleration? __________

37. Find the distance traveled (to thee decimal places) in the first 4 seconds, for a particle whose velocity is given by $v(t)=7e^{-t^2}$; where t stands for time. __________

38. $\int \tan^6 x \sec^2 x\, dx =$ ___________

<u>Section III Free Response Questions (No calculator)</u>

1. Let *f* be the function given by $f(x) = 2x^4 - 4x^2 + 1$.

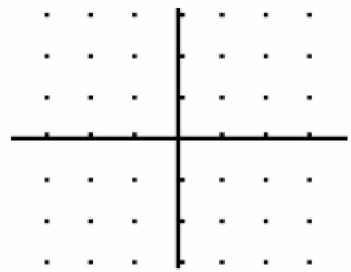
   (a) Find an equation of the line tangent to the graph at (2,17)
   (b) Find the *x*- and *y*-coordinates of the relative minima and relative maxima. Justify.
   (c) Find the *x*- and *y*-coordinates of the points of inflection. Justify.

2. A particle moves along the x-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position $x(1) = 9$.

(a) Write an expression for the velocity of the particle $v(t)$.
(b) At what values of t does the particle change direction? Justify.
(c) Write an expression for the $x(t)$ of the particle.
(d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 6$.

3. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation.



(b) Sketch a solution curve that passes through the point (0, 1) on your slope field.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

(d) Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

(e) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

Section IV Free Response (calculator may be used)

4. Sea grass grows (in tons) on a lake and the rate of growth of the sea grass is proportional to the sea grass.

   (a.) Find an expression for G, the amount of sea grass in the lake (in tons), in terms of t, the number of years, if the amount of grass is 100 tons initially, and 120 tons after one year.
   (b.) In how many years will the amount of sea grass available be 300 tons?
   (c.) If fish are now introduced into the lake and consume a constant 80 tons/year of sea grass, how long will it take the lake to be completely free of sea grass?

5. Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$

(a) Find $\frac{dy}{dx}$.

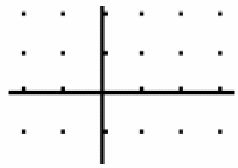
(b) Write an equation for the line tangent to the curve at the point (4,-1)

(c) There is a number k so that the point (4.2, k) is on the curve. Using the tangent line found in part (b), approximate the value of k.

(d) Write and equation that can be solved to find the actual value of k so that the point (4.2, k) is on the curve and solve for k.

6. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes provided, sketch a slope field for the given differential equation.



(b) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(d) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part (c). Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

AP Free Response Questions
Optional Problems

1.

A particle moves along the y-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t \cos t$. At time $t = 0$, the position of the particle is $y = 3$.

(a) For what values of t, $0 \leq t \leq 5$, is the particle moving upward?

(b) Write an expression for the acceleration of the particle in terms of t.

(c) Write an expression for the position $y(t)$ of the particle.

(d) For $t > 0$, find the position of the particle the first time the velocity of the particle is zero.

2.

Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

(a) Find $\frac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at the point $(4, -1)$.

(c) There is a number k so that the point $(4.2, k)$ is on the curve. Using the tangent line found in part (b), approximate the value of k.

(d) Write an equation that can be solved to find the actual value of k so that the point $(4.2, k)$ is on the curve.

(e) Solve the equation found in part (d) for the value of k.

3.

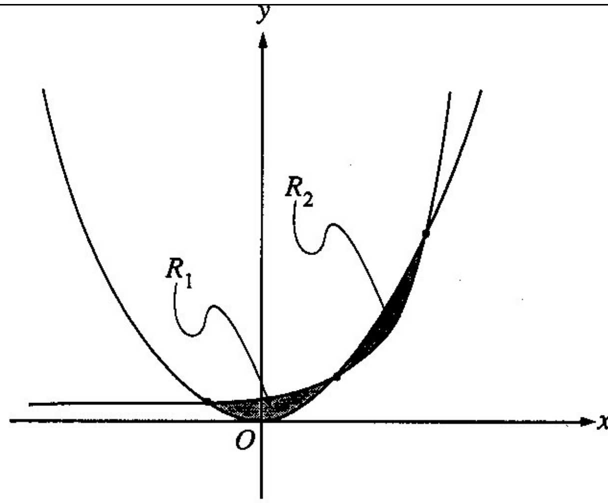
Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

(a) Write an equation for y, the amount of oil remaining in the well at any time t.

(b) At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?

(c) In order not to lose money, at what time t should oil no longer be pumped from the well?

4.



Note: Figure not drawn to scale.

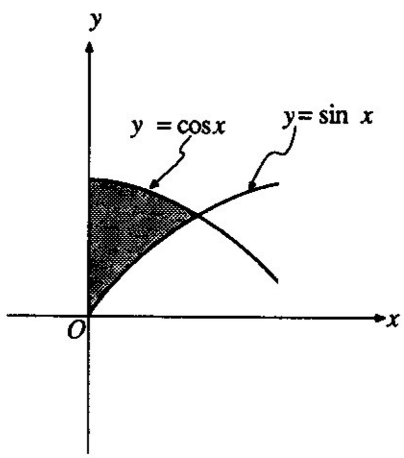
The shaded regions R_1 and R_2 shown above are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.

(a) Find the x- and y-coordinates of the three points of intersection of the graphs of f and g.

(b) Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of f and g. Do not evaluate.

(c) Without using absolute value, set up an expression involving one or more integrals that gives the volume of the solid generated by revolving the region R_1 about the line $y = 5$. Do not evaluate.

5.

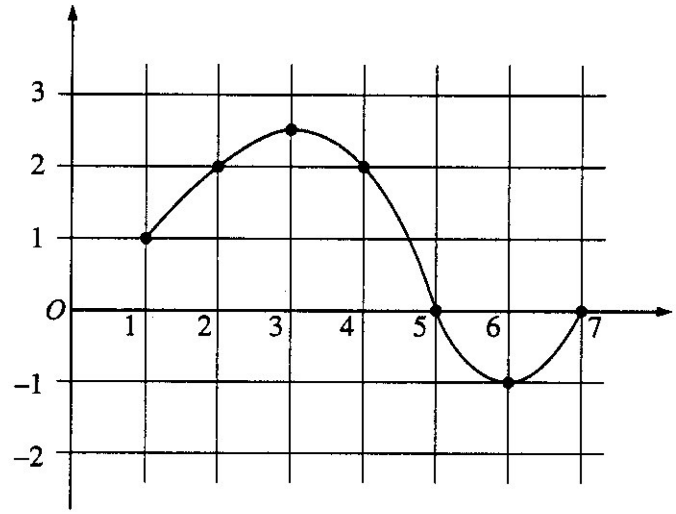


Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = \sin x$ and $y = \cos x$, as shown in the figure above.

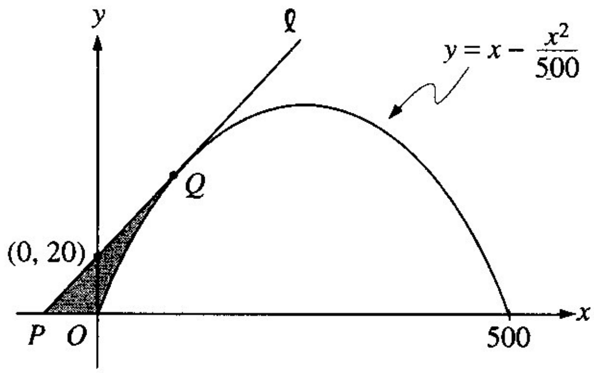
(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares.

6.	 The graph of a differentiable function f on the closed interval [1, 7] is shown above. Let $h(x) = \int_1^x f(t)\,dt$ for $1 \leq x \leq 7$. (a) Find $h(1)$. (b) Find $h'(4)$. (c) On what interval or intervals is the graph of h concave upward? Justify your answer. (d) Find the value of x at which h has its minimum on the closed interval [1, 7]. Justify your answer.
7.	Let $F(x) = \int_1^{2x} \sqrt{t^2 + t}\,dt$. (a) Find $F'(x)$. (b) Find the domain of F. (c) Find $\lim\limits_{x \to \frac{1}{2}} F(x)$.

8.



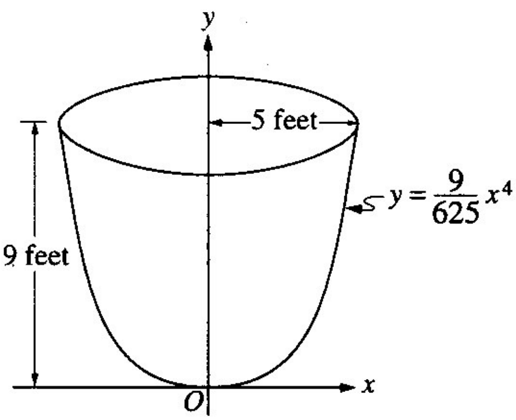
Line ℓ is tangent to the graph of $y = x - \frac{x^2}{500}$ at the point Q, as shown in the figure above.

(a) Find the x-coordinate of point Q.

(b) Write an equation for line ℓ.

(c) Suppose the graph of $y = x - \frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along line ℓ shine on any part of the tree? Show the work that leads to your conclusion.

9.



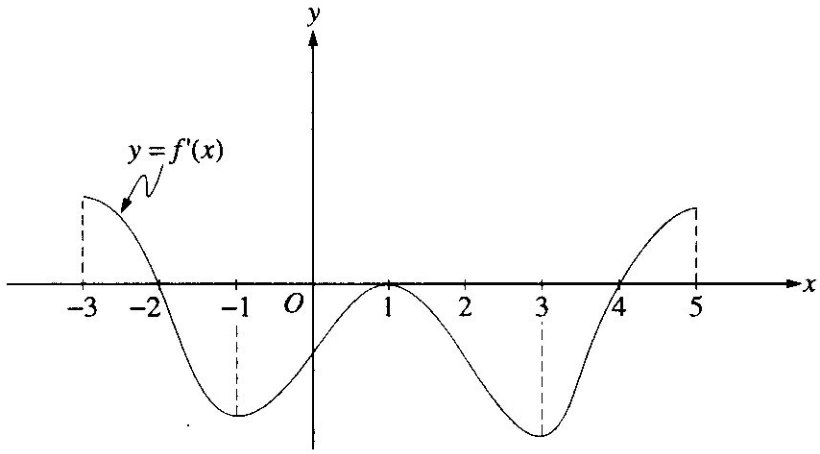
An oil storage tank has the shape shown above, obtained by revolving the curve $y = \frac{9}{625}x^4$ from $x = 0$ to $x = 5$ about the y-axis, where x and y are measured in feet. Oil flows into the tank at the constant rate of 8 cubic feet per minute.

(a) Find the volume of the tank. Indicate units of measure.

(b) To the nearest minute, how long would it take to fill the tank if the tank was empty initially?

(c) Let h be the depth, in feet, of oil in the tank. How fast is the depth of the oil in the tank increasing when $h = 4$? Indicate units of measure.

10.



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.

(a) For what values of x does f have a relative maximum? Why?

(b) For what values of x does f have a relative minimum? Why?

(c) On what intervals is the graph of f concave upward? Use f' to justify your answer.

(d) Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

Note: The axes for this graph are provided in the pink booklet only.