Calculus AB Subscore for the Calculus BC Exam

A Calculus AB subscore is reported based on performance on the portion of the exam devoted to Calculus AB topics (approximately 60 percent of the exam). The Calculus AB subscore is designed to give colleges and universities more information about the student. Although each college and university sets its own policy for awarding credit and/or placement for AP Exam scores, it is recommended that institutions apply the same policy to the Calculus AB subscore that they apply to the Calculus AB score. Use of the subscore in this manner is consistent with the philosophy of the courses, since common topics are tested at the same conceptual level in both Calculus AB and Calculus BC.

Calculus AB: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus AB are included in the following sections. Answers to the sample questions are given on page 27.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the exam.

Part A consists of 28 questions. Following are the directions for Section I, Part A, and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. What is $\lim_{h \to 0} \dfrac{\cos\left(\dfrac{3\pi}{2} + h\right) - \cos\left(\dfrac{3\pi}{2}\right)}{h}$?

   (A) 1

   (B) $\dfrac{\sqrt{2}}{2}$

   (C) 0

   (D) -1

   (E) The limit does not exist.

2. At which of the five points on the graph in the figure at the right are $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ both negative?

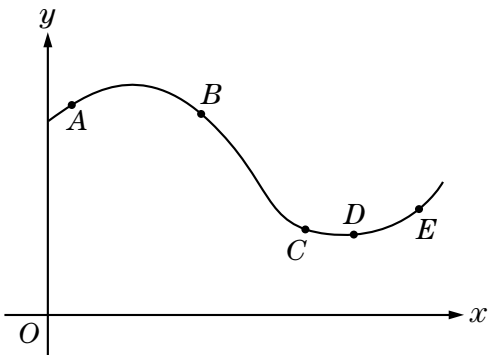
   (A) A

   (B) B

   (C) C

   (D) D

   (E) E



3. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

   (A) $-\dfrac{3}{2}$

   (B) -1

   (C) $-\dfrac{5}{14}$

   (D) $-\dfrac{3}{14}$

   (E) 0

4. Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

   (A) $\pi\int_0^1 \left(\left(3 - 2x^2\right)^2 - (3 - 2x)^2\right) dx$

   (B) $\pi\int_0^1 \left((3 - 2x)^2 - \left(3 - 2x^2\right)^2\right) dx$

   (C) $\pi\int_0^1 \left(4x^2 - 4x^4\right) dx$

   (D) $\pi\int_0^2 \left(\left(3 - \frac{y}{2}\right)^2 - \left(3 - \sqrt{\frac{y}{2}}\right)^2\right) dy$

   (E) $\pi\int_0^2 \left(\left(3 - \sqrt{\frac{y}{2}}\right)^2 - \left(3 - \frac{y}{2}\right)^2\right) dy$

5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

   (A) The function has no relative extremum.
   (B) The graph of the function has one point of inflection and the function has two relative extrema.
   (C) The graph of the function has two points of inflection and the function has one relative extremum.
   (D) The graph of the function has two points of inflection and the function has two relative extrema.
   (E) The graph of the function has two points of inflection and the function has three relative extrema.

6. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

   (A) $-2 \cos 3$
   (B) $-2 \sin 3 \cos 3$
   (C) $6 \cos 3$
   (D) $2 \sin 3 \cos 3$
   (E) $6 \sin 3 \cos 3$

7. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$, where $y(2) = -2$?

   (A) $y = 2x$ for $x > 0$

   (B) $y = 2x - 6$ for $x \ne 3$

   (C) $y = -\sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

   (D) $y = \sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

   (E) $y = -\sqrt{4x^2 - 6}$ for $x > \sqrt{1.5}$

8. What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval $[0, 3]$?

   (A) 8.5
   (B) 8.7
   (C) 22
   (D) 33
   (E) 66

9. The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

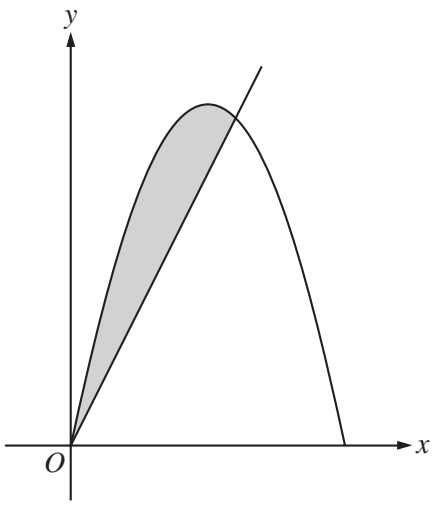
   (A) $3 < t < 4$ only
   (B) $t > 4$ only
   (C) $t > 5$ only
   (D) $0 < t < 3$ and $t > 5$
   (E) $3 < t < 4$ and $t > 5$

10. $\int (x - 1)\sqrt{x}\, dx =$

    (A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$
    (B) $\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2} + C$
    (C) $\frac{1}{2}x^2 - x + C$
    (D) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$
    (E) $\frac{1}{2}x^2 + 2x^{3/2} - x + C$

11. What is $\lim_{x \to \infty} \frac{x^2 - 4}{2 + x - 4x^2}$?

    (A) -2
    (B) $-\frac{1}{4}$
    (C) $\frac{1}{2}$
    (D) 1
    (E) The limit does not exist.



12. The figure above shows the graph of $y = 5x - x^2$ and the graph of the line $y = 2x$. What is the area of the shaded region?

(A) $\frac{25}{6}$

(B) $\frac{9}{2}$

(C) 9

(D) $\frac{27}{2}$

(E) $\frac{45}{2}$

13. If $y = 5 + \int_2^{2x} e^{-t^2}\,dt$, which of the following is true?

(A) $\frac{dy}{dx} = e^{-x^2}$ and $y(0) = 5$

(B) $\frac{dy}{dx} = e^{-x^2}$ and $y(1) = 5$

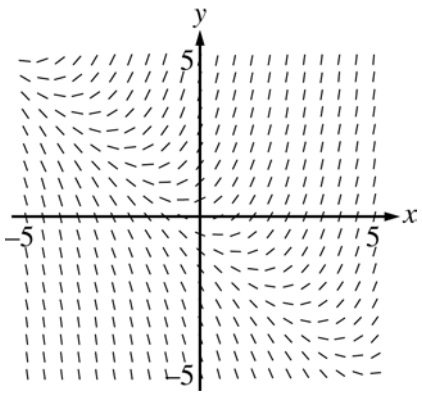
(C) $\frac{dy}{dx} = e^{-4x^2}$ and $y(1) = 5$

(D) $\frac{dy}{dx} = 2e^{-4x^2}$ and $y(0) = 5$

(E) $\frac{dy}{dx} = 2e^{-4x^2}$ and $y(1) = 5$

14. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

(A)



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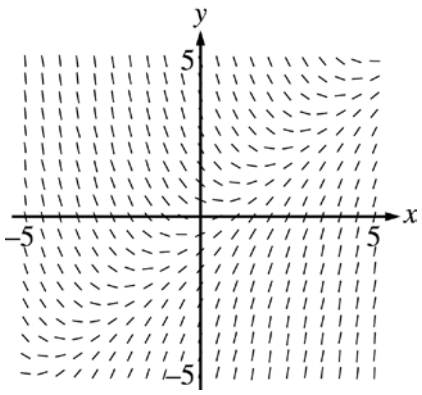

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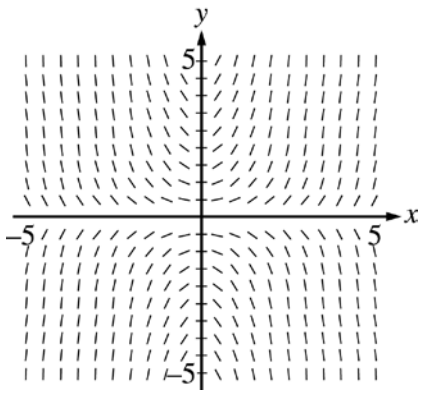

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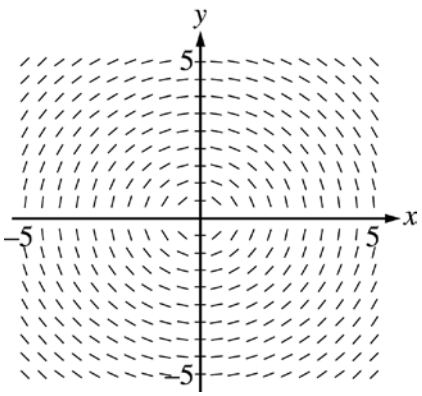
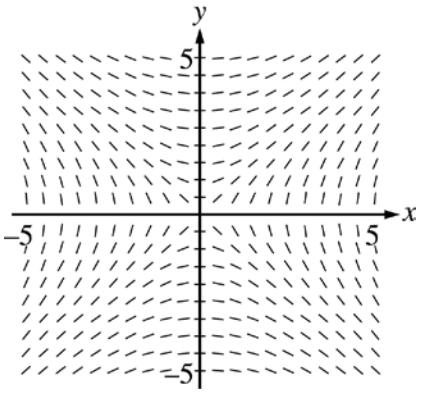


(E)



Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. Following are the directions for Section I, Part B, and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

15. A particle travels along a straight line with a velocity of $v(t) = 3e^{(-t/2)}\sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835
(B) 1.850
(C) 2.055
(D) 2.261
(E) 7.025

16. A city is built around a circular lake that has a radius of 1 mile. The population density of the city is $f(r)$ people per square mile, where r is the distance from the center of the lake, in miles. Which of the following expressions gives the number of people who live within 1 mile of the lake?

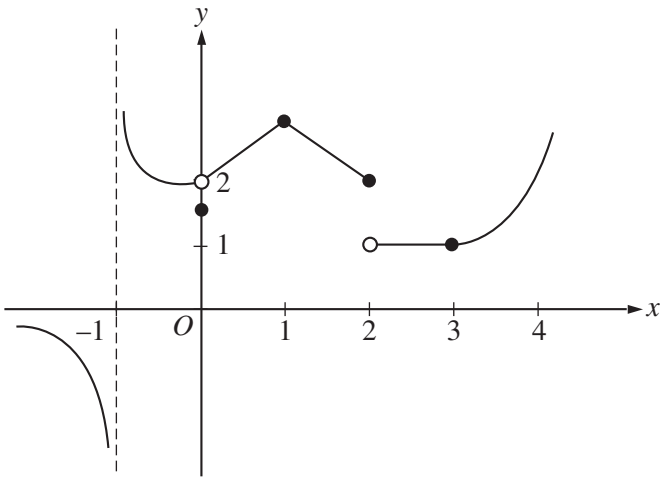
(A) $2\pi\int_0^1 r f(r)\,dr$

(B) $2\pi\int_0^1 r(1 + f(r))\,dr$

(C) $2\pi\int_0^2 r(1 + f(r))\,dr$

(D) $2\pi\int_1^2 r f(r)\,dr$

(E) $2\pi\int_1^2 r(1 + f(r))\,dr$



17. The graph of a function f is shown above. If $\lim_{x \to b} f(x)$ exists and f is not continuous at b, then $b =$

(A) –1
(B) 0
(C) 1
(D) 2
(E) 3

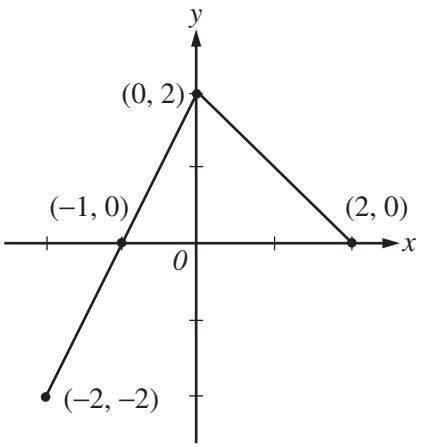
x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

18. Let f be a function such that $f''(x) < 0$ for all x in the closed interval $[1, 2]$. Selected values of f are shown in the table above. Which of the following must be true about $f'(1.2)$?

(A) $f'(1.2) < 0$
(B) $0 < f'(1.2) < 1.6$
(C) $1.6 < f'(1.2) < 1.8$
(D) $1.8 < f'(1.2) < 2.0$
(E) $f'(1.2) > 2.0$

19. Two particles start at the origin and move along the x-axis. For $0 \le t \le 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

(A) None
(B) One
(C) Two
(D) Three
(E) Four



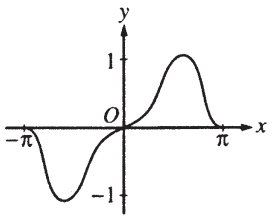
Graph of f

20. The graph of the function f shown above consists of two line segments. If g is the function defined by $g(x) = \int_0^x f(t)\,dt$, then $g(-1) =$

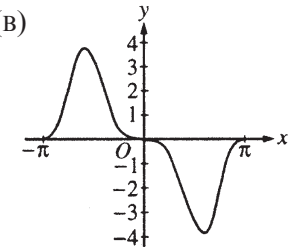
(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

21. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

(A)



(B)



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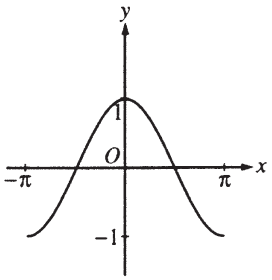

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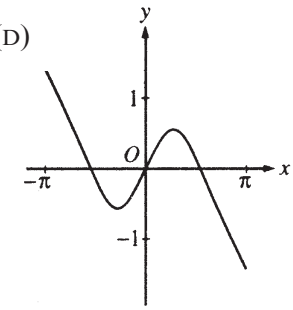
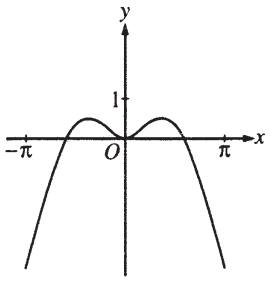


(E)



22. A differentiable function f has the property that $f(5) = 3$ and $f'(5) = 4$. What is the estimate for $f(4.8)$ using the local linear approximation for f at $x = 5$?

(A) 2.2
(B) 2.8
(C) 3.4
(D) 3.8
(E) 4.6

23. Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil leaks out of the tanker from time $t = 0$ to $t = 10$?

(A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons

24. If $f'(x) = \sin\left(\frac{\pi e^x}{2}\right)$ and $f(0) = 1$, then $f(2) =$

(A) −1.819
(B) −0.843
(C) −0.819
(D) 0.157
(E) 1.157

Answers to Calculus AB Multiple-Choice Questions

Part A		*Part B*	
1.	A	15.*	D
2.	B	16.	D
3.	C	17.	B
4.	A	18.	D
5.	C	19.*	D
6.	B	20.	B
†7.	C	21.	E
8.	C	22.	A
9.	E	23.*	D
10.	D	24.*	E
11.	B		
12.	B		
13.	E		
14.	E		

*Indicates a graphing calculator-active question.
†For resources on differential equations, see the Course Home Pages for Calculus AB and Calculus BC at AP Central.

Calculus BC: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus BC are included in the following sections. Answers to the sample questions are given on page 39.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the exam.

Part A consists of 28 questions. Following are the directions for Section I, Part A, and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. A curve is described by the parametric equations $x = t^2 + 2t$ and $y = t^3 + t^2$. An equation of the line tangent to the curve at the point determined by $t = 1$ is

   (A) $2x - 3y = 0$
   (B) $4x - 5y = 2$
   (C) $4x - y = 10$
   (D) $5x - 4y = 7$
   (E) $5x - y = 13$

2. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

   (A) $-\frac{3x + y}{y^2}$

   (B) $-\frac{3x + y}{x + y}$

   (C) $\frac{1 - 3x - y}{x + y}$

   (D) $-\frac{3x}{1 + y}$

   (E) $-\frac{3x}{x + y}$

x	$g'(x)$
−1.0	2
−0.5	4
0.0	3
0.5	1
1.0	0
1.5	−3
2.0	−6

3. The table above gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

   (A) −6.5
   (B) −1.5
   (C) 1.5
   (D) 2.5
   (E) 3

4. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ converges?

   (A) All x except $x = 0$
   (B) $|x| = 3$
   (C) $-3 \le x \le 3$
   (D) $|x| > 3$
   (E) The series diverges for all x.

5. If $\frac{d}{dx}f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx}f(h(x)) =$

   (A) $g\left(x^2\right)$

   (B) $2xg(x)$

   (C) $g'(x)$

   (D) $2xg\left(x^2\right)$

   (E) $x^2g\left(x^2\right)$

6. If F' is a continuous function for all real x, then $\lim_{h\to 0}\frac{1}{h}\int_a^{a+h}F'(x)\,dx$ is

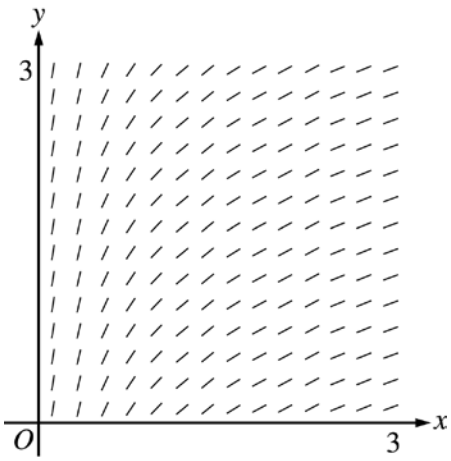
   (A) 0

   (B) $F(0)$

   (C) $F(a)$

   (D) $F'(0)$

   (E) $F'(a)$



7. The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

   (A) $y = x^2$

   (B) $y = e^x$

   (C) $y = e^{-x}$

   (D) $y = \cos x$

   (E) $y = \ln x$

8. $\displaystyle\int_0^3 \frac{dx}{(1-x)^2}$ is

(A) $-\frac{3}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

(E) divergent

9. Which of the following series converge to 2?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n+3}$

II. $\displaystyle\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

III. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

10. If the function f given by $f(x) = x^3$ has an average value of 9 on the closed interval $[0, k]$, then $k =$

(A) 3

(B) $3^{1/2}$

(C) $18^{1/3}$

(D) $36^{1/4}$

(E) $36^{1/3}$

11. Which of the following integrals gives the length of the graph $y = \sin\left(\sqrt{x}\right)$ between $x = a$ and $x = b$, where $0 < a < b$?

(A) $\displaystyle\int_a^b \sqrt{x + \cos^2\left(\sqrt{x}\right)}\, dx$

(B) $\displaystyle\int_a^b \sqrt{1 + \cos^2\left(\sqrt{x}\right)}\, dx$

(C) $\displaystyle\int_a^b \sqrt{\sin^2\left(\sqrt{x}\right) + \frac{1}{4x}\cos^2\left(\sqrt{x}\right)}\, dx$

(D) $\displaystyle\int_a^b \sqrt{1 + \frac{1}{4x}\cos^2\left(\sqrt{x}\right)}\, dx$

(E) $\displaystyle\int_a^b \sqrt{\frac{1 + \cos^2\left(\sqrt{x}\right)}{4x}}\, dx$

12. Which of the following integrals represents the area enclosed by the smaller loop of the graph of $r = 1 + 2\sin\theta$?

(A) $\displaystyle\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(B) $\displaystyle\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1 + 2\sin\theta)\, d\theta$

(C) $\displaystyle\frac{1}{2}\int_{-\pi/6}^{7\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(D) $\displaystyle\int_{-\pi/6}^{7\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(E) $\displaystyle\int_{7\pi/6}^{-\pi/6} (1 + 2\sin\theta)\, d\theta$

13. The third-degree Taylor polynomial about $x = 0$ of $\ln(1 - x)$ is

(A) $-x - \frac{x^2}{2} - \frac{x^3}{3}$

(B) $1 - x + \frac{x^2}{2}$

(C) $x - \frac{x^2}{2} + \frac{x^3}{3}$

(D) $-1 + x - \frac{x^2}{2}$

(E) $-x + \frac{x^2}{2} - \frac{x^3}{3}$

14. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$

(B) $e^{\tan x} + 5$

(C) $5e^{\tan x}$

(D) $\tan x + 5$

(E) $\tan x + 5e^x$

Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. Following are the directions for Section I, Part B, and a representative set of 10 questions.

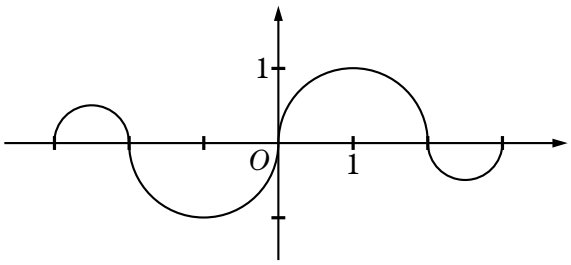
Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f

15. The graph of the function f above consists of four semicircles. If $g(x) = \int_0^x f(t)\,dt$, where is $g(x)$ nonnegative?

(A) $[-3, 3]$
(B) $[-3, -2] \cup [0, 2]$ only
(C) $[0, 3]$ only
(D) $[0, 2]$ only
(E) $[-3, -2] \cup [0, 3]$ only

16. If f is differentiable at $x = a$, which of the following could be false?

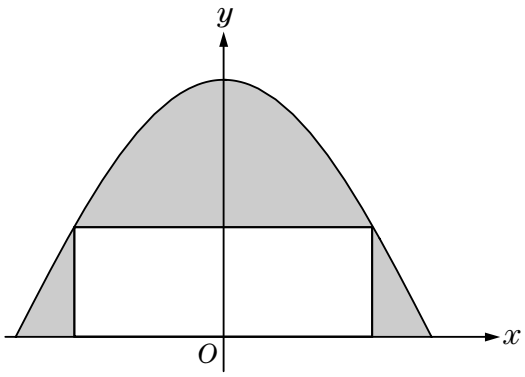
(A) f is continuous at $x = a$.

(B) $\lim_{x \to a} f(x)$ exists.

(C) $\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ exists.

(D) $f'(a)$ is defined.

(E) $f''(a)$ is defined.



17. A rectangle with one side on the x-axis has its upper vertices on the graph of $y = \cos x$, as shown in the figure above. What is the minimum area of the shaded region?

(A) 0.799
(B) 0.878
(C) 1.140
(D) 1.439
(E) 2.000

18. A solid has a rectangular base that lies in the first quadrant and is bounded by the x- and y-axes and the lines $x = 2$ and $y = 1$. The height of the solid above the point (x, y) is $1 + 3x$. Which of the following is a Riemann sum approximation for the volume of the solid?

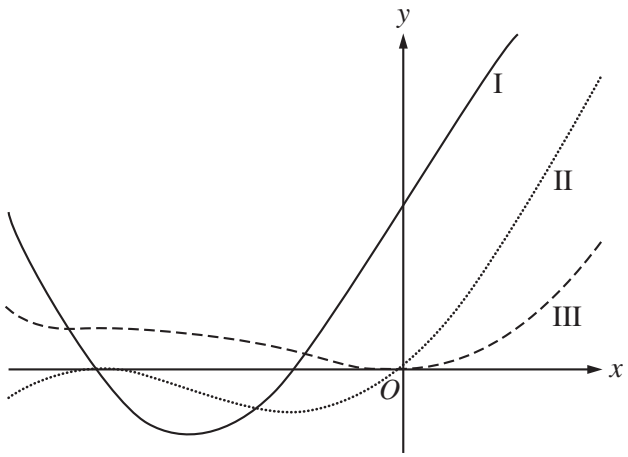
(A) $\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

(B) $2\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

(C) $2\sum_{i=1}^{n} \frac{i}{n}\left(1 + \frac{3i}{n}\right)$

(D) $\sum_{i=1}^{n} \frac{2}{n}\left(1 + \frac{6i}{n}\right)$

(E) $\sum_{i=1}^{n} \frac{2i}{n}\left(1 + \frac{6i}{n}\right)$



19. Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	I	III	II
(C)	II	I	III
(D)	II	III	I
(E)	III	II	I

20. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t+1) - 2t + 1$. The total distance traveled by the particle from $t = 0$ to $t = 2$ is

(A) 0.667
(B) 0.704
(C) 1.540
(D) 2.667
(E) 2.901

21. If the function f is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

(A) −3.268
(B) −1.585
(C) 1.732
(D) 6.585
(E) 11.585

22. Let g be the function given by $g(x) = \int_1^x 100\left(t^2 - 3t + 2\right)e^{-t^2}\,dt$.
Which of the following statements about g must be true?

I. g is increasing on (1, 2).

II. g is increasing on (2, 3).

III. $g(3) > 0$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

23. For a series S, let
$S = 1 - \frac{1}{9} + \frac{1}{2} - \frac{1}{25} + \frac{1}{4} - \frac{1}{49} + \frac{1}{8} - \frac{1}{81} + \frac{1}{16} - \frac{1}{121} + \cdots + a_n + \cdots,$

$$\text{where } a_n = \begin{cases} \dfrac{1}{2^{(n-1)/2}} & \text{if } n \text{ is odd} \\ \dfrac{-1}{(n+1)^2} & \text{if } n \text{ is even.} \end{cases}$$

Which of the following statements are true?

I. S converges because the terms of S alternate and $\lim_{n\to\infty} a_n = 0$.

II. S diverges because it is not true that $|a_{n+1}| < |a_n|$ for all n.

III. S converges although it is not true that $|a_{n+1}| < |a_n|$ for all n.

(A) None
(B) I only
(C) II only
(D) III only
(E) I and III only

24. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$.
For $0 \le t \le 8$, g is decreasing most rapidly when $t =$

(A) 0.949
(B) 2.017
(C) 3.106
(D) 5.965
(E) 8.000

Answers to Calculus BC Multiple-Choice Questions

Part A		*Part B*	
†1.	D	15.	A
2.	B	16.	E
†3.	D	17.*	B
†4.	D	18.	D
5.	D	19.	E
6.	E	20.*	C
7.	E	21.*	B
†8.	E	22.*	B
†9.	E	†23.	D
10.	E	24.*	B
†11.	D		
†12.	A		
†13.	A		
14.	C		

*Indicates a graphing calculator-active question.
†Indicates a Calculus BC–only topic.