Name: _____ **Class:** _____ **Date:** _____ **ID: A**

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CALCULUS AB

SECTION I, Part A
Time- 55 Minutes
Number of questions -28
A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.
In this test: Unless otherwise specified, the domain of the function f is assumed to be the set of all real numbers x for which f(x) is a real number.

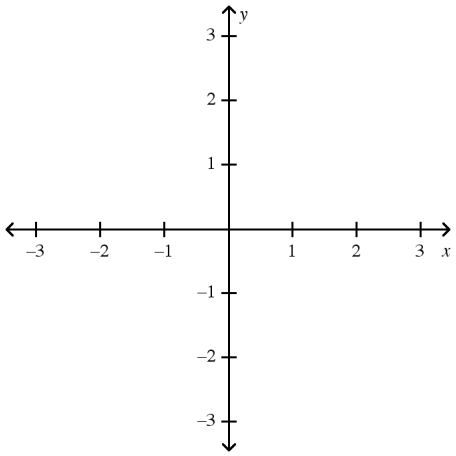
____ 1. If $f(x) = \dfrac{x+3}{x^2+1}$, then $f'(-2) =$

a. $-\dfrac{9}{25}$ b. $-\dfrac{1}{4}$ c. $\dfrac{1}{25}$ d. $\dfrac{1}{4}$ e. $\dfrac{9}{25}$

____ 2. $\lim\limits_{x \to \frac{\pi}{3}} \dfrac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$ is

a. $-\dfrac{\sqrt{3}}{2}$ b. $-\dfrac{1}{2}$ c. $\dfrac{1}{2}$ d. $\dfrac{\sqrt{3}}{2}$ e. nonexistent

____ 3.



The graph of the function f shown in the figure above has vertical tangents at (-1,-1) and (1,1) and a horizontal tangent at (0,0). For what values of x, -3,x,3, is f not differentiable?

a. 2 only
b. 0 and 2 only
c. -1 and 1 only
d. -1, 1, and 2 only
e. -1, 0,1, and 2

____ 4. If $f(x) = \sec(2x)$, then $f'(x) =$

a. $\tan^2(2x)$
b. $2\tan^2(2x)$
c. $\tan(2x)\sec(2x)$
d. $2\tan(2x)\sec(2x)$
e. $-2\sin(2x)\sec^2(2x)$

____ 5. $\int_0^1 x\left(\sqrt{x}+1\right)dx =$

a. 0 b. $\frac{5}{6}$ c. $\frac{9}{10}$ d. 1 e. 2

____ 6. What is the slope of the tangent line to the graph of $\frac{xy+1}{y+2} = 1$ at point (2,1) ?

a. -1 b. -0.5 c. 0.5 d. 1 e. nonexistent

____ 7. What is the x-coordinate of the point of inflection for the graph of $y = x^3 + 3x^2 - 1$?

a. -2 b. -1 c. 0 d. 1 e. 2

____ 8. $\int_0^x \sin t\cos^2 t\,dt =$

a. $-\cos^3 x$ c. $\frac{\cos^3 x}{3}$ e. $\frac{1-\cos^3 x}{3}$

b. $-\frac{\cos^3 x}{3}$ d. $\cos^3 x$

____ 9. A particle moves along the x-axis so that its position at time t > 0 is given by $x(t) = 3\ln t - 6t + 7$. At what time t > 0 is the velocity of the particle equal to zero?

a. ln 0.5 b. 0.5 c. ln 2 d. 2 e. 3

____ 10. If $F(x) = \int_1^{\ln x} \sqrt{\cos t}\,dt$, then $F'(x) =$

a. $\sqrt{\cos x}$

b. $\sqrt{\cos\left(\frac{1}{x}\right)}$

c. $\sqrt{\cos(\ln x)}$

d. $\frac{\sqrt{\cos(\ln x)}}{x}$

e. $\frac{\sin(\ln x)}{2\sqrt{\cos(\ln x)}}$

____ 11. The function f is differntiable on the closed interval [1,4 , and it has values as follows: f(1)=3, f(2)=k , and f(4)=5k+2. For which of the following values of k must there exist two points a and b on the open interval (1,4) with $f'(a) = f'(b)$?

a. -4 b. $-\frac{5}{3}$ c. $-\frac{1}{2}$ d. $\frac{1}{5}$ e. 3

____ 12. Which of the following is a solution to the differential equation $\frac{dy}{dx} = \frac{\sec^2 x}{y^2}$?

a. $y = \sec x$

b. $y = \sqrt[3]{3\tan x} + 1$

c. $y = \sqrt[3]{3\tan x + 1}$

d. $y = 3\sqrt[3]{\tan x} + 1$

e. $y = 3\sqrt[3]{\tan x + 1}$

____ 13. If $g(x) = f(x^2)$, what is $g''(x)$?

a. $2xf''(x^2)$

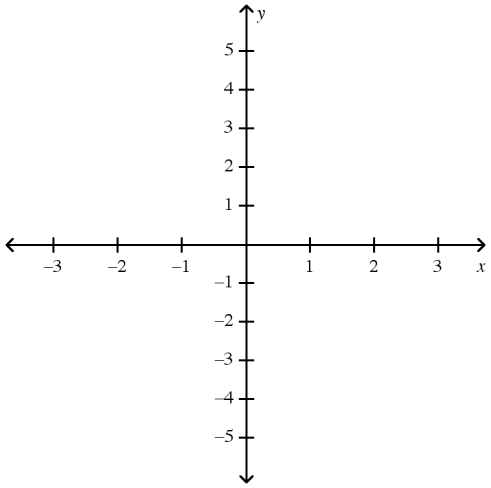
b. $4xf''(x^2)$

c. $(4x^2 + 2x)f''(x^2)$

d. $4x^2 f''(x^2) + 2f'(x^2)$

e. $4x^2 f''(x^2) + 2xf'(x^2)$

____ 14.



The graph of the derivative of the function f is shown above. If f(0)=0, then which of the following is true?

a. $f(-1) < f'(-1) < f''(-1)$
b. $f(-1) < f''(-1) < f'(-1)$
c. $f'(-1) < f''(-1) < f(-1)$
d. $f''(-1) < f(-1) < f'(-1)$
e. $f''(-1) < f'(-1) < f(-1)$

____ 15. What's the instanteneous rate of change of y=ln(cosx) at the point $x = \frac{\pi}{6}$?

a. $-\sqrt{3}$ b. $-\frac{1}{\sqrt{3}}$ c. $\frac{1}{2}$ d. $\frac{1}{\sqrt{3}}$ e. $\frac{\sqrt{3}}{2}$

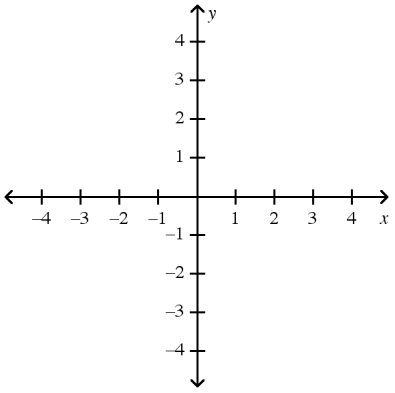
____ 16. For what values of x is the function $f(x) = 1 + x^2 - 2x^4$ increasing ?

a. $-1 < x < 1$
b. $x < -\frac{1}{2}$ *and* $0 < x < \frac{1}{2}$
c. $-\frac{1}{2} < x < 0$ *and* $x > \frac{1}{2}$
d. $x < -1$ *and* $x > 1$
e. $x > 0$

____ 17. Which of the following is an equation of the tangent line to the graph of $y = x^4 - x^3 - x^2 + x + 1$ at the point (1,1) ?

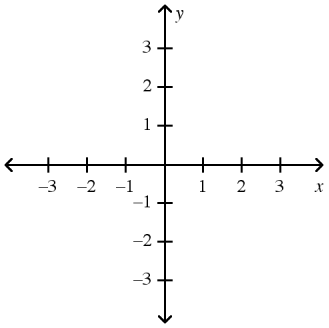
a. y = 1
b. y = x
c. y = -2x+3
d. y = 2x-1
e. y = -x+2

____ 18.

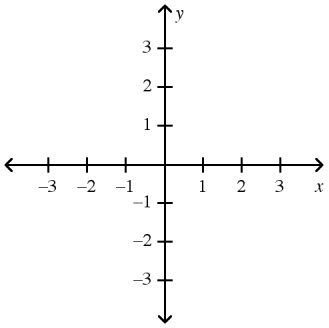


The graph of the function f' is shown in the figure above. Which of the following could be the graph of f?

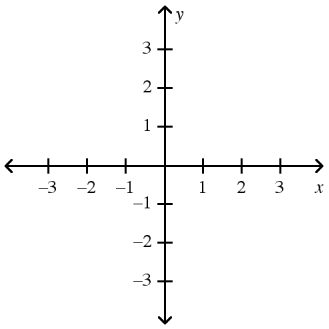
a.



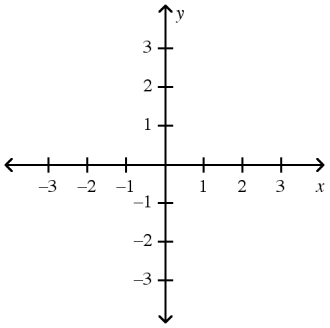
b.



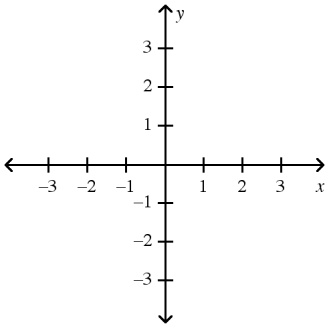
c.



d.



e.



____ 19.

x	0	1	3	7	10
$f(x)$	1	– 1	4	2	3

The function f is continuous on the closed interval [0,10] and has values given in the table above. Using the subintervals $[0,1],[1,3],[3,7]$ *and* $[7,10]$, what is the left Riemann sum estimate for $\int_0^{10} f(x)dx$?

a. 15 b. 17.5 c. 20 d. 21 e. 22.5

____ 20. The function f is given by $f(x) = \begin{cases} \ln 2x, & 0 < x < 2 \\ 2\ln x, & x \geq 2 \end{cases}$. The limit $\lim_{x \to 2} f(x)$ is

a. 0 b. 0.5 c. 1 d. 2ln2 e. nonexistent

____ 21. The differentiable function f(x) achieves its maximum when x=0. Which of the following statements must be true?

I. The function g(x)= x f(x) has a critical point when x=0.
II. The function h(x)= $(f(x))^2$ achieves its maximum at x=0.
III. The function k(x)= $f(x^2)$ achieves its maximum at x=0.

a. None
b. III only
c. I and II only
d. I and III only
e. II and III only

____ 22. $\int_1^8 \frac{dx}{\sqrt[3]{x}} =$

a. $-\frac{63}{128}$ b. $\frac{63}{128}$ c. 1 d. 3 e. $\frac{9}{2}$

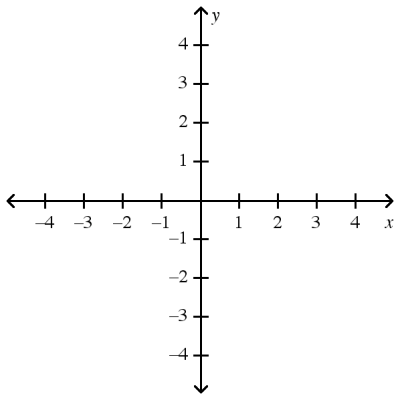
____ 23. If $x + y = 2$, what is the minimum value of $x^2 + y^2$?

a. 0 b. 1 c. 2 d. 4 e. 8

____ 24. A particle moves along the graph of $y = x + \sin x$. As it passes the point $(2\pi, 2\pi)$, the particle's y-coordinate is increasing at a rate of 2 units per second. How fast is the x-coordinate of the particle changing at this point Iin units per second)?

a. 0 b. $\frac{1}{\pi}$ c. $\frac{1}{2}$ d. 1 e. 2

____ 25.



The graph of the piecewise linear function f on the closed interval [-2,4] is shown above. What is $\int_{-1}^{2} f(2x)\,dx$?

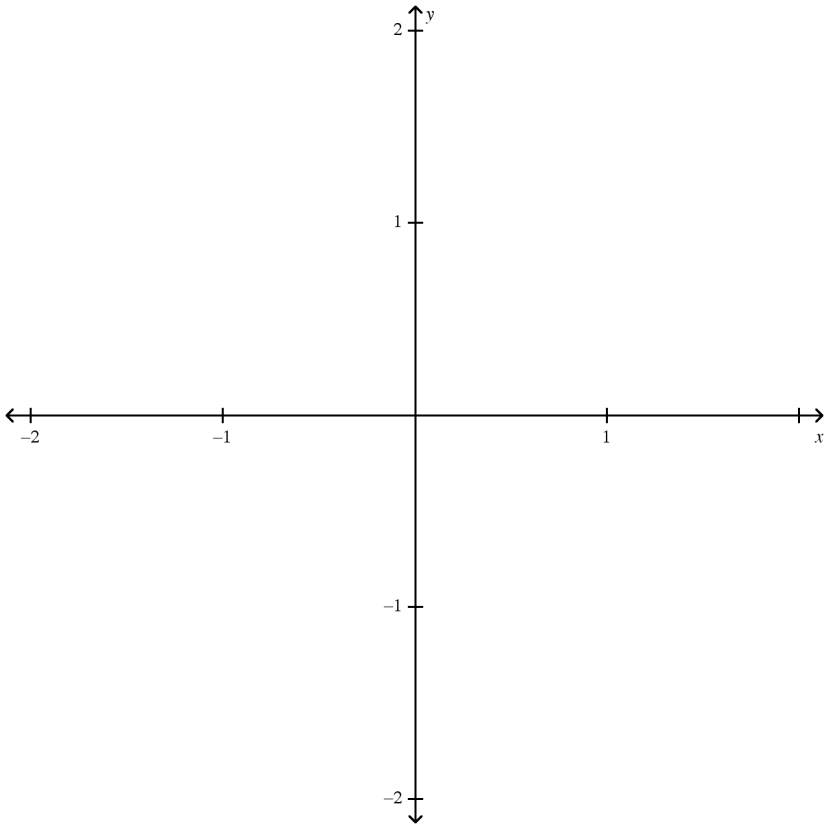
a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. 1 d. $\frac{9}{4}$ e. $\frac{9}{2}$

____ 26. A particle moves along the x-axis. Its position at time t is given by $x(t) = \frac{t^4}{24} - \frac{t^3}{2} + 2t^2 - 1$. What is the maximum acceleration of the particle on the interval $0 \le t \le 4$?

a. $\frac{8}{3}$ b. 3 c. $\frac{10}{3}$ d. $\frac{11}{3}$ e. 4

____ 27. The function f is given by $f(x) = \int_0^x (t^2 - 3)e^t\,dt$. For which value of x does f have a relative minimum?

a. -3 b. $-\sqrt{3}$ c. 0 d. 1 e. $\sqrt{3}$



____ 28.

Shown above is the slope field for which of the following differential equations?

a. $\frac{dy}{dx} = x^2 + y^2$

b. $\frac{dy}{dx} = x(x - 1)$

c. $\frac{dy}{dx} = xy$

d. $\frac{dy}{dx} = \frac{x}{2y}$

e. $\frac{dy}{dx} = y$

CALCULUS AB

.

Answer Section

MULTIPLE CHOICE

1. ANS: E
2. ANS: A
3. ANS: D
4. ANS: D
5. ANS: C
6. ANS: A
7. ANS: B
8. ANS: E
9. ANS: B
10. ANS: D
11. ANS: A
12. ANS: C
13. ANS: D
14. ANS: B
15. ANS: B
16. ANS: B
17. ANS: A
18. ANS: B
19. ANS: D
20. ANS: D
21. ANS: B
22. ANS: E
23. ANS: C
24. ANS: D
25. ANS: A
26. ANS: E
27. ANS: E
28. ANS: C

Name: _____ **Class:** _____ **Date:** _____ **ID: A**

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CALCULUS AB

SECTION I, Part B

Time- 50 Minutes

Number of questions -17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number the best approximates the exact numerical value.

2. Unless otherwise specified, the domain of the function f is assumed to be the set of all real numbers x for which f(x) is a real number.

3. In the actual AP Calculus Exam, the questions in this Part B are numbered from 76 to 92 to eliminate confusion between Part A and Part B on the bubble sheet.

____ 1. What is the average value of the function $f(x) = \sin\sqrt{x}$ on the interval [1,3] ?

a. 0.146 b. 0.914 c. 0.964 d. 0.987 e. 1.928

____ 2. Right triangle ABC has its right angle at A. Leg AB is decreasing at a constant rate of 3 cm per minute. Leg AC is increasing at a constant rate of 4 cm per minute. How is the hypotenuse BC changing at the moment when AB=12 and AC=5?

a. It is decreasing at the rate of 16 cm per minute.
b. it is decreasing at the rate of $\frac{16}{13}$ cm per minute.
c. It is not changing.
d. It is increasing at the rate of $\frac{33}{13}$ cm per minute.
e. it is increasing at a rate of 16 cm per minute.

____ 3. If $g'(x) = x(\ln x)^2$ and $g(2) = 3$, what is $g(3)$?

a. 2.163 b. 2.660 c. 2.780 d. 5.163 e. 5.660

____ 4. $\lim_{x \to -\infty} \dfrac{x^3 - 111x^2 + 3x - 2}{1 - 2x + 22x^2 + 3x^3}$

a. -2 b. $-\dfrac{2}{3}$ c. $-\dfrac{1}{3}$ d. $\dfrac{1}{3}$ e. 1

____ 5. The derivative of a function f is given by $f'(x) = \sin(\cos x) - 0.1x$. How many critical points does f have on the open interval (0,8)?

a. None b. One c. Two d. Three e. Four

____ 6. The function f is given by $f(x) = \begin{cases} \dfrac{x^3}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Which of the following statements are true?

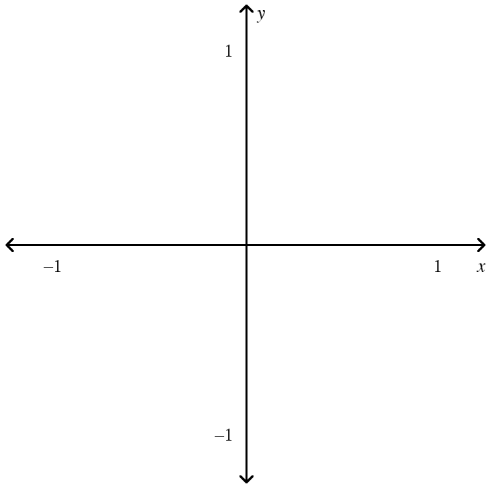
I. f is continuous at the point x=0.
II. f is differentiable at the point x=0.
III. x=0 is a point of inflection for a graph of f.

a. I only
b. III only
c. I and II only
d. I and III only
e. I,II, and III

____ 7. What is the area of the region bounded by the graphs of $y = e^{x^2}$ and y=tanx and the vertical lines $x=-\dfrac{1}{2}$ and x=1?

a. 1.523 b. 2.358 c. 2.493 d. 4.783 e. 7.409

____ 8.



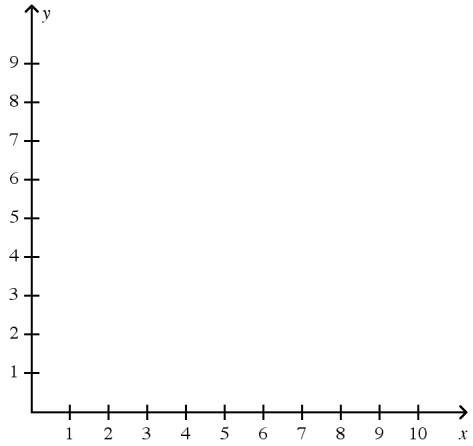
Let f be a function whose domain is the open interval (-2,2). The graph of the derivative of f is shown in the figure above. Which of the following describes the relative extrema of f and the points of inflection on the graph of f?

a. 1 relative minimum, 1 relative maximum, and 1 point of inflection.
b. 1 relative minimum and 1 point of inflection.
c. 1 relative maximum and 1 point of inflection.
d. 1 relative minimum and 2 point of inflection.
e. No relative extrema and 1 point of inflection.

____ 9. What is the x-value of the point at which the tangent line to the graph of $y = e^{x^2} + x$ perpendicular to the line $3y = 2x + 1$

a. -0.732 b. -0.589 c. -0.162 d. 0.236 e. 0.361

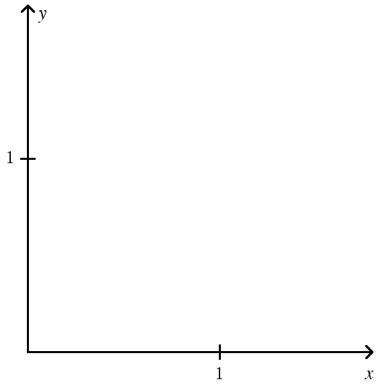
____ 10.



Water is leaking from the tank at the rate of R(t) gallons per hour. The graph R(t) from t=0 to t=10, where t is measured in hours, is shown above. Which of the following best approximates the total loss of water, in gallons, over the ten-hour period?

a. 300 b. 1000 c. 1700 d. 2400 e. 3000

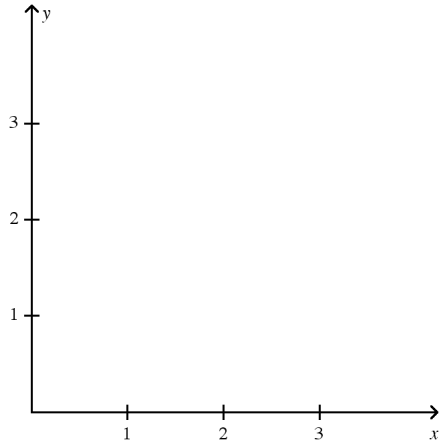
____ 11.



The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the graph of $y = 1 - x^3$, as shown in the figure above. If cross-sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

a. 0.252 b. 0.505 c. 1.010 d. 2.020 e. 2.356

____ 12.



The graph of f is shown above. Which of the following statements about f is false?

a. $\lim_{x \to b} f(x)$ exists.
b. $\lim_{x \to a^+} f(x)$ exists.
c. f has a relative minimum at x=a.
d. f has a relative maximum at x=b.
e. f' is continuous at x=b.

____ 13. The region enclosed by the graphs of $y = e^{x-1}$ and $y = -x$ and the vertical lines x=0 and x=2 is rotated about the line y=-3. Which of the following gives the volume of the generated solid?

a. $\pi\int_0^2 \left(\left(e^{x-1} - 3\right)^2 - (-x-3)^2 \right) dx$
b. $\pi\int_0^2 \left(\left(e^{x-1} + 3\right)^2 - (-x+3)^2 \right) dx$
c. $\pi\int_0^2 \left(\left(e^{x-1}\right)^2 - (-x)^2 - 3^2 \right) dx$
d. $\pi\int_{-2}^e \left(\left(\ln y - 2\right)^2 - \left(-y-3\right)^2 \right) dy$
e. $\pi\int_{-2}^e \left(\left(\ln y + 4\right)^2 - \left(-y+3\right)^2 \right) dy$

____ 14. Which of the following is an equation of the tangent line to the graph of the function $f(x) = e^x + x^2$ at the point where $f'(x) = 2$?

a. y=2x-0.630
b. y=2x+0.537
c. y=2x+0.839
d. y=2x+0.926
e. y=2x+1.469

____ 15.

x	0	1	2
$f(x)$	1	3	1

The function f is twice differentiable on the closed interval [0,2] and has values that are given in the table above. Which of the following statements is false?

a. The equation f(x)=2 has at least two solutions on the open interval (0,2).
b. The equation f'(x)= 0 has a solutions on the open interval (0,2).
c. There exists a point c on the open interval (0,2) such that f'(c)=2.
d. f'(x)> 0 for all x on the open interval (0,2).
e. f has a relative maximum on the open interval (0,2).

____ 16.

x	0	1	2	3
$f(x)$	2	5	4	3

The function f is continuous on the closed interval [0,3] and has values that are given in the table above. Using the subintervals [0,1],[1,2], and [2,3], what is the trapezoidal approximation to $\int_0^3 f(x)dx$?

a. 11 b. 11.5 c. 12 d. 12.5 e. 13

____ 17. A population of bacteria given by y(t) grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in minutes. If y(10)=10 and y(30)=25, what is the value of k?

a. -2.079 b. 0.046 c. 0.107 d. 0.125 e. 0.230

. **CALCULUS AB**

Answer Section

MULTIPLE CHOICE

1. ANS: C
2. ANS: B
3. ANS: D
4. ANS: D
5. ANS: D
6. ANS: E
7. ANS: A
8. ANS: C
9. ANS: A
10. ANS: C
11. ANS: A
12. ANS: E
13. ANS: B
14. ANS: C
15. ANS: D
16. ANS: B
17. ANS: B

Name: _____ Class: _____ Date: _____ ID: A

. AP CALCULUS - SECTION II - GENERAL INSTRUCTIONS

You may wish to look over all problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight , but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION

* You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write crearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.

* Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.

*Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.

* You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

* Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt $\left(X^2,X,1,5\right)$.

* Unless otherwise specified (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

* Unless otherwise specified, the domai of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

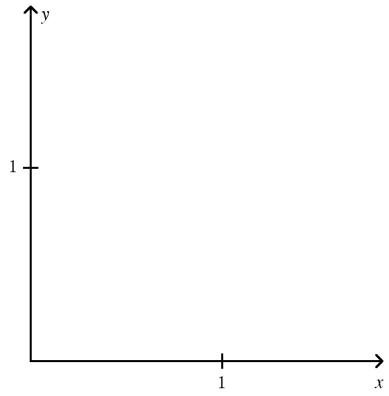
SECTION II, Part A
Time- 45 minutes
Number of problems- 3

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

____ 1.



Let R and S be regions in the first quadrant shown in the figure above. The region R is bounded by the y-axis and the graphs of y= x and y=cos x. The region S is bounded by the x-axis and the graphs of y=x and y=cos x.

a. Find the area of R.

b. Find the area of S.

c. Find the volume of the solid generated when R is revolved about the line y=1.

____ 2. Because of rainfall from 9 a.m. to 9 p.m. on given day, water enters an open reservoir at the rate of $R(t) = \dfrac{73000}{t^2 - 8t + 25}$, where R(t) is measured in gallons per hour and t is measured in hours after 9 a.m. (so $0 \le t \le 12$). To prevent overflow, water is pumped out the reservoir at the constant rate of k gallons per hour. The reservoir holds 500,000 gallons of water both at 9 a.m. and at 9 p.m.

a. What is k? Round your answer to the nearest whole number.

b. To the nearest gallon, how much water is in the reservoir at 7 p.m. (t=10)?

c. At what time t, $0 \le t \le 12$, does the reservoir hold the greatest amount of water?

____ 3.

t (*hours*)	0	2	5	7	8	10
$v(t)$ (*miles per hour*)	50	55	60	70	65	75

The table above gives the velocity v(t) at selected times t of a car traveling along a straight road.

a. Use the values of the table to approximate the acceleration of the car at time t=6. Show your work that leads to your answer and indicate units of measure.

b. Use a right Riemann sum with the subintervals given in the table to approximate $\int_0^{10} v(t)dt$. Indicate units of measure. What physical quantity does this integral represent?

c. The function v(t) is twice differentiable on the interval [0,10]. Show that there must be a moment of time when the acceleration of the car is equal to zero.

SECTION II, Part B
Time- 45 minutes
Number of problems- 3

No calculator is allowed for these problems. During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

4. Let f be a differentiable function whose graph is passing through the point $\left(2,\frac{1}{2}\right)$. For all points (x,y), the slope on the graph of y=f(x) is given by $\frac{dy}{dx} = y^3(3-2x)$.

   (a) Find $\frac{d^2y}{dx^2}$. Is the graph of f concave up, concave down, or neither at the point $\left(2,\frac{1}{2}\right)$?

   (b) Write an equation of the tangent line to the graph of f at point $\left(2,\frac{1}{2}\right)$.

   (c) Find y=f(x) by solving the separable differentiable equation $\frac{dy}{dx} = y^3(3-2x)$ with the initial condition $f(2) = \frac{1}{2}$.

5. Let f be the function defined by $f(x) = e^x(ax^2 + bx + 6)$, where a and b are constants. The function f has arelative extremum at the point x= -1 and $f(-1) = 17e^{-1}$.

(a) Find a and b.

(b) Is the point $\left(-1,17e^{-1}\right)$ a relative minimum or relative maximim of f?

(c) Find all the points of inflection of the graph of f.

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6. A spherical baloon is shrinking at a variable rate so that when its radius is r inches, its volume is decreasing at π r cubic inches per second. (The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

   (a) How fast is the radius of the baloon decreasing when r=5 inches? Include units of measure.

   (b) Show that the surface area of the baloon is decreasing at a constant rate. (The surface area of a sphere with radius r is given by S=$S = 4\pi r^2$.)

   (c) If the balloon's initial radius is 10 inches, how long will it take until its surface area is zero? Indicate units of measure.

. AP CALCULUS - SECTION II - GENERAL INSTRUCTIONS
Answer Section

MULTIPLE CHOICE

1. ANS: C
   see

   TOP: Trigonometry
2. ANS: C
3. ANS: C

SHORT ANSWER

4. ANS:
   c
5. ANS:
   see
6. ANS:
   see