ANSWER KEY: PROBLEMS 1-28

1	B	7	D	13	B	19	Skip	25	B
2	A	8	A	14	D	20	D	26	B
3	D	9	C	15	C	21	E	27	E
4	B	10	B	16	C	22	E	28	C
5	E	11	A	17	E	23	D		
6	B	12	E	18	D	24	E		

ANSWER KEY: PROBLEMS 29-45

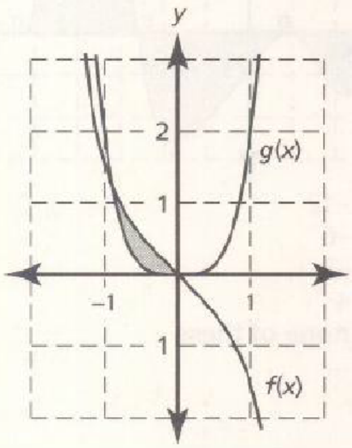
29	E	35	B	41	B				
30	A	36	A	42	C				
31	D	37	E	43	C				
32	E	38	B	44	C				
33	A	39	B	45	C				
34	A	40	C						

Last updated: 4-12-2011 (Thanks to Ben S. and X. Ruan for their help!)

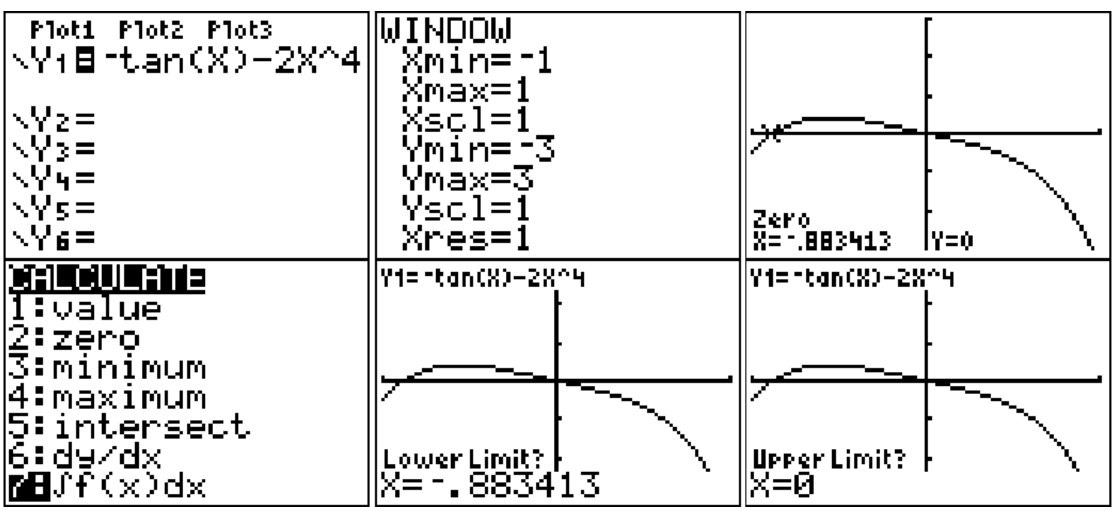
Problem 29

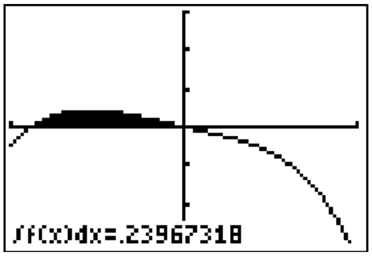
29. What is the area of the first closed region to the left of the y–axis, bounded by the curves $f(x) = -\tan x$ and $g(x) = 2x^4$?

(A) 1.206
(B) 0.931
(C) 0.691
(D) 0.452
(E) 0.240



Y = (Curve above) **minus (**Curve Below)





Final Answer: E

Problem 30

30. What is the average rate of change of $f(x) = \dfrac{e^{\frac{1}{x}}}{x^2}$ in the interval $-4 \le x \le -1$?

(A) 0.106
(B) 0.137
(C) 0.319
(D) 0.411
(E) 1.233

$$AVERAGE\ RATE_{[a,b]} = \frac{f(b) - f(a)}{b - a}$$

$$AVERAGE\ RATE_{[-4,-1]} = \frac{f(-1) - f(-4)}{-1 - (-4)} \approx 0.106$$

Use the calculator the compute the values in the numerator.

Final Answer: A

Problem 31

31. Consider the integral expression $\int_0^{\frac{\pi}{2}} \sin(2x)e^{\cos(2x)}\,dx$. If $u = \cos 2x$ then which integral below is equivalent to the given integral?

(A) $-\frac{1}{2}\int_0^{\pi} e^u\,du$

(B) $-2\int_0^{\pi} e^u\,du$

(C) $-\frac{1}{2}\int_{-1}^{1} e^u\,du$

(D) $\frac{1}{2}\int_{-1}^{1} e^u\,du$

(E) $2\int_{-1}^{1} e^u\,du$

A -2 is needed inside the integral to make the Chain Rule work; so we need to multiply by $-\frac{1}{2}$ in front of the integral sign. We should also remember to change the bounds of integration by plugging them into the expression for u. Because the new bounds of integration are 1 and -1, respectively, we swap them by multiplying by a minus sign in the front. Hence D is the correct choice.

Final Answer: D

Problem 32

32. Let $f(x) = \dfrac{1}{x}$ and $k > 1$. If the area between the x-axis and the graph of $f(x)$ in the closed interval $k \le x \le k+1$ is 0.125 where $k > 1$, then what is the value of k?

(A) 0.133
(B) 1.133
(C) 1.334
(D) 2.998
(E) 7.510

$$\int_k^{k+1} \frac{1}{x}\,dx = \ln x\Big|_k^{k+1} = \ln(k+1) - \ln k = \ln\frac{k+1}{k} = 0.125$$

$$\frac{k+1}{k} = e^{0.125}$$

$$1 + \frac{1}{k} = e^{0.125}$$

$$\frac{1}{k} = e^{0.125} - 1$$

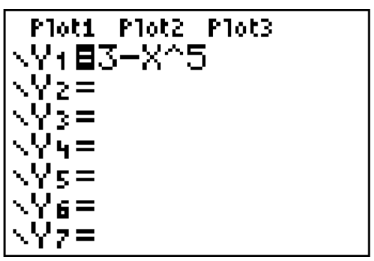
$$k = \frac{1}{e^{0.125} - 1} = 7.51041...$$

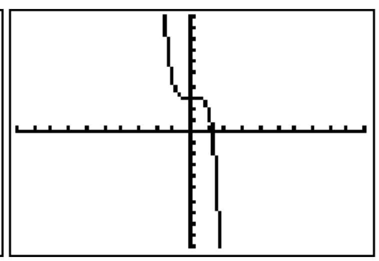
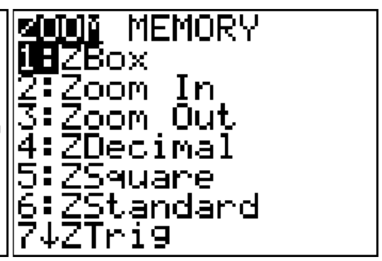
Final Answer: E

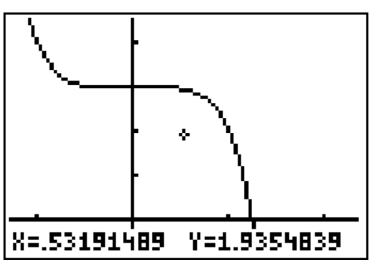
Problem 33

33. A solid has its base in the *xy*-plane, bounded by the *x*-axis, the *y*-axis, and the function $y = 3 - x^5$. If cross sections taken perpendicular to the *x*-axis are semicircles whose diameters are in the *xy*-plane, what is the volume of this solid?

(A) 3.335
(B) 4.247
(C) 5.239
(D) 6.671
(E) 13.342





Bounds of integration: $Left: x = 0 \quad Right: x = \sqrt[5]{3}$

$$r = \frac{diameter}{2} = \frac{y}{2} = \frac{3 - x^5}{2} \Rightarrow A(x) = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{3 - x^5}{2}\right)^2$$

$$V = \int_0^{\sqrt[3]{5}} \frac{\pi}{2}\left(\frac{3 - x^5}{2}\right)^2 dx \approx 3.335\,[TI - 84]$$

Final Answer: A

Problem 34

34. Shampoo drips from a crack in the side of a plastic bottle at a rate modeled by $Y(t) = \dfrac{t}{\sqrt{1 + t^{\frac{3}{2}}}}$, where $Y(t)$ is in ounces per minute. If there are 32 ounces in the bottle at $t = 0$, how many ounces are left in the bottle after 5 minutes?

(A) 26.937 ounces
(B) 24.355 ounces
(C) 7.645 ounces
(D) 5.063 ounces
(E) The bottle will be empty before 5 minutes has elapsed.

$$V(5) - V(0) = \int_0^5 -\frac{t}{\sqrt{1+t^{\frac{3}{2}}}}\,dt$$

$$V(5) = 32 + \int_0^5 -\frac{t}{\sqrt{1+t^{\frac{3}{2}}}}\,dt = 32 - 5.06306 \approx 26.937$$

- The integral was found using a graphing calculator
- The minus sign is introduced to suggest that the rate is negative. We could have computed the integral first, as long as we remember to subtract 5.06306 from 32.

Final answer: A

Problem 35

35. Consider the function $f(x) = x^3 + 2$ in the closed interval $0 < a \le c \le 2$. If the value guaranteed by the Mean Value Theorem in the closed interval is $c = 1.720$, then what is the value of a?

(A) 1.260
(B) 1.424
(C) 1.602
(D) 1.680
(E) none of these

$$\frac{f(2)-f(a)}{2-a} = f'(c) = 3c^2$$

$$\frac{10-(a^3+2)}{2-a} = 3(1.720)^2$$

$$\frac{8-a^3}{2-a} = 3(1.720)^2$$

$$4+2a+a^2 = 3(1.720)^2$$

$$Y_1 = 4+2x+x^2$$

$$Y_2 = 3(1.720)^2$$

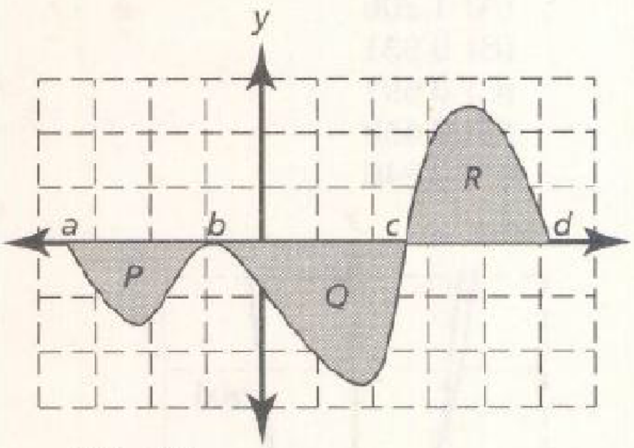
TI-84: 2nd, TRACE, INTERSECT

$$a = 1.42388 \approx 1.424$$

Final Answer: B

Problem 36

36. The sketch of $f(x)$ is shown below, with regions bounded by $f(x)$ and the x-axis indicated by P, Q, and R. If $\int_a^d f(x)\,dx = -7$, $\int_b^d f(x)\,dx = -2$ and $\int_c^a f(x)\,dx = 17$, what is $\int_b^c f(x)\,dx$?



(A) –12
(B) –6
(C) –3
(D) 4
(E) none of these

$$R - P - Q = -7$$
$$R - Q = -2$$
$$-P = -5 \Rightarrow P = 5$$
$$\int_c^a f(x)dx = -\int_a^c f(x)dx \Rightarrow -P - Q = -17$$
$$-5 - Q = -17$$
$$Q = 12$$

Final Answer: A.

Problem 37

37. Let $h(x) = xg(x)$, where $g(x) = f^{-1}(x)$. Use the table of values below to find $h'(5)$.

x	$f(x)$	$f'(x)$
2	4	−1
3	5	2
5	1	3

(A) $\frac{1}{2}$

(B) 2.5

(C) 3

(D) $4\frac{2}{3}$

(E) 5.5

$$h'(x) = g(x) + xg'(x)$$

$$g(5) = f^{-1}(5) = 3$$

$$g'(5) = \frac{1}{f'(g(5))} = \frac{1}{f'(3)} = \frac{1}{2}$$

$$h'(5) = g(5) + 5g'(5) = 3 + 5 \bullet \frac{1}{2} = 5.5$$

Final Answer: E

Problem 38

38. Let $f(x) = \sin x$ and $g(x) = p \ln x$ in the closed interval $0 \le x \le \frac{\pi}{2}$. For what value of p will the tangents to the curves at their points of intersection be perpendicular?

(A) – 0.447
(B) 0.410
(C) 1.260
(D) 1.303
(E) none of these

INTERSECTION:

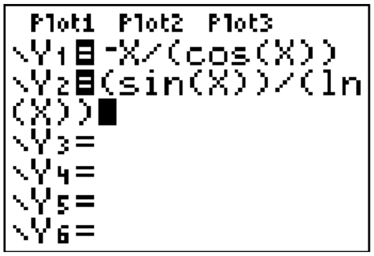
$$f(x) = g(x) \Rightarrow \sin x = p \ln x \Rightarrow p = \frac{\sin x}{\ln x}$$

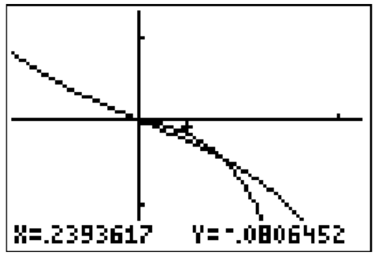
TANGENT LINES PERPENDICULAR:

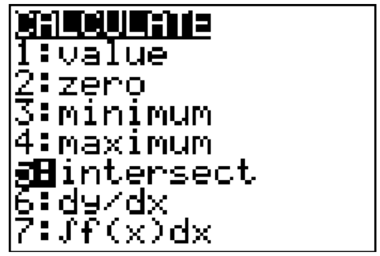
$$\begin{cases} f'(x) = \cos x \\ g'(x) = \dfrac{p}{x} \end{cases} \Rightarrow f'(x)g'(x) = -1 \Rightarrow \cos x \frac{p}{x} = -1 \Rightarrow p = \frac{-x}{\cos x}$$

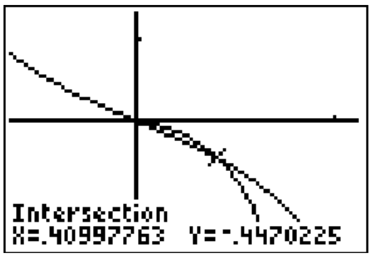
$$\frac{-x}{\cos x} = \frac{\sin x}{\ln x}$$

$$\begin{cases} Y_1 = \dfrac{-x}{\cos x} \\ Y_2 = \dfrac{\sin x}{\ln x} \end{cases} \Rightarrow x =$$









Final Answer: B

Problem 39

39. The height of a conical sand pile is always twice the radius. If sand is being added to the pile at a rate of 30π cm^3/min, how fast is the height of the pile increasing when the circumference of the base of the sand pile is 120π cm?

($V_{cone} = \frac{\pi}{3} r^2 h$)

(A) $\frac{1}{120\pi}$ cm/min

(B) $\frac{1}{120}$ cm/min

(C) $\frac{2}{15}$ cm/min

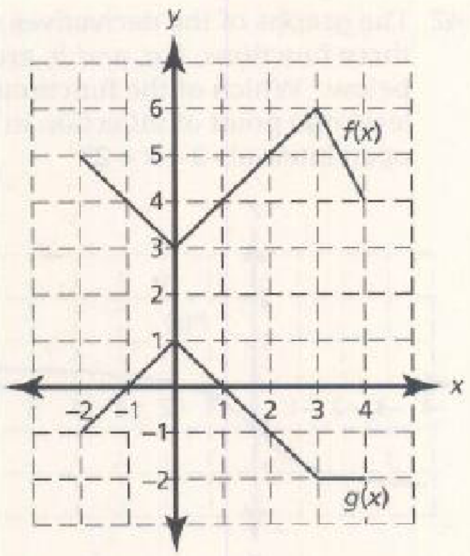
(D) $\frac{1}{4}$ cm/min

(E) none of these

$$h(t) = 2r(t)$$
$$2\pi r = \pi h = 120\pi \Rightarrow h = 120$$
$$V(t) = \frac{\pi}{3} r^2 h = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12} h^3$$
$$\frac{dV}{dt} = 30\pi = \frac{3\pi}{12} h^2 \frac{dh}{dt}$$
$$\frac{dh}{dt} = \frac{120}{h^2} = \frac{120}{120^2} = \frac{1}{120}\, cm/\min$$

Final Answer B

Problem 40



40. The graphs of $f(x)$ and $g(x)$ are shown above. If $h(x) = \dfrac{g(2x)}{f(x)}$, use the graphs to find $h'(1)$.

(A) $-\dfrac{7}{4}$

(B) $-\dfrac{9}{16}$

(C) $-\dfrac{7}{16}$

(D) $-\dfrac{5}{16}$

(E) $-\dfrac{3}{16}$

$$h'(x) = \frac{f(x)2g'(2x) - f'(x)g(2x)}{\left[f(x)\right]^2}$$

$$h'(1) = \frac{f(1)2g'(2) - f'(1)g(2)}{\left[f(1)\right]^2} = \frac{4*2*(-1) - 1*(-1)}{4^2} = \frac{-7}{16}$$

Final Answer: C

Problem 41

41. The number of home fires each day in a certain city increases as the temperature drops. The rate of home fires is modeled by

$F(t) = 4\cos\left(\frac{t}{58} - 2\right) + 6$, for

$0 \le t \le 365$ days, where midnight on January 1st corresponds to $t = 0$. Which of the following is *closest* to the approximate number of fires in the first quarter of the year?

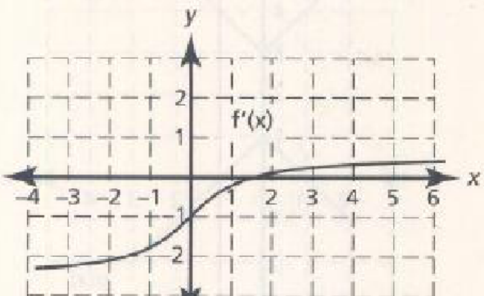
(A) 910
(B) 660
(C) 540
(D) 330
(E) 240

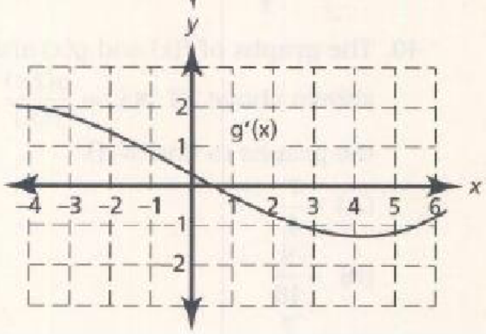
$$\int_0^{91} F(t)\,dt = \int_0^{91} 4\cos\left(\frac{t}{58} - 2\right) + 6\,dt = 660.025$$

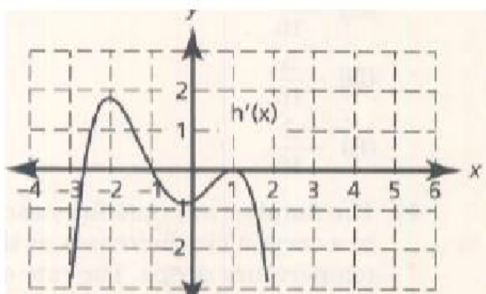
Final Answer: B

Problem 42

42. The graphs of the derivatives of three functions, f, g, and h, are given below. Which of the functions has at least one point of inflection in the open interval $-3 < x < 2$?







(A) only $f(x)$
(B) only $g(x)$
(C) only $h(x)$
(D) only $f(x)$ and $g(x)$
(E) only $g(x)$ and $h(x)$

A point of inflection occurs when the second derivative (slope of the graphs above!) changes sign while the first derivative does not change its sign. The second derivative never changes sign (always positive) for $f(x)$. The second derivative changes sign at approximately $x = 4$ for graph $g(x)$, but this outside the wanted interval. The second derivative changes sign at $x = -2, x \approx -0.1$, and $x = 1$ for graph $h(x)$. The $y-$values of the first derivative for $h(x)$ do not change sign at these points, so this agrees with the definition of inflection points.

Final Answer: C

Problem 43

x	f	g	f'	g'
1	3	4	$\frac{2}{3}$	$-\frac{5}{2}$
2	4	2	$\frac{4}{3}$	$-\frac{3}{2}$
4	8	1	$\frac{8}{3}$	$\frac{1}{2}$

43. If $f(x)$ and $g(x)$ are differentiable functions with values as given in the chart above, and $k(x) = f\left(g(x^2)\right)$, what is $k'(2)$?

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{4}{3}$

(D) $\frac{16}{3}$

(E) none of these

$$k'(x) = f'(g(x^2)) * g'(x^2) * 2x$$

$$k'(2) = f'(g(4)) * g'(4) * 2 * 2 = f'(1) * \frac{1}{2} * 4 = \frac{2}{3} * 2 = \frac{4}{3}$$

Final Answer: C

Problem 44

44. The price of a newly issued stock varies sinusoidally during the first 10 days after its initial offering and is modeled by

$P(t) = \log(2t+1)\sin t + 20$,

where t is in days. To the nearest cent, what is the price of the stock when the price of the stock is decreasing most rapidly in the interval $0 \le t \le 10$?

(A) \$7.98
(B) \$9.49
(C) \$19.91
(D) \$20.12
(E) \$21.22

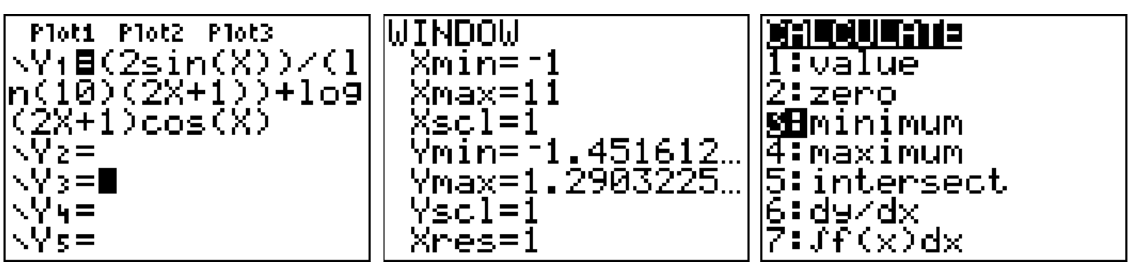
$P'(t) = \dfrac{2}{(2t+1)\ln 10}\sin t + \log(2t+1)\cos t$

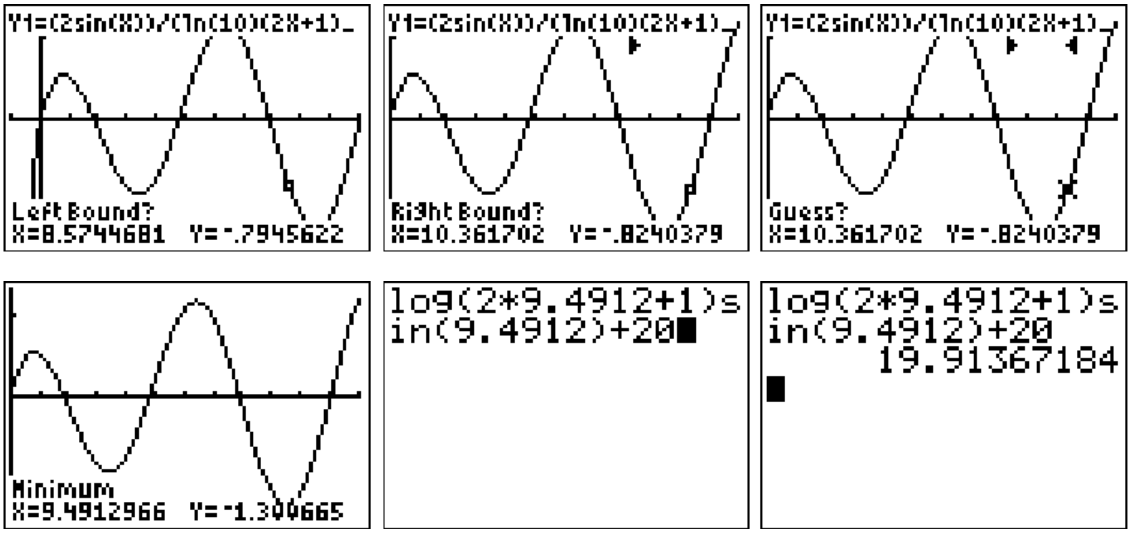
$PLOT\ \ Y_1 = P'(t)$

$XMin = 0$

$XMax = 10$

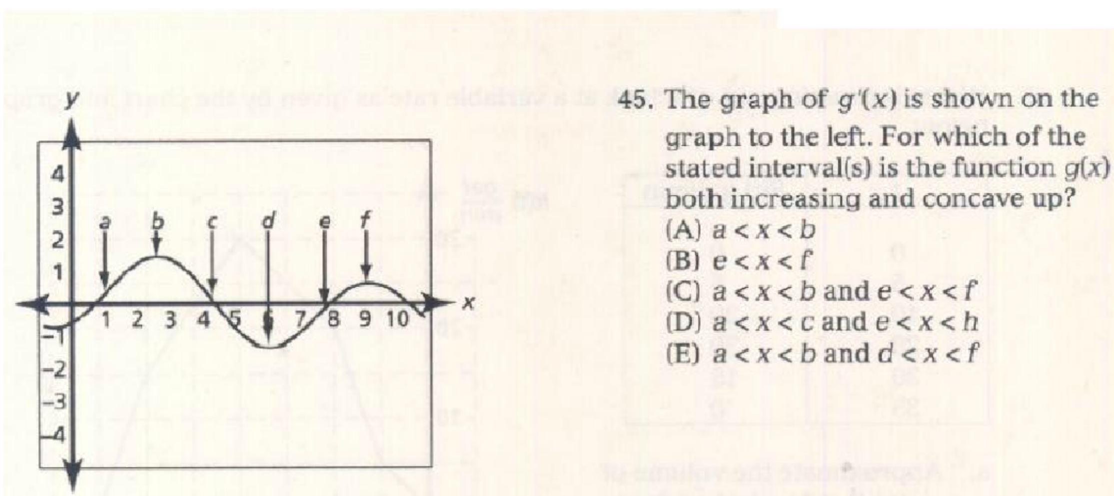
$P(9.4912966) \approx 19.91$





Final Answer: C

Problem 45


45. The graph of $g'(x)$ is shown on the graph to the left. For which of the stated interval(s) is the function $g(x)$ both increasing and concave up?

(A) $a < x < b$
(B) $e < x < f$
(C) $a < x < b$ and $e < x < f$
(D) $a < x < c$ and $e < x < h$
(E) $a < x < b$ and $d < x < f$

The given graph must have positive y-values in order for the original graph to be increasing.

The derivative of the given graph (second derivative of the original graph) must be positive in order to satisfy 'concave upward'.

This occurs in the intervals [a, b] and [e, f]. Final Answer: C.