Non Calculator Portion form A

1. If $f(x)=\frac{x^2-9}{x+3}$ is continuous at $x=-3$, then $f(-3)=$

(A) 3 (B) -3 (C) 0 (D) 6 (E) -6

2. The graph of $y=3x^2-x^3$ has a relative maximum at

(A) $(0,0)$ only (B) $(1,2)$ only (C) $(2,4)$ only

(D) $(4,-16)$ only (E) $(0,0)$ and $(2,4)$

3. $\lim_{x\to\infty}\frac{10^8x^5+10^6x^4+10^4x^2}{10^9x^6+10^7x^5+10^6x^4}=$

(A) 0 (B) 1 (C) -1 (D) $\frac{1}{10}$ (E) $-\frac{1}{10}$

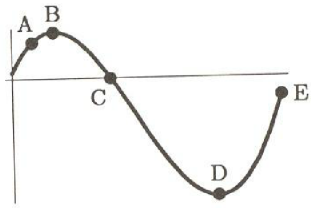
4. If $f(x)=\sqrt{4\sin x+2}$, then $f'(0)=$

(A) -2 (B) 0 (C) $\sqrt{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) 1

5, The equation of the tangent line to the curve $x^2+y^2=169$ at the point $(5,-12)$ is

(A) $5y-12x=-120$ (B) $5x-12y=119$ (C) $5x-12y=169$

(D) $12x+5y=0$ (E) $12x+5y=169$



6. The figure above shows the graph of the velocity of a moving object as a function of time. At which of the marked points is the speed the greatest?

(A) A (B) B (C) C (D) D (E) E

7. If the graph of $f(x)=2x^2+\frac{k}{x}$ has a point of inflection at $x=-1$, then the value of k is

(A) 1 (B) -1 (C) 2 (D) -2 (E) 0

8. Which of the following is an equation of the line tangent to the curve with parametric equations $x=3e^{-t}$, $y=6e^{t}$ at the point where $t=0$?

(A) $2x+y-12=0$ (B) $-2x+y-12=0$ (C) $2x+y-6=0$

(D) $-2x+y-6=0$ (E) $2x+y=0$

9. If $x=\sin t$ and $y=\cos^2 t$, then $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{4}$ (C) $-\frac{1}{4}$ (D) -2 (E) 2

10. If $y=x(\ln x)^2$, then $\frac{dy}{dx}=$

(A) $3(\ln x)^2$ (B) $(\ln x)(2x+\ln x)$ (C) $(\ln x)(2+\ln x)$

(D) $(\ln x)(2+x\ln x)$ (E) $(\ln x)(1+\ln x)$

11. A particle moves on the $x-$axis so that at any time t its velocity $v(t)=\sin 2t$ subject to the condition $x(0)=0$ where $x(t)$ is the position function. Which of the following is an expression for $x(t)$?

(A) $\cos 2t+\frac{1}{2}$ (B) $-\frac{1}{2}\sin 2t+\frac{1}{2}$ (C) $-\frac{1}{2}\cos 2t$

(D) $-\frac{1}{2}\cos 2t+\frac{1}{2}$ (E) $-\frac{1}{2}\cos 2t-\frac{1}{2}$

12. The maximum value of $f(x)=2x^3-9x^2+12x-1$ on $[-1,2]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

13. At what value(s) of x does $f(x)=x^4-8x^2$ have a relative minimum?

(A) 0 and -2 only (B) 0 and 2 only (C) 0 only

(D) -2 and 2 only (E) -2, 0, and 2

14. The function $y=x^4+bx^2+8x+1$ has a horizontal tangent and a point of inflection for the same value of x. What must be the value of b?

(A) -1 (B) 4 (C) 1 (D) 6 (E) -6

15. $\lim_{x\to 2}\frac{2^{\frac{x}{2}}-2}{2^x-4}$ is

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\ln 2$ (E) nonexistent

16. If $x+y=xy$, then $\frac{dy}{dx}=$

(A) $\frac{1}{x-1}$ (B) $\frac{y-1}{x-1}$ (C) $\frac{1-y}{x-1}$

(D) $x+y-1$ (E) $\frac{2-xy}{y}$

17. For $|x|<1$, the derivative of $y=\ln\sqrt{1-x^2}$ is

(A) $\frac{x}{1-x^2}$ (B) $\frac{x}{x^2-1}$ (C) $\frac{-x}{x^2-1}$

(D) $\frac{1}{2\left(1-x^2\right)}$ (E) $\frac{1}{\sqrt{1-x^2}}$

18. What are all values of x for which the graph of $y=x^3-6x^2$ is concave downward?

(A) $0<x<4$ (B) $x>2$ (C) $x<2$ (D) $x<0$ (E) $x>4$

19. A <u>normal</u> line to the graph of a function f at the point $\left(x,f\left(x\right)\right)$ is defined to be the line perpendicular to the tangent line at that point. The equation of the <u>normal</u> line to the curve $y=\sqrt[3]{x^2-1}$ at the point where $x=3$ is

(A) $y+12x=38$ (B) $y-4x=10$ (C) $y+2x=4$

(D) $y+2x=8$ (E) $y-2x=-4$

20. If $\int_0^6\left(x^2-2x+2\right)dx$ is approximated by three <u>inscribed</u> rectangles of equal width on the $x-$axis, then the approximation is

(A) 24 (B) 26 (C) 28 (D) 48 (E) 76

Calculator Portion – form A

21. $\lim_{x\to -3} \dfrac{x^2+3x}{\sqrt{x^2+6x+9}}$ is

(A) -3 (B) -1 (C) 1 (D) 3 (E) nonexistent

22. The cost C of producing x items is given by $C(x)=20{,}000+5(x-60)^2$. The revenue R obtained by selling x items is given by $R(x)=15{,}000+130x$. The revenue will exceed the cost for all x such that

(A) $0<x<46$ (B) $x>46$ (C) $x<100$

(D) $46<x<100$ (E) $x>100$

23.

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	20	19.5	18	15.5	12	7.5	2	-4.5	-12	-20.5	-30

Some values of a continuous function are given in the table above. The Trapezoidal Rule approximation for $\int_0^{10} f(x)\,dx$ is

(A) 30.825 (B) 32.500 (C) 33.325 (D) 33.333 (E) 35.825

24. For which pair of functions $f(x)$ and $g(x)$ below, will the $\lim_{x\to\infty} \dfrac{f(x)}{g(x)}=0$?

	$f(x)$	$g(x)$
(A)	e^x	x^2
(B)	e^x	$\ln x$
(C)	$\ln x$	e^x
(D)	x	$\ln x$
(E)	3^x	2^x

25,

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
$f(x)$	2.018	2.008	2.002	2	2.002	2.008	2.018
$g(x)$	1	1	1	2	2	2	2
$h(x)$	1.971	1.987	1.997	undefined	1.997	1.987	1.971

The table above gives the values of three functions, f, g, and h near $x-0$. Based on the values given, for which of the functions does it appear that the limit as x approaches zero is 2?

(A) f only (B) g only (C) h only

(D) f and h only (E) f, g, and h

26. If $f(x)=\left|\left(x^2-12\right)\left(x^2+4\right)\right|$, how many numbers in the interval $-2\le x\le 3$ satisfy the conclusion of the Mean Value Theorem?

(A) None (B) One (C) Two (D) Three (E) Four

27. The amount $A(t)$ of a certain item produced in a factory is given by

$$A(t)=4000+48(t-3)-4(t-3)^3$$

where t is the number of hours of production since the beginning of the workday at 8:00 am. At what time is the rate of production increasing most rapidly?

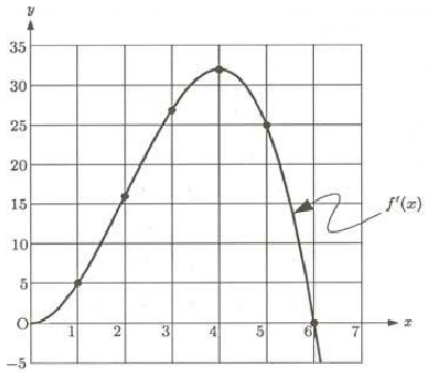
(A) 8:00 am (B) 10:00 am (C) 11:00 am (D) 12:00 am (E) 1:00 pm

28. At how many points on the curve $y=4x^5-3x^4+15x^2+6$ will the line tangent to the curve pass through the origin?

(A) One (B) Two (C) Three (D) Four (E) Five

29. A population grows according to the equation $P(t)=6000-5500e^{-0.159t}$ for $t\ge 0$, t measured in years. This population will approach a limiting value as time goes on. During which year will the population reach <u>half</u> of this limiting value?

(A) Second (B) Third (C) Fourth (D) Eighth (E) Twenty-ninth



Note: This is the graph of $f'(x)$, NOT the graph of $f(x)$.

30. Let f be a differentiable function for all x. The graph of $f'(x)$ is shown above. If $f(2)=10$, which of the following best approximates the maximum value of $f(x)$?

(A) 30 (B) 50 (C) 70 (D) 90 (E) 110

31. Of the choices given, which value is NOT in the domain of the function $f(x)=(\cos x)^x$?

(A) 1 (B) $\frac{\pi}{2}$ (C) $\frac{4\pi}{3}$ (D) 4 (E) 2π

32. Let f be a function that is everywhere differentiable. The value of $f'(x)$, is given for several values of x in the table below.

x	-10	-5	0	5	10
$f'(x)$	-2	-1	0	1	2

If $f'(x)$ is always increasing, which statement about $f(x)$ must be true?

(A) $f(x)$ has a relative minimum at $x=0$.
(B) $f(x)$ is concave downwards for all x.
(C) $f(x)$ has a point of inflection at $(0, f(0))$.
(D) $f(x)$ passes through the origin.
(E) $f(x)$ is an odd function.

33. The figure above shows the graph of a function $f(x)$ which has horizontal asymptotes of $y=3$ and $y=-3$. Which of the following statements are true?

I. $f'(x)<0$ for all $x\geq 0$

II. $\lim_{x\to+\infty} f'(x)=0$

III. $\lim_{x\to-\infty} f'(x)=2$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

34. The graph above shows the distance $s(t)$ from a reference point of a particle moving on a number line, as a function of time. Which of the points marked is closes to the point where the acceleration first becomes negative?

(A) A (B) B (C) C (D) D (E) E

35. The <u>derivative</u> of f is given by $f'(x)=e^x\left(-x^3+3x\right)-3$ for $0\leq x\leq 5$. At what value of x is $f(x)$ an absolute minimum?

(A) For no value of x (B) 0 (C) 0.618 (D) 1.623 (E) 5

36.

x	$f(x)$
3.99800	1.15315
3.99900	1.15548
4.00000	1.15782
4.00100	1.16016
4.00200	1.16250

The table above gives values of a differentiable function f. What is the approximate value of $f'(4)$?

(A) 0.00234 (B) 0.289 (C) 0.427 (D) 2.340

(E) $f'(4)$ cannot be determined from the information given..

37. If $y=7$ is a horizontal asymptote of a rational function f, then which of the following must be true?

(A) $\lim_{x\to 7} f(x)=\infty$ (B) $\lim_{x\to\infty} f(x)=7$ (C) $\lim_{x\to 0} f(x)=\infty$

(D) $\lim_{x\to 7} f(x)=0$ (E) $\lim_{x\to-\infty} f(x)=-7$

38.

x	0	1	2	3	4	5	6
$f(x)$	0	0.25	0.48	0.68	0.84	0.95	1

For the function whose values are given in the table above, $\int_0^6 f(x)dx$ is approximated by a Riemann Sum using the value at the midpoint of each of three intervals of width 2. the approximation is

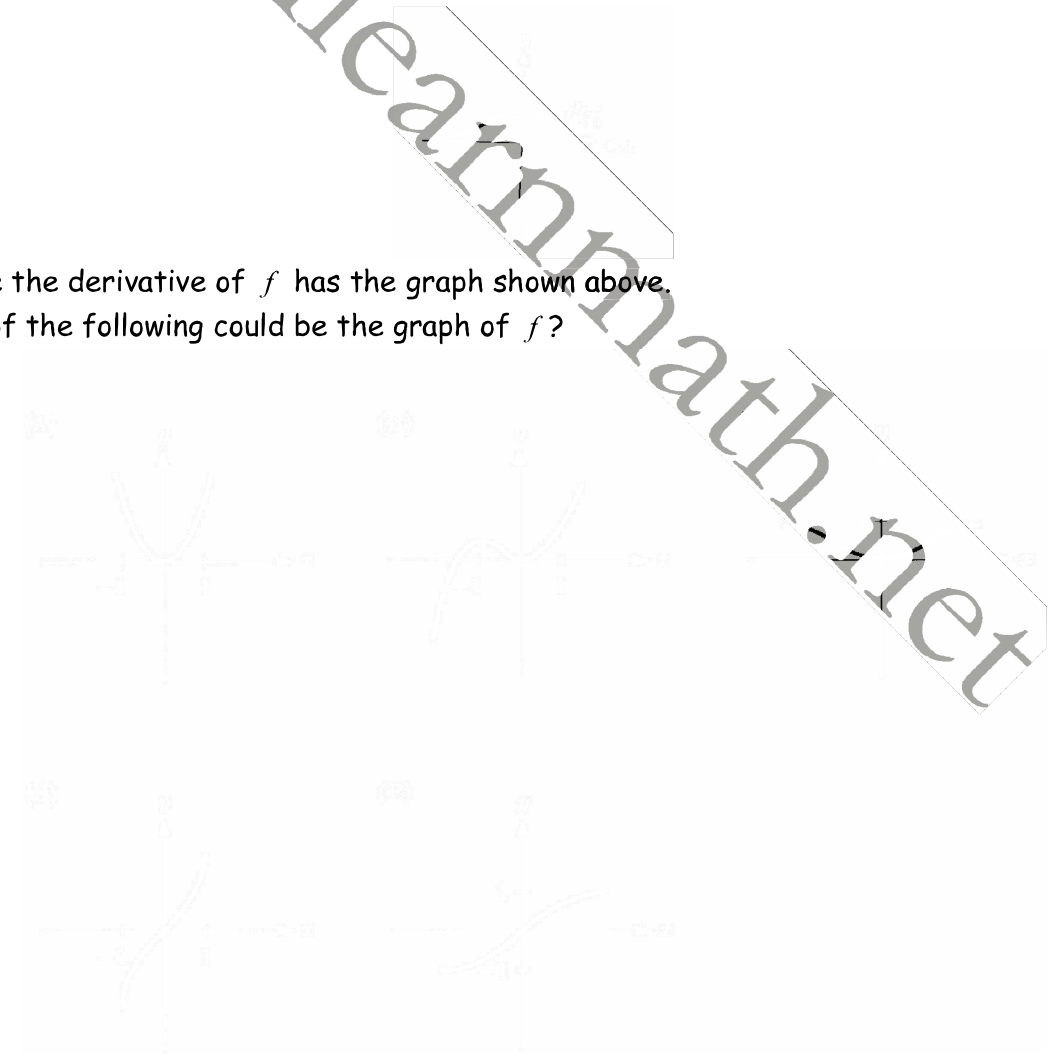
(A) 2.64 (B) 3.64 (C) 3.72 (D) 3.76 (E) 4.64

39. The tangent line to the graph $y=e^{2-x}$ at the point $(1,e)$ intersects both coordinate axes. What is the area of the triangle formed by this tangent line and the coordinate axes?

(A) $2e$ (B) e^2-1 (C) e^2 (D) $2e\sqrt{e}$ (E) $4e$

40. Which graph best represents the position of a particle, $s(t)$, as a function of time, if the particle's velocity and acceleration are both positive?

41.



Suppose the derivative of f has the graph shown above.
Which of the following could be the graph of f?