CALCULUS AB AP CHAPTER 1 TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-8: Calculator, 25 minutes. Choose the letter that best completes the statement or answers the question.

1. A population grows according to the equation $P(t) = 6000 - 5500e^{-0.159t}$ for $t \geq 0$. This population will approach a limiting value as time goes on. During which year will the population reach half its limiting value?

(a) second (b) third (c) fourth (d) eighth (e) twenty-ninth

2. Consider the function $f(x) = \begin{cases} \dfrac{\sin x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$

In order for $f(x)$ to be continuous, the value of k must be _____.

(a) 0 (b) 1 (c) -1 (d) π (e) any number greater than 1

3. $\lim\limits_{x \to -3} \dfrac{x^2 + 3x}{\sqrt{x^2 + 6x + 9}} =$ _____.

(a) -3 (b) -1 (c) 1 (d) 3 (e) nonexistent

4. For all $x > 0$, if $f(\ln x) = x^2$, then $f(x) =$ _____.

(a) $\sqrt{e^x}$ (b) $2 \ln x$ (c) $e^{\sqrt{x}}$ (d) $\sqrt{\ln x}$ (e) e^{2x}

5. How many zeros does the function $y = \sin(\ln x)$ have for $0 < x \leq 1$?

(a) One (b) Two (c) Three (d) Four (e) More than four

6. The function $f(x) = \tan(3^x)$ has one zero in the interval $[0, 1.4]$. The derivative at this point is _____.

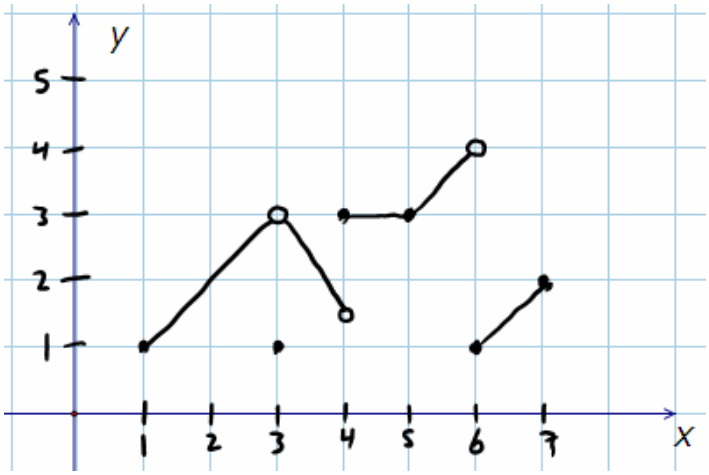
(a) 0.411 (b) 1.042 (c) 3.451 (d) 3.763 (e) undefined

7. The table below gives the values of three functions f, g, and h near $x = 0$. Based on the values given, for which of the functions does it appear that the limit as x approaches zero is 2?

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
$f(x)$	2.018	2.008	2.002	2	2.002	2.008	2.018
$g(x)$	1	1	1	2	2	2	2
$h(x)$	1.971	1.987	1.997	undefined	1.997	1.987	1.971

(a) f only (b) g only (c) h only (d) f and h only (e) f, g, and h

8. The graph of a function f whose domain is the closed interval $[1, 7]$ is shown below. Which of the following statements about $f(x)$ is true?



(a) $\lim\limits_{x \to 3} f(x) = 1$ (b) $\lim\limits_{x \to 4} f(x) = 3$ (c) $f(x)$ is continuous at $x = 3$

(d) $f(x)$ is continuous at $x = 5$ (e) $\lim\limits_{x \to 6} f(x) = f(6)$

Numbers 9-15: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

9. The $\lim_{h\to 0}\dfrac{(x+h)^3-x^3}{h}$ at the point $x=2$ is _____.

(a) 36 (b) 12 (c) 8 (d) 2 (e) 0

10. $\lim_{x\to b}\dfrac{b-x}{\sqrt{x}-\sqrt{b}}=$ _____.

(a) $-2\sqrt{b}$ (b) $-\sqrt{b}$ (c) $2b$ (d) $\sqrt{b}$ (e) $2\sqrt{b}$

11. If $f(x)=x^3-x+3$ and if c is the only real number such that $f(c)=0$, then c is between _____.

(a) -2 and -1 (b) -1 and 0 (c) 0 and 1 (d) 1 and 2 (e) 2 and 3

12. Suppose that f is a continuous function defined for all real numbers x and $f(-5)=3$ and $f(-1)=-2$. If $f(x)=0$ for one and only one value of x, then which of the following could be x?

(a) -7 (b) -2 (c) 0 (d) 1 (e) 2

13. $\lim_{x\to 3}\dfrac{(3-x)^2}{(x-3)}=$ _____.

(a) 0 (b) -2 (c) 1 (d) -1 (e) nonexistent

14. If $f(x)=\dfrac{x^2-9}{x+3}$ is continuous at $x=-3$, then $f(-3)=$ _____.

(a) 3 (b) -3 (c) 0 (d) 6 (e) -6

15. A function $f(x)$ has a vertical asymptote at $x=2$. The derivative of $f(x)$ is positive for all $x\neq 2$. Which statement is true?

I. $\lim_{x\to 2} f(x)=+\infty$ II. $\lim_{x\to 2^+} f(x)=+\infty$ III. $\lim_{x\to 2^-} f(x)=+\infty$

(a) I only (b) II only (c) III only (d) I and II only (e) I, II, and III

1. C	I-44		
2. B	II-38		
3. E	I-30		
4 .E	III-41	PC	
5. E	III-39	PC	(more TI syntax – zeroes)
6. C	II-40	PC	(s's have to use nderiv, so teach them beforehand)
7. D	I-37		
8. D	IV-3		
9. B	VI-1		
10. A	IV-1		
11. A	V-3		
12. B	II-3		
13. A	III-2		
14. E	I-1		
15. C	III-14		

---

test review probs.:

CALCULUS AB AP CHAPTER 2.1-2.4 QUIZ

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. A particle moves along the *x*-axis so that its position at any time $t > 0$ is given by $x(t) = t^4 - 10t^3 + 29t^2 - 36t + 2$. For which value of t is the speed the greatest?

(a) $t = 1$ (b) $t = 2$ (c) $t = 3$ (d) $t = 4$ (e) $t = 5$

2. If $\lim_{x \to 3} \dfrac{g(3) - g(x)}{3 - x} = -0.628$, then at the point $x = 3$, the graph of $g(x)$ _____.

(a) is decreasing (b) is increasing (c) is concave upwards
(d) is concave downwards (e) attains a relative minimum point

3. The table below gives values of a differentiable function f. What is the approximate value of $f'(4)$?

x	3.998	3.999	4	4.001	4.002
f(x)	1.15315	1.15548	1.15782	1.16016	1.1625

(a) 0.00234 (b) 0.289 (c) 0.427 (d) 2.34 (e) $f'(4)$ cannot be determined from the information given

4. At how many points on the curve $y = 4x^5 - 3x^4 + 15x^2 + 6$ will the line tangent to the curve pass through the origin?

(a) One (b) Two (c) Three (d) Four (e) Five

5. Let f and g be differentiable functions such that $f(1) = 4, g(1) = 3, f'(3) = -5, f'(1) = -4, g'(1) = -3, g'(3) = 2$. If $h(x) = f(g(x))$, then $h'(1) =$ _____.

(a) -9 (b) 15 (c) 0 (d) -5 (e) -12

Numbers 6-12: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

6. A particle moves along the *x*-axis so that its position at any time $t \geq 0$ is given by $x(t) = 3t^3 - 18t^2 + 24t$. At which time t is its average velocity zero?

(a) Never (b) 0 only (c) 2 only (d) 2 and 4 only (e) 0, 2, and 4

7. If $f(x) = (x-1)^2 \cos x$, then $f'(0) =$ _____.

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

8. $\lim_{h \to 0} \dfrac{\tan(2(x+h)) - \tan(2x)}{h} =$ _____.

(a) 0 (b) $2\cot(2x)$ (c) $\sec^2(2x)$ (d) $2\sec^2(2x)$ (e) nonexistent

9. If $f(x) = x\sqrt[3]{x}$, then $f'(x) =$ _____.

(a) $4x^3$ (b) $\frac{3}{7}x^{\frac{7}{3}}$ (c) $\frac{4}{3}x^{\frac{1}{3}}$ (d) $\frac{1}{3}x^{\frac{1}{3}}$ (e) $\frac{1}{3}x^{-\frac{2}{3}}$

10. The equation of the tangent line to the curve $y = \dfrac{3x+4}{4x-3}$ at the point $(1, 7)$ is _____.

(a) $y + 25x = 32$ (b) $y - 31x = -24$ (c) $y - 7x = 0$ (d) $y + 5x = 12$ (e) $y - 25x = -18$

11. If $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x + 1, & x > 1 \end{cases}$, then $f'(1) =$ _____.

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3 (e) nonexistent

12. What is the 20th derivative of $y = \sin(2x)$?

(a) $-2^{20}\sin(2x)$ (b) $2^{20}\sin(2x)$ (c) $-2^{19}\cos(2x)$ (d) $2^{20}\cos(2x)$ (e) $2^{21}\cos(2x)$

1. D IV-30
2. A III-38
3. D II-37
4. A I-43
5. B I-34
6. E V-9
7. A VI-3
8. D IV-26
9. C V-16
10. A VI-4
11. C V-5
12. B V-12

---

quiz review probs.:

CALCULUS AB AP CHAPTER 2 TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. If $x+y=xy$, then $\frac{dy}{dx}=$ _____.

(a) $\frac{1}{x-1}$ (b) $\frac{y-1}{x-1}$ (c) $\frac{1-y}{x-1}$ (d) $x+y-1$ (e) $\frac{2-xy}{y}$

2. A missile rises vertically from a point on the ground 75,000 feet from a radar station. If the missile is rising at the rate of 16,500 feet per minute at the instant when it is 38,000 feet high, what is the rate of change, in radians per minute, of the missile's angle of elevation from the radar station at this instant?

(a) 0.175 (b) 0.219 (c) 0.227 (d) 0.469 (e) 0.507

3. Two cars start at the same place and at the same time. One car travels west at a constant speed of 50 miles per hour and a second car travels south at a constant speed of 60 miles per hour. Approximately how fast is the distance between them changing one-half hour later?

(a) 72 miles per hour (b) 74 miles per hour (c) 76 miles per hour (d) 78 miles per hour (e) 80 miles per hour

4. The equation of the line tangent to the curve $y=\frac{kx+8}{k+x}$ at $x=-2$ is $y=x+4$. What is the value of k?

(a) -3 (b) -1 (c) 1 (d) 3 (e) 4

5. If $f(x)=(2+3x)^4$, then the fourth derivative of f is _____.

(a) 0 (b) $4!(3)$ (c) $4!(3^4)$ (d) $4!(3^5)$ (e) $4!(2+3x)$

6.: NO calculator, 10 minutes.

Consider the curve given by $x^2+4y^2=7+3xy$.

(a) Show that $\frac{dy}{dx}=\frac{3y-2x}{8y-3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. If $y^2-2xy=21$, then $\frac{dy}{dx}$ at the point $(2,-3)$ is _____.

(a) $-\frac{6}{5}$ (b) $-\frac{3}{5}$ (c) $-\frac{2}{5}$ (d) $\frac{3}{8}$ (e) $\frac{3}{5}$

8. The volume of a cube is increasing at the rate of 20 cubic centimeters per second. How fast, in square centimeters per second, is the surface area of the cube increasing at the instant when each edge of the cube is 10 centimeters long?

(a) $\frac{4}{3}$ (b) 2 (c) 4 (d) 6 (e) 8

9. If $\sin(xy)=x^2$, then $\frac{dy}{dx}=$ _____.

(a) $2x\sec(xy)$ (b) $\frac{\sec(xy)}{x^2}$ (c) $2x\sec(xy)-y$ (d) $\frac{2x\sec(xy)}{y}$ (e) $\frac{2x\sec(xy)-y}{x}$

10. The volume of an expanding sphere is increasing at a rate of 12 cubic feet per second. When the volume of the sphere is 36π cubic feet, how fast, in square feet per second, is the surface area increasing? Note: $V=\frac{4}{3}\pi r^3, S=4\pi r^2$.

(a) 8 (b) 6 (c) 8π (d) $\frac{8\pi}{3}$ (e) 10

11. If $x^2y + yx^2 = 6$, then $\frac{d^2y}{dx^2}$ at the point $(1,3)$ is _____.

(a) -18 (b) -6 (c) 6 (d) 12 (e) 18

12. If the radius of a sphere is increasing at the rate of 2 inches per second, how fast, in cubic inches per second, is the volume increasing when the radius is 10 inches?

(a) 800π (b) 800 (c) 3200π (d) 40π (e) 80π

13. The equation of the tangent line to the curve $x^2 + y^2 = 169$ at the point $(5,-12)$ is _____.

(a) $5y - 12x = -120$ (b) $5x - 12y = 119$ (c) $5x - 12y = 169$ (d) $12x + 5y = 0$ (e) $12x + 5y = 169$

14.: NO calculator, 15 minutes.

Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

1. C II-13
2. A VI-35
3. D IV-36
4. D III-12
5. C II-4
6. 2004 AB/BC4 (no calc): omitted part c – requires section 3.4; reduced time to 15 minutes too
7. E VI-11
8. E V-27
9. E V-8
10. A IV-16
11. E III-25
12. A III-4
13. C I-9
14. 2005B AB5

---

test review probs.:

CALCULUS AB AP CHAPTER 3.1-3.4 QUIZ

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. Let $f(x)$ be a differentiable function defined for all real numbers. The table below gives the value of $f(x)$ and its derivative $f'(x)$ for several values of x.

x	-3	-2	-1	0	1	2	3
$f(x)$	8	5	0	1	0	5	8
$f'(x)$	-6	-4	-2	0	2	4	6

Which of the following statements is true?

I. At $x=2$, the function is increasing.

II. There is a relative minimum in the interval $-1\le x\le 1$, but not necessarily at $x=0$.

III. There is a relative maximum in the interval $-1\le x\le 1$.

(a) I only (b) II only (c) III only (d) I and II only (e) I, II, and III

2. If the derivative of a function f is given by $f'(x)=\sin(x^x)$, then how many critical points does the function $f(x)$ have on the interval $[0.2, 2.6]$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

3. The equation of the tangent line to the curve $y=x^3-6x^2$ at its point of inflection is _____.

(a) $y=-12x+8$ (b) $y=-12x+40$ (c) $y=12x-8$ (d) $y=-12x+12$ (e) $y=12x-40$

4. If $f(x)=\left|(x^2-12)(x^2+4)\right|$, how many numbers in the interval $-2\le x\le 3$ satisfy the conclusion of the Mean Value Theorem?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. Suppose that $f(x)$, $f'(x)$, and $f''(x)$ are continuous for all real numbers x, and that f has the following properties.

(i) f is negative on $(-\infty, 6)$ and positive on $(6, \infty)$

(ii) f is increasing on $(-\infty, 8)$ and decreasing on $(8, \infty)$

(iii) f is concave up on $(-\infty, 3)$ and concave down on $(3, \infty)$

Of the following, which has the <u>smallest</u> numerical value (i.e. -5 is smaller numerically than 2)?

(a) $f''(3)$ (b) $f'(10)$ (c) $f'(4)$ (d) $f'(1)$ (e) $f'(-7)$

Numbers 6-12: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

6. What are all values of x for which the graph of $y = 6x^2 + \frac{x}{2} + 3 + \frac{6}{x}$ is concave downward?

(a) $x < -1$ (b) $x < 0$ (c) $-1 < x < 0$ (d) $0 < x < 1$ (e) $x > -1$

7. If f is a continuous function on the closed interval $[a,b]$, which of the following is NOT necessarily true?

I. f has a minimum on $[a,b]$ II. f has a maximum on $[a,b]$ III. $f'(c) = 0$ for $a < c < b$

(a) I only (b) II only (c) III only (d) I and II only (e) I, II, and III

8. For what value of k will $\frac{8x+k}{x^2}$ have a relative maximum at $x = 4$?

(a) -32 (b) -16 (c) 0 (d) 16 (e) 32

9. Suppose that $f(x)$ is a twice differentiable function on the closed interval $[a,b]$. If $f'(c) = 0$ for $a < c < b$, which of the following statements must be true?

I. $f(a) = f(b)$ II. f has a relative extremum at $x = c$ III. f has a point of inflection at $x = c$

(a) None (b) I only (c) II only (d) I and II only (e) II and III only

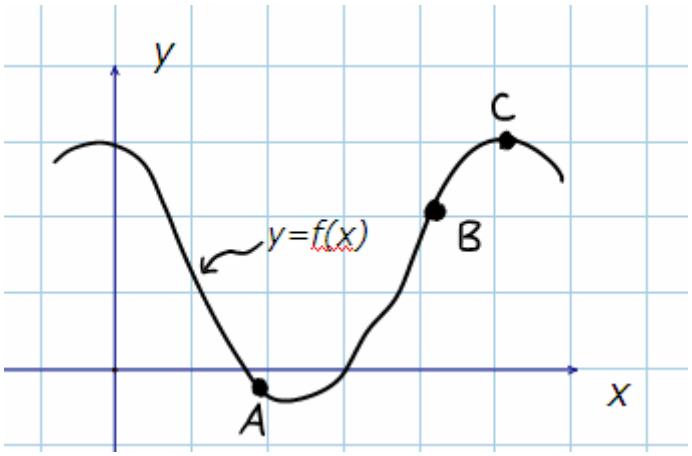
10. How many points of inflection does the graph of $y = 2x^6 + 9x^5 + 10x^4 - x + 2$ have?

(a) None (b) One (c) Two (d) Three (e) Four

11. What is the maximum value of the <u>derivative</u> of $f(x) = 3x^2 - x^3$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

12. At which of the three points on the graph of $y = f(x)$ in the figure below is $f'(x) < f''(x)$?



(a) A only (b) B only (c) C only (d) A and B (e) A and C

1. E V-43
2. D III-33
3. A III-29
4. D I-40
5. B V-38
6. C VI-15
7. C V-25
8. B V-20
9. A V-14
10. C V-10
11. D V-6
12. A IV-28

---

quiz review probs.:

CALCULUS AB AP CHAPTER 3.3-3.6 TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. How many extrema does the function $f(x)=(x+2)^3(x-5)^2$ have on the open interval $-3<x<6$?

(a) None (b) One (c) Two (d) Three (e) Four

2. If f is a continuous odd function and the $\lim_{x\to-\infty} f(x)=-3$, which of the following is true?

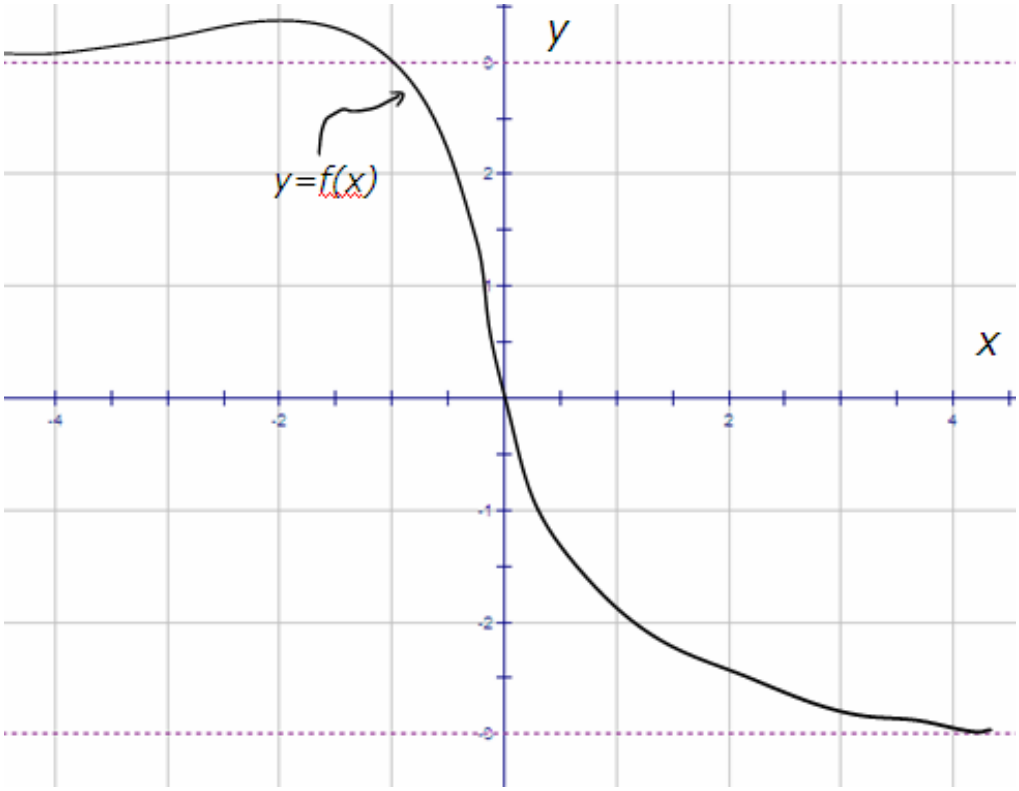
I. $\lim_{x\to\infty} f(x)=3$ II. There are no vertical asymptotes. III. The lines $y=3$ and $y=-3$ are horizontal asymptotes.

(a) I only (b) II only (c) III only (d) II and III only (e) I, II, and III

3. The derivative of a polynomial function $P(x)$ has a relative minimum at $(1,3)$ and a relative maximum at $(3,0)$ and no other critical points. The maximum number of real zeros of $P(x)$ is _____.

(a) None (b) One (c) Two (d) Three (e) Four

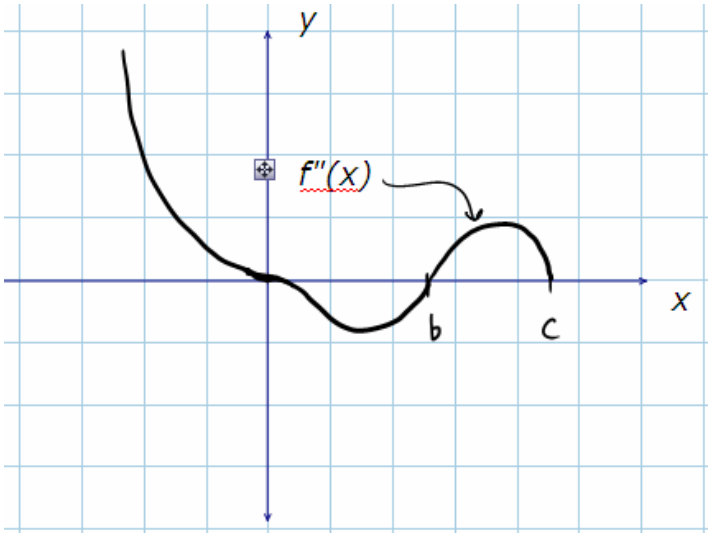
4. The figure below shows the graph of a function $f(x)$ which has horizontal asymptotes of $y=3$ and $y=-3$. Which of the following statements is true?



I. $f'(x)<0$ for all $x\geq 0$ II. $\lim_{x\to\infty} f'(x)=0$ III. $\lim_{x\to-\infty} f'(x)=2$

(a) I only (b) II only (c) III only (d) I and II only (e) I, II, and III

5. The figure below shows the graph of $f''(x)$, the second derivative of a function $f(x)$. The function $f(x)$ is continuous for all x. Which of the following statements about f is true?

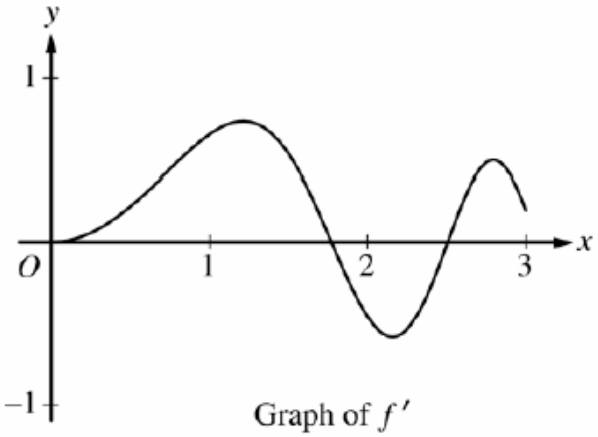


I. f is concave up for $x < 0$ and $b < x < c$ II. f has a relative minimum in the open interval $b < x < c$

III. f has points of inflection at $x = 0$ and $x = b$

(a) I only (b) II only (c) III only (d) I and III only (e) I, II, and III

6.: Calculator, 15 minutes.



Graph of f'

Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.

(a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

(b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of f at $x = 2$.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. If $\lim_{x\to 2}\dfrac{f(x)}{x-2} = f'(2) = 0$, which of the following must be true?

I. $f(2)=0$ II. $f(x)$ is continuous at $x=2$ III. $f(x)$ has a horizontal tangent line at $x=2$

(a) I only (b) II only (c) I and II only (d) II and III only (e) I, II, and III

8. $\lim_{x\to\infty}\dfrac{x-\dfrac{1}{2x}}{2x+\dfrac{1}{6x}}$ is _____?

(a) -3 (b) $-\frac{1}{2}$ (c) $-\frac{1}{3}$ (d) $\frac{1}{2}$ (e) 2

9. The graph of $y=\dfrac{x}{1-|x|}$ has _____.

(a) no horizontal asymptotes and one vertical asymptote
(b) one horizontal asymptotes and one vertical asymptote
(c) two horizontal asymptotes and one vertical asymptote
(d) one horizontal asymptotes and two vertical asymptote
(e) two horizontal asymptotes and two vertical asymptote

10. Which of the following are the equations of all horizontal and vertical asymptotes for the curve $y=\dfrac{x}{x(x^2-4)}$?

	Horizontal Asymptote	Vertical Asymptote
(a)	$y=1$	$x=-2, x=2$
(b)	$y=0$	$x=-2, x=0, x=2$
(c)	$y=0$	$x=-2, x=2$
(d)	$y=1$	$x=-2, x=0, x=2$
(e)	None	$x=-2, x=2$

11. The function $y=x^4+bx^2+8x+1$ has a horizontal tangent and a point of inflection for the same value of x. What must be the value of b?

(a) -1 (b) 4 (c) 1 (d) 6 (e) -6

12. If $y=7$ is a horizontal asymptote of a rational function f, then which of the following must be true?

(a) $\lim_{x\to 7} f(x)=\infty$ (b) $\lim_{x\to\infty} f(x)=7$ (c) $\lim_{x\to 0} f(x)=7$ (d) $\lim_{x\to 7} f(x)=0$ (e) $\lim_{x\to-\infty} f(x)=-7$

13. $\lim_{x\to\infty}\dfrac{10^8x^5+10^6x^4+10^4x^2}{10^9x^6+10^7x^5+10^5x^3} =$ _____.

(a) 0 (b) 1 (c) -1 (d) $\dfrac{1}{10}$ (e) $-\dfrac{1}{10}$

14.: NO calculator, 15 minutes.

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2-2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

1. C VI-42
2. E IV-44
3. C IV-39
4. D II-33
5. D VI-38
6. 2006B AB2
7. E VI-19
8. D VI-6
9. E V-26
10. C IV-17
11. E II-8
12. B II-9
13. A I-5
14. 2001 AB/BC4

---

test review probs.:

CALCULUS AB AP CHAPTER 4.1-4.4 QUIZ

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. $\frac{d}{dx}\int_0^{2x}\left(e^t+2t\right)dt =$ _____.

(a) $e^{2x}+4x$ (b) $e^{2x}+4x-1$ (c) $e^{2x}+4x^2-1$ (d) $2e^{2x}+4x$ (e) $2e^{2x}+8x$

2. The present price of a new car is \$14,500. The price of a new car is changing at a rate of $120+180\sqrt{t}$ dollars per year. How much will a new car cost 5 years from now?

(a) \$15,020 (b) \$15,300 (c) \$16,440 (d) \$18,120 (e) \$22,600

3. The average value of the function $f\left(x\right)=e^{-x}\sin x$ on the closed interval $\left[1,\pi\right]$ is _____.

(a) 0.129 (b) 0.145 (c) 0.155 (d) 0.276 (e) 0.310

4. If $f\left(x\right)=x^3-2x+1$ and $g\left(x\right)=x^2-2x+1$, for what values of a and b is $\int_a^b f\left(x\right)dx<\int_a^b g\left(x\right)dx$?

I. $a=-1,b=0$ II. $a=0,b=1$ III. $a=1,b=2$

(a) I only (b) II only (c) I and II only (d) I and III only (e) I, II, and III

5. For the function whose values are given in the table below, $\int_0^6 f\left(x\right)dx$ is approximated by a Midpoint Riemann Sum with three intervals of equal width. The approximation is _____.

x	0	1	2	3	4	5	6
f(x)	0	0.25	0.48	0.68	0.84	0.95	1

(a) 2.64 (b) 3.64 (c) 3.72 (d) 3.76 (e) 4.64

Numbers 6-12: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

6. If $\int_2^8 f(x)\,dx = -10$ and $\int_2^4 f(x)\,dx = 6$, then $\int_8^4 f(x)\,dx =$ _____.

(a) -16 (b) -6 (c) -4 (d) 4 (e) 16

7. If $\int_0^4 \left(x^2 - 6x + 9\right)dx$ is approximated by 4 inscribed rectangles of equal width on the *x*-axis, then the approximation is _____.

(a) 14 (b) 10 (c) 6 (d) 5 (e) 4

8. Let f be a differentiable function for all x. Which of the following must be true?

I. $\frac{d}{dx}\int_0^3 f(x)\,dx = f(x)$ II. $\int_3^x f'(x)\,dx = f(x)$ III. $\frac{d}{dx}\int_3^x f(x)\,dx = f(x)$

(a) II only (b) III only (c) I and II only (d) II and III only (e) I, II, and III

9. $\int_2^4 \left[\frac{d}{dt}\left(3t^2 + 2t - 1\right)\right]dt =$ _____.

(a) 12 (b) 40 (c) 46 (d) 55 (e) 66

10. $\int_{-3}^3 |x+2|\,dx =$ _____.

(a) 0 (b) 8 (c) 13 (d) 17 (e) 21

11. The velocity of a particle moving along the *x*-axis is given by a third-degree polynomial $P(t)$. The roots of $P(t)$ are all in the open interval $0 < t < a$. Which of the following statements must be true?

I. The velocity of the particle will be zero at least once and at most three times for $0 < t < a$.
II. In the interval $0 < t < a$, the particle moves both left and right.
III. The total distance traveled by the particle from $t = 0$ to $t = a$ is given by $\int_0^a P(t)\,dt$.

(a) I only (b) II only (c) III only (d) I and II only (e) I, II, and III

12. Which of the following properties of the definite integral is true?

I. $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$, *k* is constant
II. $\int_a^b xf(x)\,dx = x\int_a^b f(x)\,dx$
III. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

(a) I only (b) I and II only (c) I and III only (d) II and III only (e) I, II, and III

1. E VI-41
2. C V-39
3. A V-37 (on BC assessment too)
4. C III-37
5. D II-41
6. E V-22
7. D V-11 (on BC assessment too)
8. B V-7
9. B IV-18
10. C IV-9 (on BC assessment too)
11. D IV-19
12. C III-23

---

quiz review probs.:

CALCULUS AB AP CHAPTER 4 TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. If three subdivisions of $[0,3]$ are used, what is the trapezoidal approximation of $\int_0^3 (x^2-6x+9)\,dx$?

(a) 3 (b) 9 (c) 9½ (d) 10 (e) 19

2. Suppose that f is a continuous function and $\int_2^3 f(2x)\,dx = 8$. What is $\int_4^6 f(x)\,dx = ?$

(a) 4 (b) 8 (c) 12 (d) 16 (e) 24

3. Let $f(x)$ be a function and let $f(x)$ and its first and second derivatives all be positive on a closed interval $[a,b]$. The interval $[a,b]$ is partitioned into n equal length subintervals and these are used to compute an Upper Sum (U), a Lower Sum (L), and a Trapezoidal approximation (T) to the exact area $I = \int_a^b f(x)\,dx$. Which statement below is true?

(a) L<U<T<I (b) L<U<I<T (c) L<T<I<U (d) L<I<T<U (e) L<I<U<T

4. In the interval $0 \le x \le 5$ the graphs of $y = \cos 2x$ and $y = \sin 3x$ intersect four times. Let A, B, C, and D be the *x*-coordinates of these points so that 0<A<B<C<D<5. Which of the definite integrals below represents the largest number?

(a) $\int_0^A (\cos 2x - \sin 3x)\,dx$ (b) $\int_A^B (\sin 3x - \cos 2x)\,dx$ (c) $\int_B^C (\sin 3x - \cos 2x)\,dx$

(d) $\int_C^D (\cos 2x - \sin 3x)\,dx$ (e) $\int_C^D (\sin 3x - \cos 2x)\,dx$

5.Some values of a continuous function are given in the table below. The Trapezoidal Rule approximation for $\int_0^{10} f(x)\,dx$ is _____.

x	0	1	2	3	4	5	6	7	8	9	10
F(x)	20	19.5	18	15.5	12	7.5	2	-4.5	-12	-20.5	-30

(a) 30.825 (b) 32.500 (c) 33.325 (d) 33.333 (e) 35.825

6.: Calculator, 15 minutes.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. If f and g are continuously differentiable functions for all real numbers, which of the following definite integrals is equal to $f\left(g\left(4\right)\right)-f\left(g\left(2\right)\right)$?

(a) $\int_2^4 f'\left(g\left(x\right)\right)dx$ (b) $\int_2^4 f\left(g\left(x\right)\right)f'\left(x\right)dx$ (c) $\int_2^4 f\left(g\left(x\right)\right)g'\left(x\right)dx$

(d) $\int_2^4 f'\left(g\left(x\right)\right)g'\left(x\right)dx$ (e) $\int_2^4 f\left(g'\left(x\right)\right)g'\left(x\right)dx$

8. $\int_0^3 \frac{x}{\sqrt{x^2+16}}dx =$ _____.

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

9. $\int \csc^2 x\,dx =$ _____.

(a) $\sec^2 x+C$ (b) $\frac{\csc^3 x}{3}+C$ (c) $\tan x+C$ (d) $-2\csc^2 x\cot x+C$ (e) $-\cot x+C$

10. If the substitution $u=25-x^2$ is made, the integral $\int_0^3 x\sqrt{25-x^2}dx =$ _____.

(a) $\frac{1}{2}\int_0^3 \sqrt{u}\,du$ (b) $\frac{1}{2}\int_{25}^{16} \sqrt{u}\,du$ (c) $-\frac{1}{2}\int_0^3 \sqrt{u}\,du$ (d) $\frac{1}{2}\int_{16}^{25} \sqrt{u}\,du$ (e) $2\int_{16}^{25} \sqrt{u}\,du$

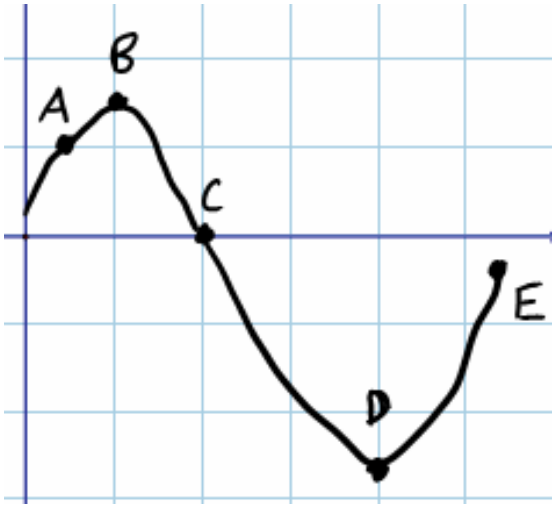
11. $\int \left(3x+5\right)^2 dx =$ _____.

(a) $\frac{1}{3}\left(3x+5\right)^3+C$ (b) $2\left(3x+5\right)+C$ (c) $6\left(3x+5\right)+C$ (d) $\frac{1}{9}\left(3x+5\right)^3+C$ (e) $\frac{1}{9}\left(3x+5\right)+C$

12. If $0<k<\pi$, then $\int_0^k \cos\left(2x\right)dx=\frac{1}{2}$, when $k =$ _____.

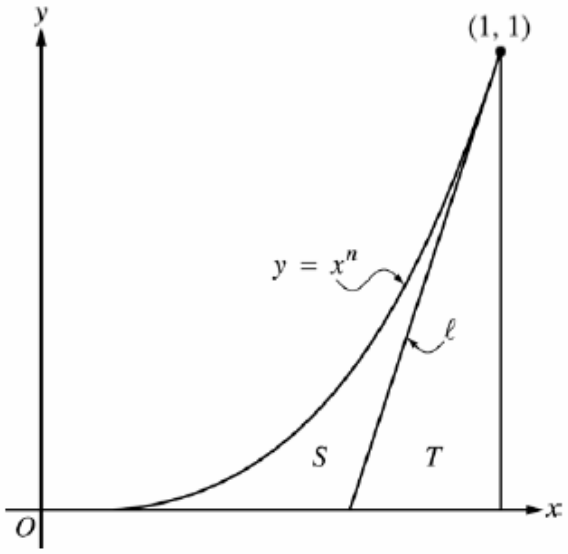
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{12}$ (d) $\frac{3\pi}{4}$ (e) $\frac{5\pi}{12}$

13. The figure below shows the graph of velocity of an object moving on the *x*-axis as a function of time. At which of the marked points is the object farthest to the right?



(a) A (b) B (c) C (d) D (e) E

14.: NO calculator, 15 minutes.



Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

(a) Find $\int_0^1 x^n \, dx$ in terms of n.

(b) Let T be the triangular region bounded by ℓ, the x-axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.

(c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.

1. C VI-36
2. D VI-29
3. D IV-33
4. D II-39 (on BC quiz too)
5. B I-32 (on BC test too)
6. 2005AB 3
7. D II-14
8. A VI-8
9. E VI-2
10. D IV-22
11. D IV-5
12. A III-22
13. C VI-20
14. 2004B AB/BC 6

---

test review probs.:

CALCULUS AB AP CHAPTER 5.1-5.2 QUIZ

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. If $y = \ln(3x+5)$, then $\dfrac{d^2y}{dx^2} =$ _____.

(a) $\dfrac{3}{3x+5}$ (b) $\dfrac{3}{(3x+5)^2}$ (c) $\dfrac{9}{(3x+5)^2}$ (d) $\dfrac{-9}{(3x+5)^2}$ (e) $\dfrac{-3}{(3x+5)^2}$

2. Let f be the function defined by $f(x) = \ln(3x+2)^k$ for some positive constant k. If $f'(2) = 3$, what is the value of k?

(a) $\dfrac{\ln 3}{\ln 8}$ (b) $\ln 8$ (c) 4 (d) 8 (e) 16

3. Which statement is true for the function $f(x) = \ln(\tan x)$ on the open interval $\pi < x < \dfrac{5\pi}{4}$?

(a) $f(x)$ is increasing at an increasing rate
(b) $f(x)$ is increasing at an decreasing rate
(c) $f(x)$ has an absolute maximum in the open interval
(d) $f(x)$ has a point of inflection in the open interval
(e) $f(x)$ has a point of symmetry in the open interval

4. If $\displaystyle\int_a^b \frac{f'(t)\,dt}{f(t)} = \ln[f(b)]$, then $f(a) =$ _____.

(a) 0 (b) $\dfrac{1}{e}$ (c) 1 (d) e (e) undefined

5. The position of a particle on the *x*-axis at time $t, t > 0,$ is $\ln t$. The average velocity of the particle for $1 \le t \le e$ is _____.

(a) 1 (b) $\dfrac{1}{e} - 1$ (c) $\dfrac{1}{e-1}$ (d) e (e) $e-1$

Numbers 6-12: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

6. $\int_e^{e^2} \frac{dx}{x \ln x} =$ _____.

(a) $\ln 2$ (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) e

7. If $k > 0$ and $\int_k^6 \frac{dx}{x+2} = \ln k$, then $k =$ _____.

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

8. If $f(x) = 3x \ln x$, then $f'(x) =$ _____.

(a) $3 + \ln(x^3)$ (b) $1 + \ln(x^3)$ (c) $\frac{3}{x} + 3 \ln x$ (d) $\frac{3}{x^2}$ (e) $\frac{1}{x}$

9. For $0 \le x \le \frac{\pi}{2}$, an antiderivative of $2 \tan x$ is _____.

(a) $\ln(\sec 2x)$ (b) $2 \sec^2 x$ (c) $\ln(\sec^2 x)$ (d) $2 \ln(\cos x)$ (e) $\ln(2 \sec x)$

10. $\int_1^3 \frac{x}{x^2+1} dx =$ _____.

(a) $\ln 5$ (b) $\ln 10$ (c) $2 \ln 2$ (d) $\frac{1}{2} \ln 5$ (e) $\ln\left(\frac{5}{2}\right)$

11. $\int_0^3 \frac{x}{x+1} dx =$ _____.

(a) $2 \ln 2$ (b) $6 \ln 2$ (c) $3 - 2 \ln 2$ (d) $3 + 2 \ln 2$ (e) $3 + \ln 3$

12. If $y = x(\ln x)^2$, then $\frac{dy}{dx} =$ _____.

(a) $3(\ln x)^2$ (b) $(\ln x)(2x + \ln x)$ (c) $(\ln x)(2 + \ln x)$ (d) $(\ln x)(2 + x \ln x)$ (e) $(\ln x)(1 + \ln x)$

1. D VI-9
2. D VI-26
3. B VI-40
4. C VI-28
5. C IV-6
6. A V-18
7. B V-17
8. A IV-8
9. C III-10
10. D III-8
11. C I-23
12. C I-18

---

quiz review probs.:

CALCULUS AB AP CHAPTER (5.1),5.2 TEST NO FREE-RESPONSE

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-7: Calculator, 21 minutes. Choose the letter that best completes the statement or answers the question.

1. A particle with velocity at any time t given by $v(t) = 2e^{2t}$ moves in a straight line. How far does the particle travel during the time interval when its velocity increases from 2 to 4?

(a) 1 (b) 2 (c) 3 (d) e^4 (e) $e^8 - e^4$

2. If $e^{xy} = 2$, then at the point $(1, \ln 2)$, $\dfrac{dy}{dx} =$ _____.

(a) $-\ln 2$ (b) $2\ln 2$ (c) $\ln 2$ (d) $-2e$ (e) $-4\ln 2$

3. The graphs of the functions $f(x) = 5 + x^4$ and $g(x) = 5e^{0.2x}$ intersect _____.

(a) Never (b) Once (c) Twice (d) Three times (e) Four times

4. If $f(x) = \begin{cases} e^{-x} + 2, & x < 0 \\ ax + b, & x \geq 0 \end{cases}$ is differentiable at 0, then $a + b =$ _____.

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. The tangent line to the graph of $y = e^{2-x}$ at the point $(1, e)$ intersects both coordinate axes. What is the area of the triangle formed by this tangent line and the coordinate axes?

(a) $2e$ (b) $e^2 - 1$ (c) e^2 (d) $2e\sqrt{e}$ (e) $4e$

6. The derivative of f is given by $f'(x) = e^x(-x^3 + 3x) - 3$ for $0 \leq x \leq 5$. At what value of x is $f(x)$ an absolute minimum?

(a) For no value of x (b) 0 (c) 0.618 (d) 1.623 (e) 5

7. A population grows according the equation $P(t) = 6000 - 5500e^{-0.159t}$ for $t \geq 0$. This population will approach a limiting value as time goes on. During which year will the population reach half of this limiting value?

(a) second (b) third (c) fourth (d) eighth (e) twenty-ninth

Numbers 8-17: NO calculator, 22 minutes. Choose the letter that best completes the statement or answers the question.

8. If $f(x)=e^x \ln x$, $f'(e)=$ _____.

(a) $e^{e-1}+e^e$ (b) $e^{e+1}+e^e$ (c) e^e+e (d) $e^e+\frac{1}{e}$ (e) e^{e-1}

9. Let $f(x)=\begin{cases} 1+e^{-x}, & 0\le x\le 5 \\ 1+e^{x-10}, & 5\le x\le 10 \end{cases}$. Which of the following statements is true?

I. $f(x)$ is continuous for all values of x in the interval $[0,10]$.

II. $f'(x)$, the derivative of $f(x)$, is continuous for all values of x in the interval $[0,10]$.

III. The graph of $f(x)$ is concave upwards for all values of x in the interval $[0,10]$.

(a) I only (b) II only (c) III only (d) I and III only (e) I, II, and III

10. The minimum value of $f(x)=e^x-2x$ is _____.

(a) $\ln 2$ (b) e^2-4 (c) $\sqrt{e}-1$ (d) $2(1-\ln 2)$ (e) 2

11. If $f(x)=\sqrt{e^{2x}+1}$, then $f'(0)=$ _____.

(a) $\frac{\sqrt{2}}{4}$ (b) $\sqrt{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) 1 (e) $-\frac{\sqrt{2}}{2}$

12. If $y=2xe^{-x}$, then y has a point of inflection at $x=$ _____.

(a) 0 (b) 1 (c) 2 (d) -2 (e) 4

13. If $y=xe^x$, then $\frac{d^n y}{dx^n}=$ _____.

(a) e^x (b) e^{nx} (c) $(x+n)e^x$ (d) $x^n e^x$ (e) $(x+n^2)e^x$

14. Suppose that $f(x)=\ln 3x$ and $f^{-1}(x)$ denotes the inverse of f. Then $\int f^{-1}(x)\,dx=$ _____.

(a) $3e^x+C$ (b) $\frac{1}{3}e^x+C$ (c) $\frac{1}{x}+C$ (d) $\frac{1}{3x}+C$ (e) $\frac{1}{3}e^{3x}+C$

15. $\frac{d}{dx}\left(e^{3\ln x}\right)=$ _____.

(a) $e^{3\ln x}$ (b) $\frac{e^{3\ln x}}{x}$ (c) x^3 (d) $3x^2$ (e) 3

16. The graph of which function has $y=-1$ as an asymptote?

(a) $y=e^{-x}$ (b) $y=\frac{-x}{1-x}$ (c) $y=\ln(x+1)$ (d) $y=\frac{x}{x+1}$ (e) $y=\frac{x}{1-x}$

17. If $f(x)=e^{\sin x}$, how many zeros does $f'(x)$ have on the closed interval $[0,2\pi]$?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

1. A IV-34
2. A IV-29 (on BC 5bt)
3. D III-44
4. C III-43
5. A II-45
6. E II-36
7. C I-44
8. A VI-18
9. D IV-20
10. D IV-11
11. C III-24
12. C III-3
13. C II-24
14. B I-17
15. D I-8
16. E I-6
17. B I-4

---

test review probs.:

CALCULUS AB AP CHAPTER 5.6,6.1-6.3 TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. If $\frac{dy}{dx} = 2xy$, and if $y = 2$ when $x = 0$, then when $y = e$, $x =$ _____.

(a) 0.307 (b) 0.554 (c) 0.693 (d) 1.000 (e) 2.718

2. At any time $t \geq 0$, in days, the rate of growth of bacteria population is given by $y' = ky$, where y is the number of bacteria present and k is a constant. The initial population is 1500 and the population is quadrupled during the first 2 days. By what factor will the population have increased during the first 3 days.

(a) 4 (b) 5 (c) 6 (d) 8 (e) 10

3. If $\frac{dy}{dx} = xy^2$ and $y(1) = 1$, then $y =$ _____.

(a) x^2 (b) $\frac{-2}{x^2 - 3}$ (c) $x^2 - 3$ (d) $\frac{2}{x^2 + 1}$ (e) $\frac{x^2 - 3}{2}$

4. If f is a continuous function on the closed interval $[a,b]$, which of the following is NOT necessarily true?

I. f has a minimum on $[a,b]$ II. f has a maximum on $[a,b]$ III. $f'(c) = 0$ for $a < c < b$

(a) I only (b) II only (c) III only (d) I and II only (e) I, II, and III

5. A function whose derivative is a constant multiple of itself must be _____.

(a) periodic (b) linear (c) exponential (d) quadratic (e) logarithmic

6.: Calculator, 15 minutes.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. If $\frac{dy}{dx}=\frac{x}{y}$ and $y(3)=4$, then _____.

(a) $x^2-y^2=-7$ (b) $x^2+y^2=7^2$ (c) $x^2-y^2=7$ (d) $y^2-x^2=5$ (e) $y^2-x^2=7^2$

8. At each point (x,y) on a certain curve, the slope of the curve is $4xy$. If the curve contains the point $(0,4)$, then its equation is _____.

(a) $y=e^{2x^2}+4$ (b) $y=e^{2x^2}+3$ (c) $y=4e^{2x^2}$ (d) $y^2=2x^2+4$ (e) $y=2x^2+4$

9. The rate of decay of a radioactive substance is proportional to the amount of substance present. Four years ago there were 12 grams of substance. Now there are 8 grams. How many grams will there be 8 years from now?

(a) 0 (b) $\frac{8}{3}$ (c) $\frac{32}{9}$ (d) $\frac{81}{16}$ (e) $\frac{16}{3}$

10. If $y=\arcsin\left(\frac{3x}{4}\right)$, then $\frac{dy}{dx}=$ _____.

(a) $\frac{-3}{\sqrt{16-9x^2}}$ (b) $\frac{12}{16+9x^2}$ (c) $\frac{4}{\sqrt{16-9x^2}}$ (d) $\frac{12}{\sqrt{16-9x^2}}$ (e) $\frac{3}{\sqrt{16-9x^2}}$

11. $\frac{d}{dx}\int_x^0 \frac{du}{1+u^2}=$ _____.

(a) $\frac{1}{x^2+1}$ (b) $\frac{-1}{x^2+1}$ (c) x^2+1 (d) $-x^2+1$ (e) $\arctan x$

12. If $f(x)=\arctan\left(\frac{1}{x}\right)$, then $f'(x)=$ _____.

(a) $\frac{-1}{x^2+x}$ (b) $\frac{x}{\sqrt{x^2-1}}$ (c) $\frac{x^2}{x^2+1}$ (d) $\frac{1}{x^2+1}$ (e) $\frac{-1}{x^2+1}$

13. $\int \frac{dx}{\sqrt{4-x^2}}=$ _____.

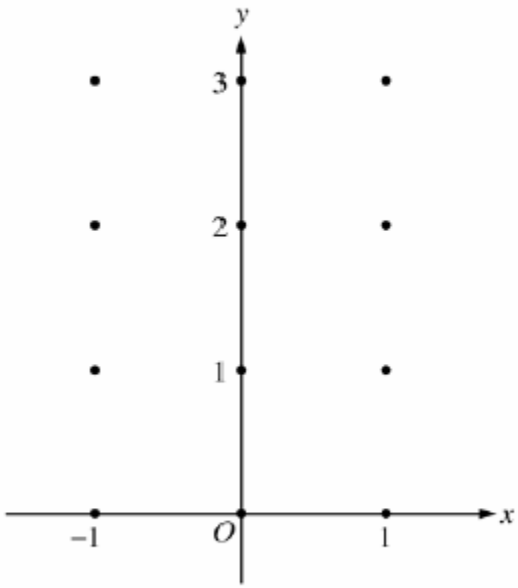
(a) $\arcsin\left(\frac{x}{2}\right)+C$ (b) $2\sqrt{4-x^2}+C$ (c) $\arcsin(x)+C$ (d) $\sqrt{4-x^2}+C$ (e) $\frac{1}{2}\arcsin\left(\frac{x}{2}\right)+C$

14.: NO calculator, 15 minutes.

Copy the axes and points for Part a, and draw the slope field on your own paper.

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

1. B IV-41
2. D III-36
3. B I-38
4. C [V-25]; MVT/Rolle's
5. C II-21
6. 2001 AB/BC2
7. A V-28
8. C III-20
9. C II-23
10. E IV-23
11. B III-18
12. E III-9
13. A I-12
14. 2004B AB5

---

test review probs.:

CALCULUS AB AP CHAPTER 7.1,7.2 TEST

Don't write on the test materials. Put all answers on a separate sheet of paper.

Numbers 1-5: Calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

1. The base of a solid is the region enclosed by the graph of $y = 3(x-2)^2$ and the coordinate axes. If every cross section perpendicular to the *x*-axis is a square, then the volume of the solid is _____.

(a) 8.0 (b) 19.2 (c) 24.0 (d) 25.6 (e) 57.6

2. The region enclosed by the line $x + y = 1$ and the coordinate axes is rotated about the line $y = -1$. What is the volume of the solid generated?

(a) $\frac{17\pi}{2}$ (b) $\frac{17\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{4}$ (e) $\frac{4\pi}{3}$

3. The region in the first quadrant enclosed by the *x*-axis, the line $x = \pi$, and the curve $y = \cos(\cos x)$ is rotated about the *x*-axis. What is the volume of the solid generated?

(a) 1.92 (b) 3.78 (c) 6.04 (d) 8.13 (e) 23.73

4. The region in the first quadrant enclosed by the graphs of $y = x$ and $y = 2\sin x$ is evolved about the *x*-axis. The volume of the resulting figure is _____.

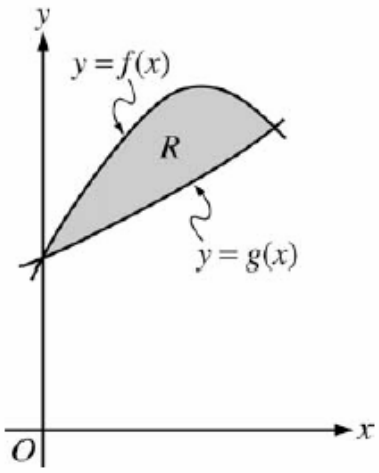
(a) 1.895 (b) 2.126 (c) 5.811 (d) 6.678 (e) 13.355

5. The volume of the solid formed by revolving the region bounded by the graph of $y = (x-3)^2$ and the coordinate axes about the x-axis is given by which of the following integrals?

(a) $\pi\int_0^3 (x-3)^2\,dx$ (b) $\pi\int_0^3 (x-3)^4\,dx$ (c) $2\pi\int_0^3 (x-3)^2\,dx$

(d) $2\pi\int_0^3 x(x-3)^2\,dx$ (e) $2\pi\int_0^3 x(x-3)^4\,dx$

6.: Calculator, 15 minutes.



Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

Numbers 7-13: NO calculator, 15 minutes. Choose the letter that best completes the statement or answers the question.

7. The total area between the curves $y = x^3$ and $y = x$ is _____.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{2}$ (e) 4

8. If the definite integral $\int_a^b f(x)\,dx$ represents the area of the region bounded by $y = f(x)$, the *x*-axis, and the lines $x = a$ and $x = b$, which of the following must be true?

(a) $a > b$ and $f(x) > 0$ (b) $a > b$ and $f(x) < 0$ (c) $a < b$ and $f(x) > 0$

(d) $a < b$ and $f(x) < 0$ (e) None of the above

9. A solid has a circular base of radius 3. If every plane cross section perpendicular to the *x*-axis is an equilateral triangle, then its volume is _____.

(a) 36 (b) $12\sqrt{3}$ (c) $18\sqrt{3}$ (d) $24\sqrt{3}$ (e) $36\sqrt{3}$

10. Let R be the region in the fourth quadrant enclosed by the *x*-axis and the curve $y = x^2 - 2kx$, where $k > 0$. If the area of the region R is 36, then the value of k is _____.

(a) 2 (b) 3 (c) 4 (d) 6 (e) 9

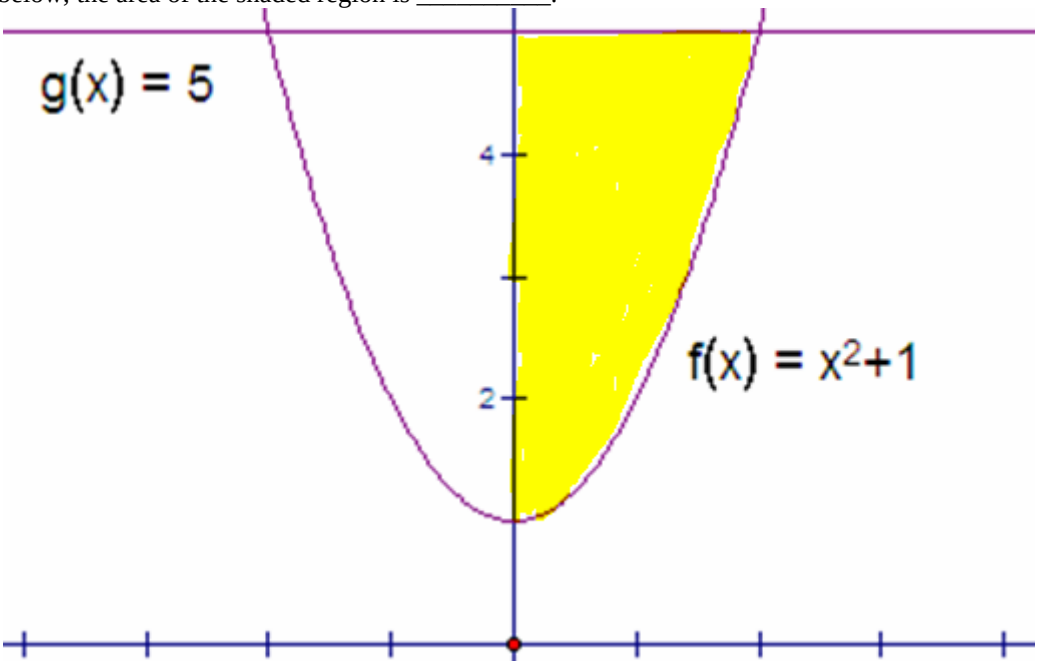
11. The base of a solid is the region in the first quadrant bounded by the line $x + 2y = 4$ and the coordinate axes. What is the volume of the solid if every cross section perpendicular to the *x*-axis is a semicircle?

(a) $\frac{2\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{8\pi}{3}$ (d) $\frac{32\pi}{3}$ (e) $\frac{64\pi}{3}$

12. The area of the region between the graph of $y = 3x^2 + 2x$ and the *x*-axis from $x = 1$ to $x = 3$ is _____.

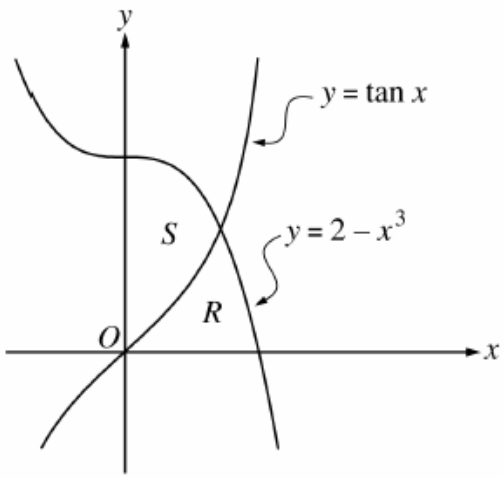
(a) 36 (b) 34 (c) 31 (d) 26 (e) 12

13. For the region below, the area of the shaded region is _____.



(a) $\frac{14}{3}$ (b) $\frac{16}{3}$ (c) $\frac{28}{3}$ (d) $\frac{32}{3}$ (e) $\frac{65}{3}$

14.: Calculator, 15 minutes.



Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.

1. E V-34
2. E IV-40
3. C III-30
4. D II-44
5. B I-29
6. 2005B AB/BC1
7. B VI-10
8. E V-13
9. E IV-21
10. B IV-10
11. A III-26
12. B III-1
13. B I-11
14. 2001 AB1

---

test review probs.: