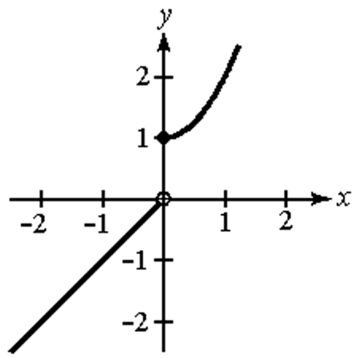
Calculus 1st Semester Final Review

1. Use the graph to find $\lim_{x \to 0} f(x)$ (if it exists).



2. Find the limit: $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x - 3}$.

3. Find the limit: $\lim_{x \to 1} f(x)$ if $f(x) = \begin{cases} x^2 + 4, & x \neq 1 \\ 2, & x = 1 \end{cases}$.

4. Find the limit: $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x - 1}$.

5. Find the limit: $\lim_{x \to 1} \dfrac{1 - \sqrt{2x^2 - 1}}{x - 1}$.

6. Let $f(x) = \begin{cases} x^2 + 1, & x \leq 0 \\ 2x - 3, & x > 0 \end{cases}$. Find each limit (if it exists).

   a. $\lim_{x \to 0^-} f(x)$

   b. $\lim_{x \to 0^+} f(x)$

   c. $\lim_{x \to 0^+} f(x)$

7. Find the values of x for which $f(x) = \dfrac{x - 2}{x^2 - 4}$ is discontinuous and label these discontinuities as removable or nonremovable.

8. Let $f(x) = \dfrac{5}{x - 1}$ and $g(x) = x^4$.

   a. Find $f(g(x))$.

   b. Find all values of x for which $f(g(x))$ is discontinuous.

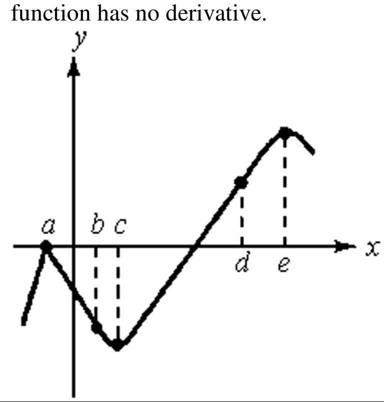
9. Determine the value of c so that $f(x)$ is continuous on the entire real line if $f(x) = \begin{cases} x^2, & x \leq 3 \\ c / x, & x > 3 \end{cases}$.

10. Find the limit: $\lim_{x \to 3} \dfrac{3}{x^2 - 6x + 9}$.

11. Find all vertical asymptotes of the graph of $f(x) = \dfrac{x^2 + 3x - 1}{x + 7}$.

12. Find the vertical asymptote(s) of $f(x) = \dfrac{x^2 - x - 2}{x^2 + x - 6}$.

13. At each point indicated on the graph, determine whether the value of the derivative is positive, negative, zero, or if the function has no derivative.



14. Use the definition of a derivative to calculate the derivative of $f(x) = x^2 + 2$.

15. Find an equation of the tangent line to the graph of $f(x) = 3x^3 + 2x$ when $x = 1$.

16. Find the values of x for all points on the graph of $f(x) = x^3 - 2x^2 + 5x - 16$ at which the slope of the tangent line is 4.

17. Find all points at which the graph of $f(x) = x^3 - 3x$ has horizontal tangent lines.

18. The position function for an object is given by $s(t) = 6t^2 + 240t$, where s is measured in feet and t is measured in seconds. Find the velocity of the object when $t = 2$ seconds.

19. Differentiate: $y = \dfrac{2x}{(1 - 3x^2)^2}$.

20. Calculate $\dfrac{d^2y}{dx^2}$ if $y = \dfrac{1 - x}{2 - x}$.

21. Find the derivative of $y = \sqrt[3]{x^2 + x}$.

22. Find the derivative of $y = (x^2 + 2x + 5)^6$.

23. Find $\frac{dy}{dx}$ for $y = \sqrt{2x+1}(x^3)$.

24. The position equation for the movement of a particle is given by $s = (t^3 + 1)^2$ where s is measured in feet and t is measured in seconds. Find the acceleration of this particle at 1 second.

25. Find $\frac{dy}{dx}$ if $y = \frac{x}{x+y}$.

26. Use implicit differentiation to find $\frac{dy}{dx}$ for $x^2 + xy + y^2 = 5$.

27. Find the slope of the curve $y^4 - xy^2 = x$ at the point $\left(\frac{1}{2}, 1\right)$.

28. The radius of a circle is increasing at the rate of 5 inches per minute. At what rate is the area increasing when the radius is 10 inches?

29. Air is being pumped into a spherical balloon at a rate of 28 cubic feet per minute. At what rate is the radius changing when the radius is 3 feet?
$\left(V = \frac{4}{3}\pi r^3\right)$

30. A point moves along the curve $y = \frac{-x + 4x - 3}{10}$ so that the y value is decreasing at a rate of 3 units per second. Find the instantaneous rate of change of x with respect to time at the point on the curve where $x = 5$.

31. Find all critical numbers for the function: $f(x) = x\sqrt{2x+1}$.

32. Find all critical numbers for the function: $f(x) = 3x^4 - 4x^3$.

33. Find the minimum and maximum values of $f(x) = x^2 - 2x + 1$ on the interval $[0, 3]$.

34. Consider $f(x) = \frac{1}{(x-3)^2}$.
a. Sketch the graph of $f(x)$.
b. Calculate $f(2)$ and $f(4)$.
c. State why Rolle's Theorem does not apply to f on the interval $[2, 4]$.

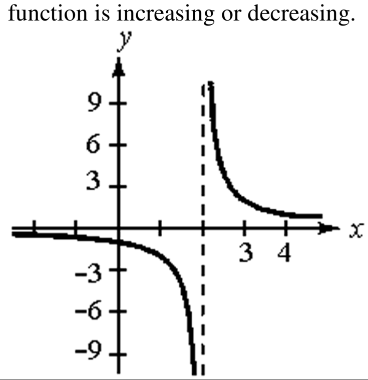
35. Decide whether Rolle's Theorem can be applied to $f(x) = x^4 - 4x^3 + 4x^2 + 1$ on the interval $[-1, 3]$. If Rolle's Theorem can be applied, find all value(s), c, in the interval such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, state why.

36. Determine whether the Mean Value Theorem applies to $f(x) = 3x - x^2$ on the interval $[2, 3]$. If the Mean Value Theorem can be applied, find all value(s) of c in the interval such that $f'(c) = \frac{f(3) - f(2)}{3 - 2}$. If the Mean Value Theorem does not apply, state why.

37. Find the open intervals on which $f(x) = \frac{1}{x^2}$ is increasing or decreasing.

38. Find the open intervals on which $f(x) = x^3 - 3x^2$ is increasing or decreasing.

39. Use the graph to identify the open intervals on which the function is increasing or decreasing.



40. Find all relative extrema of $y = \frac{2x}{(x+4)^3}$.

41. Find the relative minimum and relative maximum for $f(x) = 2x^3 + 3x^2 - 12x$.

42. Use the first derivative test to find the x-values that give relative extrema for $f(x) = -x^4 + 2x^3$.

43. Let $f(x) = \frac{x}{1-x}$. Show that f has no critical numbers.

44. A differentiable function f has only one critical number: $x = -3$. Identify the relative extrema of f at $(-3, f(-3))$ if $f'(-4) = \frac{1}{2}$ and $f'(-2) = -1$.

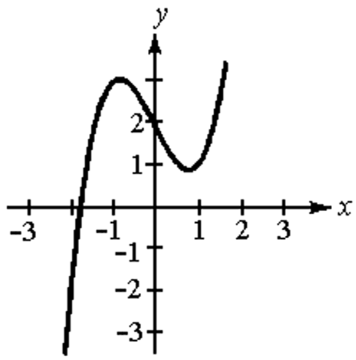
45. Find the intervals on which the graph of the function $f(x) = x^4 - 4x^3 + 2$ is concave upward or downward. Then find all points of inflection for the function.

46. Find all points of inflection of the graph of the function $f(x) = 2x(x - 4)^3$.

47. Find all points of inflection of the graph of the function $f(x) = x^3 - 3x^2 - x + 7$.

48. Let $f(x) = x^3 - x^2 + 3$. Use the Second Derivative Test to determine which critical numbers, if any, give relative extrema.

49. The graph of a polynomial function, f, is given. On the same coordinate axes sketch f' and f''.



$y = f(x)$

50. Find the horizontal asymptote for $f(x) = \dfrac{2x^2 + x - 7}{x^2 - 1}$.

51. Find the limit: $\lim\limits_{x \to \infty} \dfrac{x^2}{\sqrt{4x^2 - 1}}$.

52. An open box is to be made from a square piece of material, 12 inches on each side, by cutting equal squares from each corner and turning up the sides. Find the volume of the largest box that can be made in this manner.

53. Use the techniques learned in this chapter to sketch the graph of $f(x) = x^3 + x^2 - 6x$.

54. A rancher has 300 feet of fencing to enclose a pasture bordered on one side by a river. The river side of the pasture needs no fence. Find the dimensions of the pasture that will produce a pasture with a maximum area.

55. A manufacturer determines that x employees on a certain production line will produce y units per month where $y = 75x^2 - 0.2x^4$. To obtain maximum monthly production, how many employees should be assigned to the production line?

56. The volume of a cube is claimed to be 27 cubic inches, correct to within 0.027 in.^{3}. Use differentials to estimate the propagated error in the measurement of the side of the cube.

Calculus 1st Semester Final Review

Reference: [6.6]

[1] The limit does not exist.

Reference: [7.8]

[2] 0

Reference: [7.14]

[3] 5

Reference: [7.39]

[4] 3

Reference: [7.46]

[5] -2

Reference: [8.10]

a. 1
b. -3
[6] **c.** The limit does not exist.

Reference: [8.20]

[7] $x = 2$, removable; $x = -2$, nonremovable

Reference: [8.23]

a. $\dfrac{5}{x^4 - 1}$
[8] **b.** -1, 1

Reference: [8.26]

[9] 27

Reference: [9.8]

[10] ∞

Reference: [9.23]

[11] $x = -7$

Reference: [9.25]

[12] $x = -3$

Reference: [10.5]

a. no derivative
b. negative
c. zero
d. positive
[13] **e.** zero

Reference: [10.11]

[14] $-2x$

Reference: [11.23]

[15] $y = 11x - 6$

Reference: [11.26]

[16] $\dfrac{1}{3}$, 1

Reference: [11.28]

[17] $(1, -2), (-1, 2)$

Reference: [11.40]

[18] 264 ft/sec

Reference: [12.2]

[19] $\dfrac{6x^2 + 2}{(1 - 3x^2)^2}$

Reference: [12.25]

[20] $\dfrac{-2}{(2 - x)^3}$

Reference: [13.2]

[21] $\dfrac{2x + 1}{3(x^2 + x)^{2/3}}$

Reference: [13.3]

[22] $12(x + 1)(x^2 + 2x + 5)^5$

Reference: [13.11]

[23] $\dfrac{x^2(7x + 3)}{\sqrt{2x + 1}}$

Reference: [13.34]

[24] 42 ft/sec^2

Reference: [14.3]

[25] $\dfrac{y}{(x + y)^2 + x}$

Reference: [14.10]

[26] $\dfrac{-2x - y}{x + 2y}$

Reference: [14.29]

[27] $\frac{2}{3}$

Reference: [15.5]

[28] 100π in.^{2}/min

Reference: [15.14]

[29] $\frac{7}{9\pi}$ ft/min

Reference: [15.19]

[30] 5 units/sec

Reference: [16.3]

[31] $-\frac{1}{3},\ -\frac{1}{2}$

Reference: [16.4]

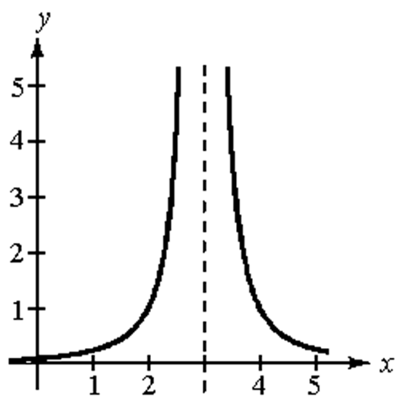
[32] $x = 0, 1$

Reference: [16.17]

[33] Minimum at (1, 0); Maximum at (3, 4)

Reference: [17.3]

a.



b. $f(2) = f(4) = 1$

[34] c. f is not continuous on [2, 4]

Reference: [17.7]

[35] Rolle's Theorem applies; $c = 0$, 1, and 2.

Reference: [17.14]

[36] The Mean Value Theorem applies; $c = \frac{5}{2}$.

Reference: [18.3]

[37] Increasing $(-\infty, 0)$; decreasing $(0, \infty)$

Reference: [18.4]

[38] Increasing $(-\infty, 0)$ and $(2, \infty)$; decreasing (0, 2)

Reference: [18.10]

[39] Decreasing $(-\infty, 2)$ and $(2, \infty)$

Reference: [18.16]

[40] $\left(2, \frac{1}{54}\right)$, relative maximum

Reference: [18.17]

[41] Relative maximum: (–2, 20); relative minimum: (1, –7)

Reference: [18.22]

[42] Relative maximum at $x = \frac{3}{2}$.

Reference: [18.27]

$$f'(x) = \frac{1}{(1-x)^2} \neq 0 \text{ for all } x \neq 1.$$

[43] $f'(x)$ is undefined at $x = 1$, a vertical asymptote.

Reference: [18.28]

[44] Relative maximum

Reference: [19.4]

Concave upward: $(-\infty, 0)$, $(2, \infty)$
Concave downward: (0, 2)

[45] Points of inflection: (0, 2) and (2, –14)

Reference: [19.14]

[46] (4, 0), (2, –32)

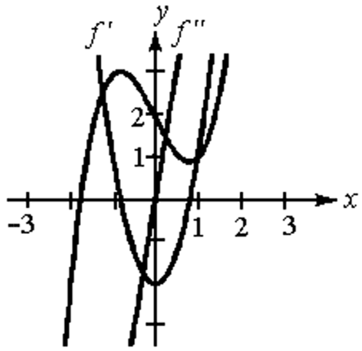
Reference: [19.18]

[47] (1, 4)

Reference: [19.21]

[48] $x = 0$, relative maximum; $x = \frac{2}{3}$, relative minimum

Reference: [19.32]

[49] 

Reference: [20.3]

[50] $y = 2$

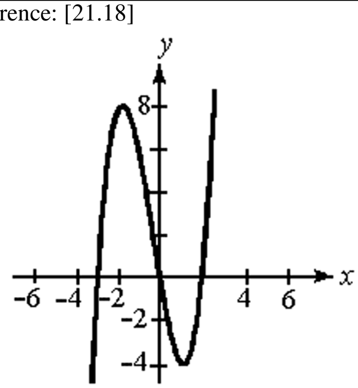
Reference: [20.17]

[51] ∞

Reference: [22.17]

[52] $8^2(2) = 128$ cubic inches

Reference: [21.18]

[53] 

Reference: [22.8]

[54] 75 feet by 150 feet

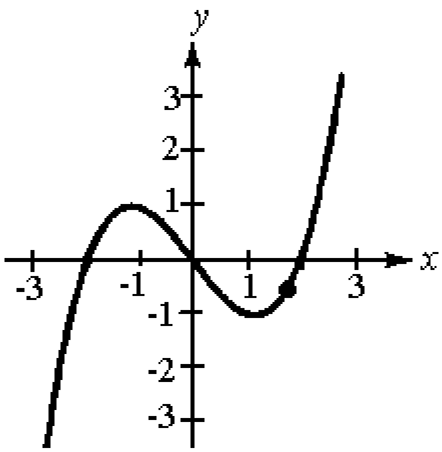
Reference: [22.28]

[55] 14

Reference: [24.3]

[56] ±0.001 in.

1. Determine whether the slope at the indicated point is positive, negative, or zero.
   a. None of these
   b. Zero
   c. No slope
   d. Negative
   e. Positive



2. If $f(x) = 2x^2 + 4$, which of the following will calculate the derivative of $f(x)$?
   a. $\lim\limits_{\Delta x \to 0} \dfrac{[2(x + \Delta x)^2 + 4] - (2x2 + 4)}{\Delta x}$
   b. $\dfrac{[2(x + \Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$
   c. $\lim\limits_{\Delta x \to 0} \dfrac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$
   d. $\dfrac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$
   e. None of these

3. Find an equation of the tangent line to the graph of $f(x) = x^2 - 2x - 3$ at the point $(-2, 5)$.

4. Find $f'(x)$: $f(x) = 4x^4 - 5x^3 + 2x - 3$.
   a. $16x^3 - 15x^2 + 2$
   b. None of these
   c. $16x^3 - 15x^2 + 2x - 3$
   d. $4x^4 - 5x^3 + 2x$
   e. $4x^3 - 5x^2 + 2$

5. Find $f'(x)$: $f(x) = \frac{1}{x^2}$.

a. $\frac{2}{x}$

b. None of these

c. $\frac{1}{x}$

d. $-\frac{1}{x^3}$

e. $-\frac{2}{x^3}$

6. Let $g(x) = 9f(x)$ and let $f'(-6) = -6$. Find $g'(-6)$.

a. −6

b. 9

c. −54

d. None of these

e. 0

7. Find the instantaneous rate of change of w with respect to z for $w = \frac{1}{z} + \frac{z}{2}$.

a. $\frac{z^2 - 2}{2z^2}$

b. $\frac{3}{2}$

c. None of these

d. −2

e. $\frac{-1}{z^2}$

8. Find an equation for the tangent line to the graph of $f(x) = -2x^2 + 2x + 3$ at the point where $x = 1$.

a. $y = -4x + 2$

b. $y = -4x^2 + 2x + 1$

c. None of these

d. $2x + y - 1 = 0$

e. $2x + y = 5$

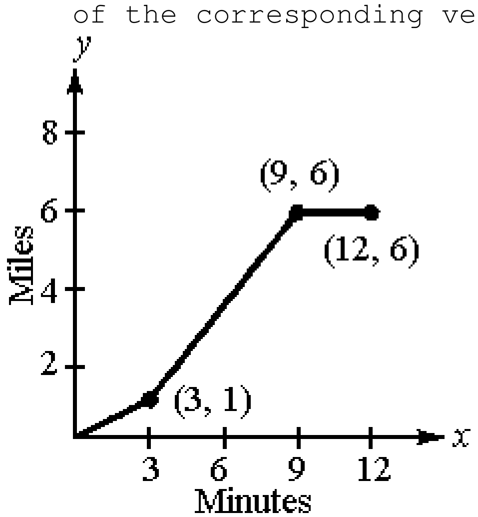
9. Find the point(s) on the graph of the function $f(x) = x^3 - 2$ where the slope is 3.
   a. (1, 3), (−1, 3)
   b. (1, −1), (−1, −3)
   c. $(\sqrt[3]{2}, 0)$
   d. (1, 3)
   e. None of these

10. Suppose the position equation for a moving object is given by $s(t) = 3t^2 + 2t + 5$ where s is measured in meters and t is measured in seconds. Find the velocity of the object when $t = 2$.
    a. 10 m/sec
    b. 6 m/sec
    c. None of these
    d. 14 m/sec
    e. 13 m/sec

11. Find the average rate of change of y with respect to x on the interval [0, 5], where $y = 2x^2 + x - 3$.

12. Find $\frac{dy}{dx}$: $y = 4 \sin x - 5 \cos x + x$.
    a. $4 \cos x + 5 \sin x$
    b. $-4 \cos x + 5 \sin x + 1$
    c. None of these
    d. $4 \cos x + 5 \sin x + 1$
    e. $4 \cos x - 5 \sin x + 1$

13. The given graph of a position function represents the distance in miles that a person drives during a 12-minute drive to school. Make a sketch of the corresponding velocity function.



14. Differentiate: $y = \dfrac{3x}{x^2 + 1}$.

a. $\dfrac{3(1 - x^2)}{(1 + x^2)^2}$

b. $\dfrac{3}{2x}$

c. None of these

d. $\dfrac{3x^2 - 3}{(1 + x^2)^3}$

e. $\dfrac{3}{1 + x^2}$

15. Let $f(7) = 0$, $f'(7) = 14$, $g(7) = 1$ and $g'(7) = \dfrac{1}{7}$. Find $h'(7)$ if $h(x) = f(x)/g(x)$.

a. −2
b. 14
c. −14
d. 98
e. None of these

16. If $f''(x) = -2x^2 + 7x - 2$, find $f^{(4)}(x)$.

a. $-4x + 7$
b. −4
c. $-2x + 7$
d. None of these
e. 0

17. Find y'' for $y = \dfrac{\csc x}{2}$.

18. Find $\dfrac{dy}{dx}$ for $y = \sqrt{x}(3x - 1)$.

a. $\dfrac{3}{2\sqrt{x}}$

b. $3\sqrt{x}$

c. $\dfrac{9}{2}\sqrt{x} - 1$

d. $\dfrac{9x - 1}{2\sqrt{x}}$

e. None of these

19. Find the derivative of $y = (x^2 + 2x + 5)^6$.

20. A particle moves along the curve given by $y = \sqrt{t^3 + 1}$. Find the acceleration when $t = 2$ seconds.

a. $-\dfrac{1}{9}$ units/sec^2

b. $\dfrac{2}{3}$ units/sec^2

c. None of these

d. 3 units/sec^2

e. $-\dfrac{1}{108}$ units/sec^2

21. Let $f(x) = \sqrt{x^2 + 1}$.

a. Calculate $f'(x)$.
b. Use a graphing utility to graph f and f' on the same axes.
c. Use the graph to determine those point(s) where f has a horizontal tangent line.
d. Give the value of f' at each of the points found in part **c**.

22. Find $\frac{dy}{dx}$ for $2x^2 + xy + 3y^2 = 0$.

a. $-\frac{4x + y}{x + 6y}$

b. None of these

c. $\frac{4x + 6y}{-x}$

d. $4x + y + 6y$

e. $-\frac{4x + y}{6y}$

23. Find $\frac{dy}{dx}$ if $x\sqrt{y} + y^2 = x$.

24. Find the slope of the curve $y^4 - xy^2 = x$ at the point $\left(\frac{1}{2}, 1\right)$.

25. A point moves along the curve $y = 2x^2 - 1$ in such a way that the y value is decreasing at the rate of 2 units per second. At what rate is x changing when $x = -\frac{3}{2}$?

a. Decreasing $\frac{1}{3}$ unit/sec

b. Decreasing $\frac{7}{2}$ unit/sec

c. None of these

d. Increasing $\frac{7}{2}$ unit/sec

e. Increasing $\frac{1}{3}$ unit/sec

26. As a balloon in the shape of a sphere is being blown up, the radius is increasing $\frac{1}{\pi}$ inches per second. At what rate is the volume increasing when the radius is 1 inch?

a. 3 in.^{3}/sec

b. 4π in.^{3}/sec

c. 3π in.^{3}/sec

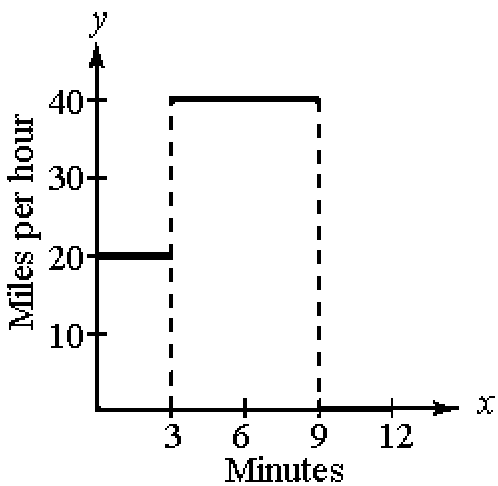
d. None of these

e. 4 in.^{3}/sec

27. Two boats leave the same port at the same time with one boat traveling north at 15 knots per hour and the other boat traveling west at 12 knots per hour. How fast is the distance between the two boats changing after 2 hours?
   a. 19.2 knots/hr
   b. 38.4 knots/hr
   c. 26.8 knots/hr
   d. 17.7 knots/hr
   e. None of these

28. The height of a cylinder with a radius of 4 cm is increasing at a rate of 2 centimeters per minute. Find the rate of change of the volume of the cylinder with respect to time when the height is 10 centimeters.
   a. $\frac{1}{16\pi}$ cm^3/min
   b. None of these
   c. 16π cm^3/min
   d. $\frac{5}{8\pi}$ cm^3/min
   e. 160π cm^3/sec

1. e
2. a
3. $6x + y + 7 = 0$
4. a
5. e
6. c
7. a
8. e
9. b
10. d
11. 11
12. d
13. See graph below.

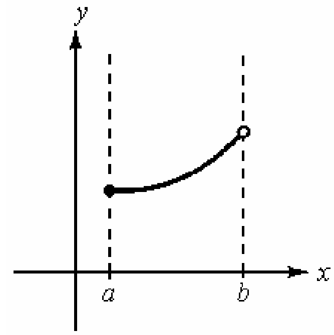


14. a
15. b
16. b
17. $\frac{1}{2}(\csc x)(2\csc^2 x - 1)$
18. d
19. $12(x + 1)(x^2 + 2x + 5)^5$
20. b
21. **a.** $\frac{x}{\sqrt{x^2 + 1}}$
    b. student to sketch graph
    c. (0, 1)
    d. 0
22. a
23. $\frac{2\sqrt{y} - 2y}{2y\sqrt{y} + x}$

24. $\frac{2}{3}$
25. e
26. e
27. a
28. b

AP Calculus Review
Chapter 3

1. Determine from the graph whether f possesses absolute extrema on the interval $[a, b]$.



   A. None of these
   B. Maximum at $x = b$; minimum at $x = a$
   C. Maximum at $x = a$; minimum at $x = b$
   D. No extrema
   E. No maximum; minimum at $x = a$

2. Decide whether Rolle's Theorem can be applied to $f(x) = x^2 + 3x$ on the interval $[0, 2]$. If the Rolle's Theorem can be applied, find all value(s) of c in the interval such that $f'(c) = 0$. If the Mean Value Theorem cannot be applied, state why.
   A. None of these
   B. Rolle's Theorem can be applied; $c = 0, -3$.
   C. Rolle's Theorem does not apply because $f(0) \neq f(2)$.
   D. Rolle's Theorem can be applied; $c = -\frac{3}{2}$
   E. Rolle's Theorem does not apply because $f(x)$ is not continuous on $[0, 2]$.

3. Which of the following statements is true of $f(x) = -x^3 - 6x^2 - 9x - 2$?
   A. f is decreasing on $(-3,-1)$.
   B. f is increasing on $(-\infty,-3)$.
   C. f is increasing on $(-2,\infty)$.
   D. None of these
   E. f is increasing on $(-3,-1)$.

4. Find the values of x that give relative extrema for the function $f(x) = (x + 1)^2(x - 2)$.
   A. Relative maxima: $x = 1$, $x = 3$; Relative minimum: $x = -1$
   B. Relative maximum: $x = -1$; Relative minimum: $x = 2$
   C. Relative minimum: $x = 2$
   D. None of these
   E. Relative maximum: $x = -1$; Relative minimum: $x = 1$

5. Given that $f(x) = -x^2 + 18x - 78$ has a relative maximum at $x = 9$, choose the correct statement.
   A. f' is positive on the interval $(-\infty,\infty)$.
   B. f' is positive on the interval $(9,\infty)$.
   C. f' is negative on the interval $(9,\infty)$.
   D. None of these
   E. f' is negative on the interval $(-\infty,9)$.

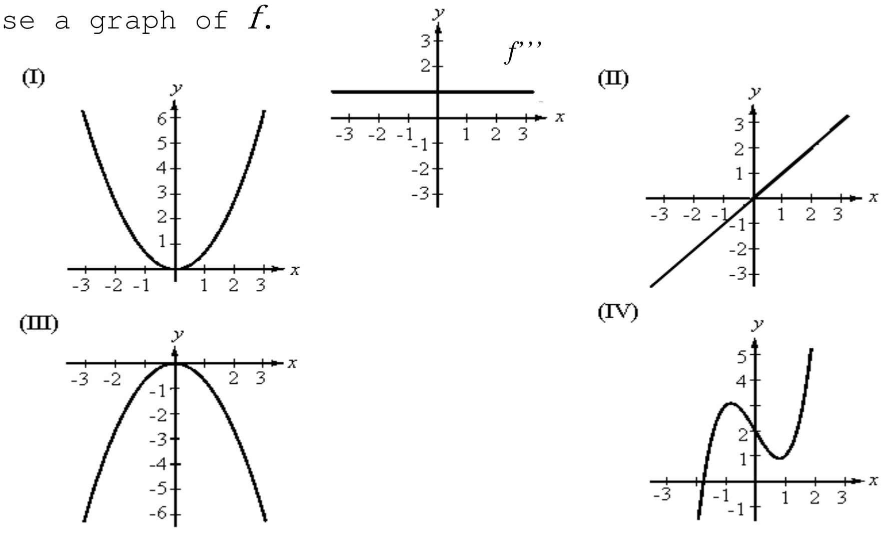
6. Find all intervals on which the graph of the function is concave upward:

$$f(x) = \frac{x^2+1}{x^2}$$

A. $(1, \infty)$
B. None of these
C. $(-\infty, -1)$ and $(1, \infty)$
D. $(-\infty, \infty)$
E. $(-\infty, 0)$ and $(0, \infty)$

7. The figure given in the graph is the **third** derivative of a polynomial function, f. Choose a graph of f.

A. graph II
B. graph III
C. graph I
D. graph IV
E. None of these.



8. Let $f(x)$ be a polynomial function such that $f(4) = -1$, $f'(4) = 0$, $f''(4) = 0$. If $x < 4$, then $f''(x) > 0$ and if $x > 4$, then $f''(x) < 0$. The point $(4, -1)$ is a ____________ of the graph of f.

A. Relative maximum
B. Critical number
C. Point of inflection
D. None of these
E. Relative minimum

9. Find $\lim\limits_{x\to\infty} \dfrac{2x^3+6x^2+5}{3x+2x^3}$

A. ∞
B. 2
C. None of these
D. $\dfrac{2}{3}$
E. 1

10. Find the limit: $\lim\limits_{x\to\infty} \dfrac{\sin x}{3x}$

A. ∞
B. 0
C. None of these
D. $\dfrac{1}{3}$
E. 3

11. Find the horizontal asymptotes (if any) for $f(x)=\dfrac{ax^3}{b+cx+dx^2}$

A. $y = 0$

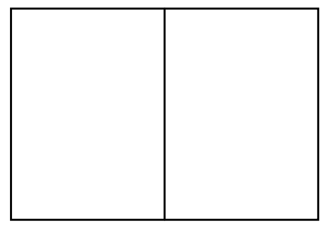
B. $y=\dfrac{a}{b}$

C. There are no horizontal asymptotes.

D. None of these

E. $y=\dfrac{a}{d}$

12. A farmer has 160 feet of fencing to enclose 2 adjacent rectangular pigpens. What dimensions should be used for each pigpen so that the enclosed area will be a maximum?



A. None of these

B. $20\,ft$ x $\dfrac{80}{3}\,ft$

C. $40\,ft$ x $\dfrac{80}{3}\,ft$

D. $40\,ft$ x $40\,ft$

E. $4\sqrt{15}\,ft$ x $\dfrac{8}{5}\sqrt{15}\,ft$

13. A nursery has determined that the demand in June for potted plants is $p = 2.00 - (x/30{,}000)$. The cost of growing x plants is $C = 2000 + 0.20x$, $0 \le x \le 100{,}000$. Find the marginal profit function.

A. $1.8x-\dfrac{x^2}{30{,}000}-2000$

B. None of these

C. $1.8-\dfrac{x}{15{,}000}$

D. $-\dfrac{6001}{30{,}000}$

E. $\dfrac{53{,}900}{30{,}000}$

14. Let $f''(x) = 4x^3 - 2x$ and let $f(x)$ have critical numbers −1, 0, and 1. Use the Second Derivative Test to determine which critical numbers, if any, give a relative maximum.

A. None of these

B. −1 and 1

C. −1

D. 0

E. 1

15. Find all points of inflection: $f(x)= x^3 - 12x$.

A. (2,0), (−2,0)

B. (0,0), $\left(\pm\sqrt{12},0\right)$

C. (2,−16),(−2,16)

D. (0,0)

E. None of These

1. e
2. c
3. e
4. e
5. c
6. e
7. d
8. c
9. e
10. b
11. c
12. b
13. c
14. c
15. d

Chapter 4 Review

1. Evaluate the integral: $\displaystyle\int (3x^2 - 2x + 5)\, dx$.

2. Evaluate the integral: $\displaystyle\int \frac{x^3 - x^2}{x^2}\, dx$.

3. Find the function, $y = f(x)$, if $f'(x) = 2x - 1$ and $f(1) = 3$.

4. Use $a(t) = -32$ feet per second squared as the acceleration due to gravity. An object is thrown vertically downward from the top of a 480-foot building with an initial velocity of 64 ft/s. With what velocity does the object hit the ground?

5. Evaluate the integral: $\displaystyle\int \frac{ax^2 + bx^3}{\sqrt{x}}\, dx$.

6. Use a graphing utility to graph $f(x) = x^4 - 6x^3 + 11x^2 - 6x$. Then use the upper sums to approximate the area of the region in the first quadrant bounded by f and the x-axis using four subintervals. (Round your answer to three decimal places.)

7. Write the definite integral that represents the area of the region enclosed by $y = 4x - x^2$ and the x axis.

8. Sketch the region whose area is indicated by the integral:
$$\int_0^3 \sqrt{9 - x^2}\, dx.$$

9. Determine if $f(x) = \dfrac{5}{2x - 3}$ is integrable on [0, 2]. Give a reason for your answer.

10. If $\int_2^5 f(x)\,dx = 5$ and $\int_4^5 f(x)\,dx = \pi$, find:

a. $\int_5^5 f(x)\,dx$

b. $\int_5^4 f(x)\,dx$

c. $\int_2^4 f(x)\,dx$

11. Evaluate: $\int_{\pi/4}^{\pi/3} \sec^2 x\, dx$.

12. Evaluate: $\int_0^3 |x - 2|\, dx$.

13. Find the average value of $f(x) = \sin x$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

14. Evaluate: $\frac{d}{dx}\int_3^x (2t^2 + 5)^2\, dt$.

15. Evaluate the integral: $\int_{\pi/4}^{3\pi/4} (-\csc^2 t)\, dt$.

16. Use a graphing utility to graph $f(x) = \cos x - \sin x$ on the interval $[0, \pi]$. Calculate the area in the first quadrant bounded by the x-axis and f.

17. Consider $F(x) = \int_2^x \frac{1}{1 + t^4}\, dt$. Find $F'(x)$ and $F'(2)$.

18. Consider $F(x) = \int_{x}^{1} \sqrt{1 + t^2}\, dt$. Find $F'(x)$.

19. Consider $F(x) = \int_{1}^{x^2} \left(t^3 + \sqrt{t}\right) dt$. Find $F'(x)$.

20. Find the value of c guaranteed by the Mean Value Theorem for Integrals for $f(x) = \dfrac{4}{x^2}$ on the interval [1, 4].

21. Evaluate the integral: $\int_{0}^{1} x\sqrt{1 - x^2}\, dx$.

22. Evaluate the integral: $\int_{0}^{1} x\sqrt{x^2 + 1}\, dx$.

23. Find the indefinite integral: $\int \left(x - \dfrac{1}{x}\right)^2 dx$.

24. Find the indefinite integral: $\int \dfrac{x}{\sqrt{x - 1}}\, dx$.

25. Evaluate the integral: $\int \dfrac{\sec^2 x}{\sqrt{\tan x}}\, dx$.

26. Evaluate the integral: $\int \dfrac{\cos \sqrt{x}}{\sqrt{x}}\, dx$.

27. Consider the integral, $\int_{\sqrt{2}}^{5} x(4x^2 - 3)^{19}\, dx$. Determine new upper and lower limits of integration using the substitution $u = 4x^2 - 3$.

28. Evaluate the definite integral: $\int_{-8}^{0} \frac{1}{\sqrt{1 - x}}\, dx$.

29. Use the Trapezoidal Rule, with $n = 4$, to approximate the area of the region bounded by the graphs of $y = \sin x$ and $y = 0$ on the interval $[0, \pi]$.

30. Let $f(x) = (x - 1)^2$.

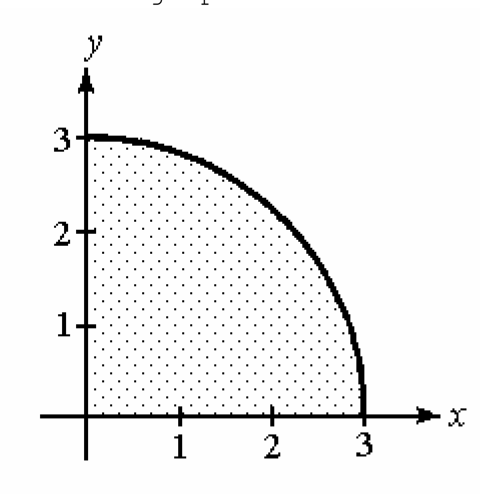
a. Sketch a graph of $f(x)$.

b. Divide the interval [1, 3] into four equal subintervals and label the markings on the x-axis.

c. Evaluate f at each of the values found in part **b**.

d. Use the Trapezoidal Rule to approximate $\int_{1}^{3} (x - 1)^2\, dx$.

1. $x^3 - x^2 + 5x + C$
2. $\frac{x^2}{2} - x + C$
3. $y = x^2 - x + 3$
4. -186.6 ft/s
5. $\frac{2a}{5}x^{5/2} + \frac{2b}{7}x^{7/2} + C$
6. 0.486
7. $\int_0^4 (4x - x^2)\,dx$
8. See graph below.



9. No, because $f(x)$ is not continuous at $x = \frac{3}{2}$.
10. **a.** 0
    b. $-\pi$
    c. $5 - \pi$
11. $\sqrt{3} - 1$
12. $\frac{5}{2}$
13. $\frac{2\sqrt{2}}{\pi}$
14. $(2x^2 + 5)^2$
15. -2
16. $\sqrt{2} - 1 \approx 0.414$
17. $F'(x) = \frac{1}{1 + x^4}$, $F'(2) = \frac{1}{17}$

18. $-\sqrt{1 + x^2}$

19. $\left[x^3 + \sqrt{x}\right](2x)$

20. 2

21. $\frac{1}{3}$

22. $\frac{1}{3}[2^{3/2} - 1]$

23. $\frac{x^3}{3} - 2x - \frac{1}{x} + C$

24. $\frac{2}{3}\sqrt{x - 1}(x + 2) + C$

25. $2\sqrt{\tan x} + C$

26. $2 \sin \sqrt{x} + C$

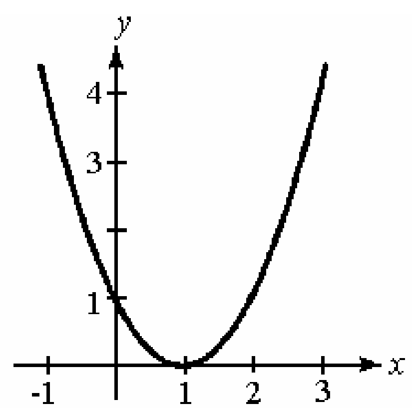
27. Upper, 97; lower, 5

28. 4

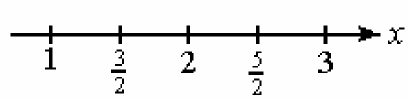
29. $\frac{\pi}{4}\left[1 + \sqrt{2}\right] \approx 1.896$

30. See graph below.

a.



b.



c. $f(1) = 0,\ f\left(\frac{3}{2}\right) = \frac{1}{4},\ f(2) = 1,\ f\left(\frac{5}{2}\right) = \frac{9}{4},\ f(3) = 4$

d. $\frac{11}{4}$

Calculus Chapter 5 Part A (Sections 1 through 5) Review

1. Expand completely: $\ln\left(\dfrac{3x^2}{7y}\right)$

2. Find the derivative: $y = \ln\left(\dfrac{\sqrt{x}}{5-x}\right)$.

   Note: Do not worry about getting into one fraction, just solve for y' and leave it.

3. Find an equation for the tangent to the graph of $f(x) = \ln(x^2 - 1)$ at the point where $x = 2$.

4. Evaluate the definite integral: $\displaystyle\int_2^{e+1} \frac{1}{x-1}\,dx$

5. Evaluate the integral: $\displaystyle\int \frac{x^2+x+1}{x^2+1}\,dx$

6. Determine the area in the region in the first quadrant bounded by $f(x) = \dfrac{16-x^2}{x}$, the x-axis, and $x = 1$.

7. Solve the differential equation: $\dfrac{dy}{dx} = \sec(2x)$

8. Given $f(x) = x^3 + 5$, find $f^{-1}(x)$.

9. Find $\left(f^{-1}\right)'(16)$ given $f(x) = \frac{1}{3}x^3 + \frac{5}{3}x + 2$ and $f(3) = 16$.

10. Find $\dfrac{dy}{dx}$ if $xe^y - x = y^2$.

11. Find the slope of the normal line to the graph of $y = \ln\left(xe^x\right)$ at the point where $x = 3$.

12. Evaluate the integral: $\displaystyle\int \frac{e^{3/x}}{x^2}\,dx$

13. Find $\dfrac{dy}{dx}$: $y = 5^{2x-3}$.

14. Find $\dfrac{dy}{dx}$ if $y = \log_5(5x+6)$.

15. Evaluate the integral: $\displaystyle\int 2x^2 5^{x^3}\,dx$

16. If an annual rate of salary increase averages 3.75% over the next 5 years, then the approximate salary S during any year in that period is $S(t) = P(1.0375)^t$ where t is the time in years and P is the present salary, \$30,000. Find the rate of change of S with respect to t when $t = 4$.

Key

Calculus Chapter 5 Part A (Sections 1 through 5) Review

1. Expand completely: $\ln\left(\frac{3x^2}{7y}\right)$ $= \ln 3 - \ln 7 + 2\ln x - \ln y$

2. Find the derivative: $y = \ln\left(\frac{\sqrt{x}}{5-x}\right)$. $= \frac{1}{2x} + \frac{1}{5-x}$

   Note: Do not worry about getting into one fraction, just solve for y' and leave it.

3. Find an equation for the tangent to the graph of $f(x) = \ln(x^2 - 1)$ at the point where $x = 2$.

   $y - \ln 3 = \frac{4}{3}(x-2)$

4. Evaluate the definite integral: $\int_2^{e+1} \frac{1}{x-1}\,dx$ $= 1$

5. Evaluate the integral: $\int \frac{x^2 + x + 1}{x^2 + 1}\,dx$ $= x + \frac{1}{2}\ln(x^2+1) + C$

6. Determine the area in the region in the first quadrant bounded by $f(x) = \frac{16 - x^2}{x}$, the x-axis, and $x = 1$.

   ≈ 14.681

7. Solve the differential equation: $\frac{dy}{dx} = \sec(2x)$ $= \frac{1}{2}\ln\left|\sec(2x) + \tan(2x)\right| + C$

8. Given $f(x) = x^3 + 5$, find $f^{-1}(x)$. $= \sqrt[3]{x-5}$

9. Find $\left(f^{-1}\right)'(16)$ given $f(x) = \frac{1}{3}x^3 + \frac{5}{3}x + 2$ and $f(3) = 16$. $3/32$

10. Find $\frac{dy}{dx}$ if $xe^y - x = y^2$. $\frac{e^y - 1}{2y - xe^y}$

11. Find the slope of the normal line to the graph of $y = \ln\left(xe^x\right)$ at the point where $x = 3$. $-3/4$

12. Evaluate the integral: $\int \frac{e^{3/x}}{x^2}\,dx$ $= -\frac{1}{3}e^{3/x} + C$

13. Find $\frac{dy}{dx}$: $y = 5^{2x-3}$. $2\ln(5)\cdot 5^{2x-3}$

14. Find $\frac{dy}{dx}$ if $y = \log_5(5x+6)$. $\frac{5}{(5x+6)\ln(5)}$

15. Evaluate the integral: $\int 2x^2 5^{x^3}\,dx$ $\frac{2\cdot 5^{x^3}}{3\ln(5)} + C$

16. If an annual rate of salary increase averages 3.75% over the next 5 years, then the approximate salary S during any year in that period is $S(t) = P(1.0375)^t$ where t is the time in years and P is the present salary, \$30,000. Find the rate of change of S with respect to t when $t = 4$. \$1279.64

1. Choose the expression equivalent to $\ln \dfrac{9x^2}{2y}$.

a. $\ln 9 - \ln 2 + 2 \ln x - \ln y$
b. $2 \ln(9x) - \ln(2y)$
c. $\ln(9x^2) + \ln(2y)$
d. None of these
e. $\dfrac{\ln 9 + \ln x^2}{\ln 2 + \ln y}$

2. Find the derivative: $f(x) = \ln \dfrac{x^2\sqrt{4x + 1}}{(x^3 + 5)^3}$.

a. $\dfrac{x}{9x^2(x^3 + 5)^2 \sqrt{4x + 1}}$
b. $\dfrac{2}{x} - \dfrac{2}{4x + 1} - \dfrac{9x^2}{x^3 + 5}$
c. $\dfrac{2}{x} + \dfrac{1}{2(4x + 1)} - \dfrac{3}{x^3 + 5}$
d. $\dfrac{2}{x} + \dfrac{2}{4x + 1} - \dfrac{9x^2}{x^3 + 5}$
e. None of these

3. Find the derivative: $f(x) = \ln \dfrac{x(x^2 + 2)}{\sqrt{x^3 - 7}}$.

a. $\dfrac{x^2 + 2}{x} + \dfrac{2x^2}{x^2 + 2} - \dfrac{3x^2}{2(x^3 - 7)}$
b. None of these
c. $\dfrac{1}{x} + \dfrac{2x}{x^2 + 2} - \dfrac{3x^2}{2(x^3 - 7)}$
d. $\dfrac{x^2 + 2}{x} + \dfrac{2x^2}{x^2 + 2} + \dfrac{3x^2}{2(x^3 - 7)}$
e. $\dfrac{1}{x} + \dfrac{2x}{x^2 + 2} + \dfrac{3x^2}{2(x^3 - 7)}$

4. Solve for x: $\ln(5x + 1) + \ln x = \ln 4$.
   a. e^4, $e^{3/5}$
   b. None of these
   c. $\frac{4}{5}$
   d. $\frac{3}{5}$, 4
   e. -1, $\frac{4}{5}$

5. Use logarithmic differentiation to find $\frac{dy}{dx}$: $y = \frac{x^3\sqrt{2x + 3}}{(x - 2)^2}$.

6. Find an equation for the tangent line to the graph of $f(x) = \ln(x^2 - 1)$ at the point where $x = 2$.
   a. $4x - 3y = -1$
   b. $4x - 3y = 8 - \ln 27$
   c. None of these
   d. $4x - y = 8 - \ln 3$
   e. $4x - 3y = 8$

7. Evaluate the definite integral: $\int_1^{\sqrt{e}} \frac{-4x}{x^2}\, dx$.
   a. -2
   b. -4
   c. None of these
   d. -6
   e. -1

8. Evaluate the integral: $\int \frac{2x + 1}{x + 1}\, dx$.
   a. None of these
   b. $2x + C$
   c. $x^2 + \ln|x + 1| + C$
   d. $\frac{2x^2 + 2x}{x^2 + 2x} + C$
   e. $2x - \ln|x + 1| + C$

9. Evaluate the integral: $\int \frac{8x^2 + 9x + 8}{x^2 + 1}\,dx$.

a. None of these

b. $8x + 9\ln(x^2 + 1) + C$

c. $8x + \frac{9}{2}\ln(x^2 + 1) + C$

d. $8 + \frac{9}{2}\ln(x^2 + 1) + C$

e. $8 + 9\ln(x^2 + 1) + C$

10. Evaluate the integral: $\int \tan 3x\,dx$.

a. $\frac{1}{3}\ln|\sec 3x| + C$

b. None of these

c. $\ln|\cos 3x| + C$

d. $3\sec^2 3x + C$

e. $\frac{1}{3}\sec^2 3x$

11. Evaluate the integral: $\int \frac{\sin^2 x - \cos^2 x}{\sin x}\,dx$.

a. $-2\cos x + \ln|\csc x + \cot x| + C$

b. $-\ln|\csc x + \cot x| + C$

c. $-\sec x + C$

d. None of these

e. $\cos x + \ln|\csc x + \cot x| + C$

12. Evaluate the integral: $\int \frac{t^2 + 1}{t + 1}\,dt$.

a. $\frac{t^2 - 2t - 1}{(t + 1)^2} + C$

b. $\frac{1}{2}t^2 - t + \ln(t + 1)^2 + C$

c. None of these

d. $t^2 - 2t + \ln(t + 1)^4 + C$

e. $t + C$

13. Solve the differential equation: $\dfrac{ds}{dt} = \dfrac{\sec t \tan t}{\sec t + 5}$.

a. $s = \dfrac{1}{5} \ln \left|\sec t\right| + C$

b. None of these

c. $s = \ln \left|\sec t + 5\right| + C$

d. $s = \dfrac{1}{5} \tan t + C$

e. $s = 2 \sec^3 t - \sec t + C$

14. A population of bacteria is changing at the rate of $\dfrac{dP}{dt} = \dfrac{2000}{1 + 0.2t}$, where t is the time in days. The initial population is 1000.

a. Write an equation that gives the population at any time t.

b. Find the population after 10 days.

15. Determine whether the function $f(x) = \dfrac{7}{x + 2}$ is one-to-one. If it is, find its inverse.

a. $f^{-1}(x) = \dfrac{7}{x + 2}$

b. Not one-to-one

c. None of these

d. $f^{-1}(x) = \dfrac{7 - 2x}{x}$

e. $f^{-1}(x) = \dfrac{x + 2}{7}$

16. Let $f(x) = \sqrt{3x^3 - 1}$. Calculate $f^{-1}(x)$.

a. $\sqrt{\dfrac{3}{x^3} - 1}$

b. $\sqrt[3]{\dfrac{x^2}{3} + 1}$

c. $\dfrac{1}{\sqrt{3x^3 - 1}}$

d. None of these

e. $\sqrt[3]{\dfrac{x^2 + 1}{3}}$, $x \geq 0$

17. Determine whether $f(x) = \dfrac{x - b}{a}$ is one-to-one; if it is, find f^{-1}.

a. $\dfrac{x - a}{b}$

b. f is not one-to-one

c. $ax + b$

d. $\dfrac{a}{x - b}$

e. None of these

18. Find $(f^{-1})'(12)$ for the function $f(x) = \dfrac{1}{3}x^3 + \dfrac{5}{3}x + 2$.

19. Find $f'(x)$ for $f(x) = \sqrt{4 + e^{2x}}$.

a. $\dfrac{1}{2\sqrt{2e^{2x}}}$

b. $\dfrac{e^{2x}}{\sqrt{4 + e^{2x}}}$

c. e^{x}

d. $\dfrac{xe^{2x-1}}{\sqrt{4 + e^{2x}}}$

e. None of these

20. Find the slope of the tangent line to the graph of $y = (\ln x)e^{x}$ at the point where $x = 2$.

a. $e^{2}\left(\ln 2 + \dfrac{1}{2}\right)$

b. None of these

c. $e(2 \ln 2 + 1)$

d. e

e. $\dfrac{1}{2}e^{2}$

21. Evaluate the integral: $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

a. $2e^{\sqrt{x}} + C$

b. $\sqrt{x}\,e^{\sqrt{x}} + C$

c. $\dfrac{1}{2}e^{\sqrt{x}} + C$

d. $\sqrt{x}\,e^{\sqrt{x}+1} + C$

e. None of these

22. Evaluate the indefinite integral: $\displaystyle\int \frac{1}{x^2 e^{2/x}}\, dx$.

a. $\frac{1}{2}e^{2/x} + C$
b. $\frac{1}{2}e^{-2/x} + C$
c. $\frac{1}{2}xe^{-2/x} + C$
d. $\frac{1}{2}xe^{2/x} + C$
e. None of these

23. Calculate the area of the region bounded by $y = e^{2x}$, $y = 0$, $x = 1$, $x = 4$.

24. Evaluate the integral: $\displaystyle\int e^{(ax+b)}\, dx$.

a. $ae^{(ax+b)} + C$
b. None of these
c. $\frac{1}{a}e^{(ax+b)} + C$
d. $e^{(ax+b)} + C$
e. $e^{(ax^2/2+bx+c)}$

25. Find $\frac{dy}{dx}$ if $y = \frac{x^3}{3^x}$.

a. $\dfrac{3x^2}{3^x(\ln 3)}$
b. None of these
c. $\dfrac{x}{3^{x-2}}$
d. $\dfrac{x^2(9 - x^2)}{3^{x+1}}$
e. $\dfrac{x^2[3 - x(\ln 3)]}{3^x}$

26. Differentiate: $y = x^{e^x}$.

a. $x^{e^x}\left[\dfrac{e^x}{x} + (\ln x)(e^x)\right]$

b. $e^x x^{e^x - 1}$

c. e^x

d. $xe^x + e^x$

e. None of these

27. Find the area bounded by the function $f(x) = 2^{-x}$, the x-axis, $x = -2$, and $x = 1$.

28. If an annual rate of salary increase averages 4.5% over the next 5 years, then the approximate salary, S, during any year in that period is $S(t) = P(1.045)^t$ where t is the time in years and P is the present salary.

a. If a person's salary is $30,000 now, use the function S to estimate her salary 5 years from now.

b. Use the model given to estimate how long it will be before this individual earns $50,000.

c. Find the rate of change of S with respect to t when $t = 1$ and when $t = 4$.

29. A certain type of bacteria increases continuously at a rate proportional to the number present. If there are 500 present at a given time and 1000 present 2 hours later, how many will there be 5 hours from the initial time given?

a. None of these
b. 1750
c. 2828
d. 3000
e. 2143

30. A certain type of bacteria increases continuously at a rate proportional to the number present. If there are 500 present at a given time and 1000 present 2 hours later, how many hours (from the initial given time) will it take for the numbers to be 2500? Round your answer to 2 decimal places.

31. Solve the differential equation: $2y' = y$.

a. $y = Ce^{x/2}$

b. $y = e^{2x} + C$

c. $2y = \dfrac{y^2}{2} + C$

d. None of these

e. $y = e^{x/2} + C$

32. Find the function $y = f(x)$ passing through the point (0, 6) that has the first derivative $\dfrac{dy}{dx} = y - 2$.

33. Use integration to find a general solution to the differential equation $y' = \dfrac{2}{\sqrt{1 - x^2}} + x$.

a. $\arcsin\left(\dfrac{x}{2}\right) + C$

b. $2 \arcsin x + x^2 + C$

c. None of these

d. $2 \arcsin x + \dfrac{x^2}{2} + C$

e. $2 \arcsin x^2 + \dfrac{x^2}{2} + C$

34. Use integration to find a general solution to the differential equation $\dfrac{dy}{dx} = \dfrac{3}{1 + x^2}$.

35. A colony of bacteria increases at a rate proportional to the number present. If there were 1000 bacteria present in the beginning of the experiment and the number triples in four hours, determine the number present as a function of time.

36. Find the general solution of the differential equation $y' = \dfrac{\sin x}{\cos y}$.

a. $\sin y = C \cos x$

b. $\sin y + \cos x = C$

c. $\sin y - \cos x = C$

d. $\tan y = C$

e. None of these

37. Find the particular solution of the differential equation $\dfrac{dy}{dx} = 500 - y$ that satisfies the initial condition $y(0) = 7$.

38. Evaluate: $\arccos\left(-\dfrac{1}{2}\right)$.

a. $\dfrac{2\pi}{3}$

b. $\dfrac{\pi}{3}$

c. $\dfrac{\pi}{6}$

d. None of these

e. $-\dfrac{\pi}{3}$

39. Evaluate: $\cos\left[\arctan\left(-\dfrac{2}{3}\right)\right]$.

a. $-\dfrac{2\sqrt{13}}{13}$

b. None of these

c. $\dfrac{2\sqrt{13}}{13}$

d. $-\dfrac{3\sqrt{13}}{13}$

e. $\dfrac{3\sqrt{13}}{13}$

40. Find the exact value: $\cos\left[\arctan\left(-\dfrac{3}{10}\right)\right]$.

41. Write an algebraic expression for tan [arcsin x].

a. $\dfrac{x\sqrt{1 + x^2}}{1 + x^2}$

b. $\dfrac{x\sqrt{1 - x^2}}{1 - x^2}$

c. None of these

d. $\dfrac{1}{x}$

e. $\dfrac{\sqrt{1 - x2}}{x}$

42. Differentiate: $f(x) = \arcsin \sqrt{1 - 36x^2}$.

a. $-\dfrac{6}{1 - 36x^2}$

b. $\dfrac{6x}{|x|\sqrt{1 - 36x^2}}$

c. $\dfrac{6x}{|x|\sqrt{1 - 36x^2}}$

d. $\dfrac{6}{1 - 36x^2}$

e. None of these

43. Find the derivative: $g(x) = \operatorname{arcsec} \frac{x}{2}$.

a. $\dfrac{1}{\sqrt{4 - x^2}}$

b. $\dfrac{1}{\sqrt{x^2 - 4}}$

c. $\dfrac{4}{x\sqrt{x^2 - 4}}$

d. None of these

e. $\dfrac{2}{x\sqrt{x^2 - 4}}$

44. Evaluate: $\displaystyle\int \frac{x + 2}{\sqrt{4 - x^2}}\, dx$.

a. $x^2 + 2x + \arcsin \frac{x}{2} + C$

b. $\ln|2 - x| + C$

c. $-\frac{1}{2}\sqrt{4 - x^2} + 2 \arcsin \frac{x}{2} + C$

d. $-\sqrt{4 - x^2} + 2 \arcsin \frac{x}{2} + C$

e. None of these

45. Evaluate: $\displaystyle\int \frac{5}{x^2 + 6x + 13}\, dx$.

a. $5\left(\frac{x^3}{3} + 3x^2 + 13x\right) + C$

b. $\frac{5}{2} \arctan \frac{x + 3}{2} + C$

c. None of these

d. $5 \ln|x^2 + 6x + 13| + C$

e. $-\frac{5}{x} + \frac{5}{6} \ln|x| + \frac{5}{13}x + C$

46. Find the indefinite integral: $\int \frac{x}{16 + x^4}\, dx$.

a. None of these

b. $\frac{1}{8} \operatorname{arcsec} \frac{x^2}{4} + C$

c. $\frac{1}{2} \arcsin \frac{x^2}{4} + C$

d. $\frac{1}{4} \arctan \frac{x^2}{4} + C$

e. $\frac{1}{8} \arctan \frac{x^2}{4} + C$

47. Evaluate the integral: $\int \frac{1}{2x\sqrt{4x^2 - 1}}\, dx$.

a. $\frac{1}{2} \arcsin \left|2x\right| + C$

b. $\frac{1}{2} \operatorname{arcsec} \left|2x\right| + C$

c. $\frac{1}{8}\sqrt{4x^2 - 1} + C$

d. $\operatorname{arcsec} \left|2x\right| + C$

e. None of these

48. Evaluate the definite integral: $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{3}{\sqrt{4 - 9x^2}}\, dx$.

49. Evaluate the integral: $\displaystyle\int \frac{(\arctan x)^3}{1 + x^2}\,dx$.

a. $\dfrac{3(\arctan x)^2}{(1 + x^2)^2} + C$
b. None of these
c. $\dfrac{1}{4}(\arctan x)^5 + C$
d. $\dfrac{1}{4}(\arctan x)^4 + C$
e. $\dfrac{3(\arctan x)^2}{2x(1 + x^2)} + C$

50. Evaluate the definite integral: $\displaystyle\int_1^4 \frac{1}{x^2 - 2x + 10}\,dx$.

a. $-\dfrac{\pi}{4}$
b. $\dfrac{\pi}{12}$
c. 1.249
d. -0.0419
e. None of these

51. Consider $f(x) = \dfrac{1}{4 + 9x^2}$.

a. Use a graphing utility to graph f.
b. Sketch the region bounded by f, the x-axis, and the line $x = 1$ and $x = 2$.
c. Write the integral that represents the area of the region.
d. Calculate the area.

1. a
2. d
3. d
4. c
5. $$y\left[\frac{3}{x}+\frac{1}{2x+3}-\frac{2}{x-2}\right]=\frac{x^3\sqrt{2x+3}}{(x-2)^2}\left[\frac{3}{x}+\frac{1}{2x+3}-\frac{2}{x-2}\right]$$
6. b
7. a
8. e
9. c
10. a
11. a
12. b
13. c
14. **a.** $P = 10{,}000 \ln(1 + 0.2t) + 1000$
    b. $10{,}000 \ln 3 + 1000 \approx 11{,}986$
15. d
16. e
17. c
18. $\frac{3}{32}$
19. b
20. a
21. a
22. b
23. $\frac{1}{2}e^2(e^6 - 1)$
24. c
25. e
26. a
27. $\frac{7}{2 \ln 2}$
28. **a.** \$37,385.46
    b. 11.6 years
    c. 1379.93, 1574.73
29. c
30. 4.64
31. a
32. $y = 4e^x + 2$
33. d
34. $y = 3 \arctan x + C$
35. $y = 1000e^{(t \ln 3)/4}$
36. b
37. $y = 500 - 493e^{-x}$
38. a
39. e

40. $\dfrac{10\sqrt{109}}{109}$

41. b
42. b
43. e
44. d
45. b
46. e
47. b
48. $\dfrac{2\pi}{3}$

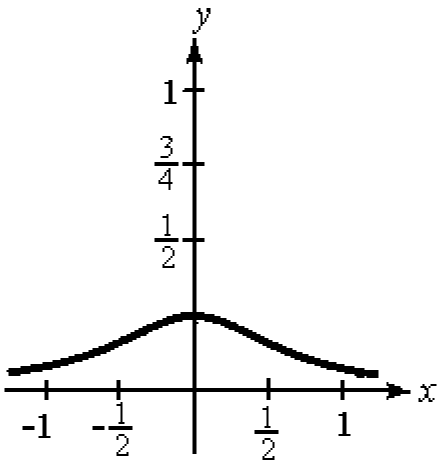
49. d
50. b
51.

a. See graph below.
b. See graph below.

c. $\displaystyle\int_1^2 \frac{1}{4 + 9x^2}dx$

d. $\dfrac{1}{6}\left[\arctan 3 - \arctan \dfrac{3}{2}\right] \approx 0.0444$

a.



b.