Multiple Choice Set #1

Find the following algebraically.

1. $\lim_{x\to 3}\frac{x-3}{x^2-2x-3}$ is

(A) 0 (B) 1 (C) $\frac{1}{4}$ (D) ∞ (E) none of these

2. $\lim_{x\to 0}\frac{|x|}{x}$ is

(A) 0 (B) nonexistent (C) 1 (D) -1 (E) none of these

3. $\lim_{x\to 7}\frac{x-7}{\sqrt{x}-7}$ is

(A) $2\sqrt{7}$ (B) $\sqrt{7}$ (C) 0 (D) $-2\sqrt{7}$ (E) nonexistent

Multiple Choice Set #2

Find the following WITHOUT the use of a calculator.

1. $\lim_{x\to 1}\frac{x}{\ln x}$ is

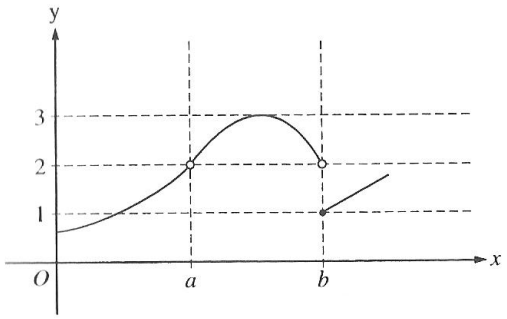
(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent

2. If $a \neq 0$, then $\lim_{x\to a}\frac{x^2-a^2}{x^4-a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

3. $\lim_{x\to\infty}\frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}=$

(A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) -1



4. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$

(B) $\lim_{x\to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

Multiple Choice Set #3

Find the following WITHOUT the use of a calculator.

1. Let $f(x) = 4 - 3x$. Which of the following is equal to $f'(-1)$?

(A) -7 (B) 7 (C) -3 (D) 3 (E) nonexistent

2. Which of the following is true about the graph of $f(x) = x^{\frac{4}{5}}$ at x = 0?

(A) It has a corner.
(B) It has a cusp.
(C) It has a vertical tangent.
(D) It has a discontinuity.
(E) $f(0)$ does not exist.

3. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at x = 0.
II. f is differentiable at x = 0.
III. f has an absolute minimum at x = 0.

(A) I only (B) II only (C) III only
(D) I and III only (E) II and III only

Multiple Choice Set #4

Find the following WITHOUT the use of a calculator.

1. If the line normal to the graph of f at the point (1,2) passes through the point (-1,1), then which of the following gives the value of $f'(1) = ?$

   (A) -2 (B) 2 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 3

2. Find $\frac{dy}{dx}$ if $y = \frac{4x-3}{2x+1}$.

   (A) $\frac{10}{(4x-3)^2}$

   (B) $-\frac{10}{(4x-3)^2}$

   (C) $\frac{10}{(2x+1)^2}$

   (D) $-\frac{10}{(2x+1)^2}$

   (E) 2

3. Let $f(x) = 1-3x^2$. Which of the following is equal to $f'(1)$?

   (A) -6 (B) -5 (C) 5 (D) 6 (E) Does not exist

Multiple Choice Set #5

Find the following WITHOUT the use of a calculator.

1. If the nth derivative of y is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(7)}$ is the same as

(A) y (B) $\frac{dy}{dx}$ (C) $\frac{d^2y}{dx^2}$ (D) $\frac{d^3y}{dx^3}$ (E) none of the above

2. Find $\frac{dy}{dx}$ if $y = \frac{4}{x^3}$.

(A) $-4x^2$

(B) $-\frac{12}{x^2}$

(C) $\frac{12}{x^2}$

(D) $\frac{12}{x^4}$

(E) $-\frac{12}{x^4}$

3. Use the table below to find the value of $\frac{d}{dx}(f \bullet g)$ at $x = 3$.

(A) $\frac{5}{2}$ (B) $-\frac{3}{2}$ (C) -13 (D) 12 (E) $\frac{21}{2}$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	2	5	$\frac{1}{2}$
3	7	-4	$\frac{3}{2}$	-1

Multiple Choice Set #6

Find the following WITHOUT the use of a calculator.

1. What does the limit statement $\lim_{x\to 1}\frac{\ln(x+1)-\ln 2}{x-1}$ represent?

(A) 0 (B) $\frac{d}{dx}[\ln(x+1)]$ (C) $f'(1)$, if $f(x)=\ln(x+1)$ (D) 1

(E) The limit does not exist

2. Find $\frac{d^2y}{dx^2}$ if $f(x)=(2x+3)^4$.

(A) $4(2x+3)^3$

(B) $8(2x+3)^3$

(C) $12(2x+3)^2$

(D) $24(2x+3)^2$

(E) $48(2x+3)^2$

3. Find $\frac{dy}{dx}$ for $y=4\sin^2(3x)$.

(A) $8\sin(3x)$ (B) $24\sin(3x)$ (C) $8\sin(3x)\cos(3x)$ (D) $12\sin(3x)\cos(3x)$ (E) $24\sin(3x)\cos(3x)$

Multiple Choice Set #7

Find the following WITHOUT the use of a calculator.

1. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

2. What is the instantaneous rate of change at x = 2 of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$.

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

3. Find $\frac{dy}{dx}$ if $3xy = 4x + y^2$.

(A) $\frac{4-3y}{2y-3x}$ (B) $\frac{3x-4}{2x}$ (C) $\frac{3y-x}{2}$ (D) $\frac{3y-4}{2y-3x}$ (E) $\frac{4+3y}{2y+3x}$

4. The function f is continuous on the closed interval [0,2] and has values that are given in the table below. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0,2] if k =

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

x	0	1	2
$f(x)$	1	k	2

Multiple Choice Set #8

Find #1-3 WITHOUT the use of a calculator.

1. $\lim_{x\to 0}\dfrac{x^3+x^2-2x}{x^3-x}=$

(A) -1 (B) 0 (C) 1 (D) 2 (E) ∞

2. What is the slope of the line tangent to the curve $y=\arctan(4x)$ at the point at which $x=\frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

3. Let f be a differentiable function such that $f(3)=15, f(6)=3, f'(3)=-8,$ and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

A Calculator may be used for #4

4. A particle moves along a straight line with velocity given by $v(t)=7-(1.01)^{-t^2}$ at time $t\geq 0$. What is the acceleration of the particle at time t = 3?

(A) –0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

Multiple Choice Set #9

You may use a graphing calculator to solve the following.

1. Which of the following gives $\frac{dy}{dx}$ at $x = 1$ if $x^3 + 2xy = 9$?

(A) $\frac{11}{2}$ (B) $\frac{5}{2}$ (C) $\frac{3}{2}$ (D) $-\frac{5}{2}$ (E) $-\frac{11}{2}$

2. Which of the following gives $\frac{dy}{dx}$ if $y = \cos^3(3x-2)$?

(A) $-9\cos^2(3x-2)\sin(3x-2)$

(B) $-3\cos^2(3x-2)\sin(3x-2)$

(C) $9\cos^2(3x-2)\sin(3x-2)$

(D) $-9\cos^2(3x-2)$

(E) $-3\cos^2(3x-2)$

3. If $y = \sin^{-1}(2x)$ find $\frac{dy}{dx}$.

(A) $-\frac{2}{\sqrt{1-4x^2}}$ (B) $-\frac{1}{\sqrt{1-4x^2}}$ (C) $\frac{2}{\sqrt{1-4x^2}}$ (D) $\frac{1}{\sqrt{1-4x^2}}$

(E) $\frac{2x}{1+4x^2}$

4. For what value of k is the function below continuous?

$$g(x)=\begin{cases} kx-2, & x\le -1 \\ kx^2+3, & x>-1 \end{cases}$$

(A) $-\frac{5}{2}$ (B) $\frac{5}{2}$ (C) -1 (D) $\frac{1}{2}$ (E) $-\frac{1}{2}$

Multiple Choice Set #10

NO calculator may be used.

1. f is continuous for $a \le x \le b$ but not differentiable for some c such that $a < c < b$. Which of the following could be true?

(A) $x = c$ is a vertical asymptote of the graph of f.

(B) $\lim_{x \to c} f(x) \neq f(c)$

(C) The graph of f has a cusp at $x = c$.

(D) $f(c)$ is undefined.

(E) None of the above

2. What is the instantaneous rate of change at $x = 3$ of the function $f(x) = \frac{x^2 - 2}{x + 1}$?

(A) $-\frac{17}{16}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{8}$ (D) $\frac{13}{16}$ (E) $\frac{17}{16}$

3. If $f(x) = \begin{cases} \ln 3x, & 0 < x \le 3 \\ x \ln 3, & 3 < x \le 4 \end{cases}$ then $\lim_{x \to 3} f(x)$ is

(A) $\ln 9$ (B) $\ln 27$ (C) $3 \ln 3$ (D) $3 + \ln 3$ (E) nonexistent

4. If $f(x) = \tan 3x$, then $f'\left(\frac{\pi}{9}\right) =$

(A) $\frac{4}{3}$ (B) 4 (C) 6 (D) 12 (E) $6\sqrt{3}$

Multiple Choice Set #11

NO calculator may be used.

1. If $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=3x^2+x$, then $\lim_{h\to 0}\frac{f(x+h)-f(x-h)}{h}=$ =

(A) $-3x^2-x$

(B) $3x^2+x$

(C) $-6x^2-2x$

(D) $6x^2+2x$

(E) None of the above

2. $\lim_{x\to 3}\frac{x^3-2x^2-3x}{x^3-9x}=$?

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) 1 (E) ∞

3. If $f(x)=\frac{x^2}{e^x}$, then $f'(1)=$

(A) 0 (B) $\frac{1}{e}$ (C) $\frac{2}{e}$ (D) 2 (E) $2e$

4. The tangent line to the curve $t=3x^4-10x+3$ at $x=1$, intersects the x -axis at the point

(A) $(-6,0)$ (B) $(-4,0)$ (C) $(0,-6)$ (D) $(3,0)$ (E) $(4,0)$

Multiple Choice Set #12

A graphing calculator may be used.

1. If $f(x)=\dfrac{e^{2x}}{2x}$, then $f'(x)=$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

2. If the derivative of f is given by $f'(x)=e^x-3x^2$, at which of the following values of x does f have a relative maximum value?

(A) -0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

3. Let f be the function give by $f(x)=2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

Multiple Choice Set #13

NO calculator.

1. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

2. Let f be the function with the derivative given by $f'(x) = x^2 - \frac{2}{x}$.

On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only (B) $(-\infty, 0)$ (C) $[-1, 0)$ only

(D) $(0, \sqrt[3]{2}]$ only (E) $[\sqrt[3]{2}, \infty)$

3. $\lim_{x \to \infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

(A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) -1

4. The graph of $f'(x)$, the derivative of the function f, is shown below.

Which of the following statements is true about f?

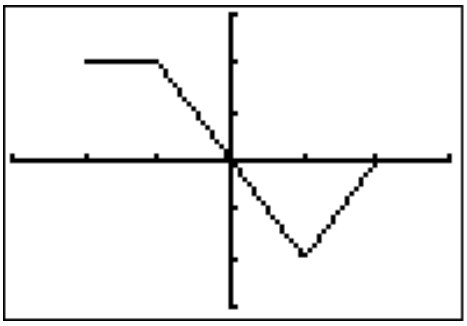
(A) f is decreasing for $-1 \le x \le 1$

(B) f is increasing for $-2 \le x \le 0$

(C) f is increasing for $1 \le x \le 2$

(D) f has local minimum at $x = 0$

(E) f is not differentiable at x = -1 and x = 1



Multiple Choice Set #14

NO CALCULATOR

1. $\lim_{x\to 0}\dfrac{x^2-1}{1-2x^2}=$

(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1 (E) $non-existent$

2. The sum of two non-negative numbers is 6. If the square of one of the numbers is multiplied by the second number, then the largest possible product is

(A) 32 (B) 36 (C) 38 (D) 45 (E) 64

3. The minimum value of the function $y=\sqrt{x^2+2ax+10a^2}$, where $a > 0$, is

(A) $-a$ (B) a (C) $3a$ (D) $6a$ (E) $9a^2$

4. If $y=\sin x+e^{-x}$, then $y+y'=$

(A) 0

(B) $\sin x+\cos x$

(C) $2e^{-x}$

(D) $2\sin x+2e^{-x}$

(E) $2\sin x-2e^{-x}$

Multiple Choice Set #15

NO CALCULATOR

1. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}}+\sqrt{\frac{2}{50}}+\sqrt{\frac{3}{50}}+\bullet\bullet\bullet+\sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for

(A) $\int_0^1 \sqrt{\frac{x}{50}}\,dx$ (B) $\int_0^1 \sqrt{x}\,dx$ (C) $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(D) $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$ (E) $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$

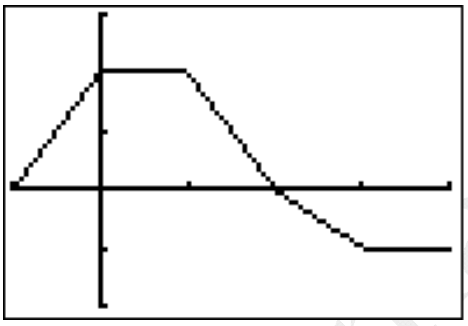
2. Let f be a function defined for all real numbers x. If $f'(x)=\frac{|4-x^2|}{x-2}$ then f is decreasing on the interval

(A) $(-\infty,2)$ (B) $(-\infty,\infty)$ (C) $(-2,4)$ (D) $(-2,\infty)$ (E) $(2,\infty)$

3. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown below.

What is the value of the $\int_{-1}^{4} f(x)dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8



Multiple Choice Set #16

NO CALCULATOR

1. If $f(x) = 6x^2 + \frac{16}{x^2}$, then $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} =$

(A) 0 (B) 20 (C) 24 (D) 32 (E) ∞

2. If $y = \frac{2x+3}{3x+2}$, then $\frac{dy}{dx} =$

(A) $\frac{12x+13}{(3x+2)^2}$ (B) $\frac{12x-13}{(3x+2)^2}$ (C) $\frac{5}{(3x+2)^2}$ (D) $\frac{-5}{(3x+2)^2}$

(E) $\frac{2}{3}$

3. $\int_0^{\frac{\pi}{4}} \sin x dx =$

(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2} - 1$ (D) $-\frac{\sqrt{2}}{2} + 1$ (E) $\frac{\sqrt{2}}{2} - 1$

Multiple Choice Set #17

CALCULATOR MAY BE USED

1. A rectangle is inscribed in the semicircle $y = \sqrt{4 - x^2}$. Find the largest possible area.

(A) 1.4 (B) $\sqrt{3}$ (C) $2\sqrt{3}$ (D) 4 (E) undefined

2. Find the value of c guaranteed by the Mean Value Theorem for $f(x) = \dfrac{2x}{x^2 + 1}$ on the interval [0, 1].

(A) 0.475 (B) 0.486 (C) 0.488 (D) 0.577 (E) 1.000

NO CALCULATOR

3. If $F(x) = \int_0^x \sin^2(2t)\,dt$, then $F'(x) =$

(A) $-\cos^2(2x)$ (B) $\cos^2(2x)$ (C) $\sin^2(2x)$

(D) $\dfrac{1}{2}\sin^2(2x)$ (E) $4\sin(2x)\cos(2x)$

4. Find the average value of $f(x) = \sqrt{x}$ on the interval [1, 4].

(A) $\dfrac{1}{3}$ (B) $\dfrac{7}{9}$ (C) $\dfrac{14}{9}$ (D) $\dfrac{7}{2}$ (E) $\dfrac{14}{3}$

Multiple Choice Set #18

CALCULATOR MAY BE USED

1. The rate of natural gas sales for the year 1993 at a certain gas company is given by $P(t) = t^2 - 400t + 160000$, where $P(t)$ is measured in gallons per day and t is the number of days in 1993 (from day 0 to day 365). To the nearest gallon, what is the total number of gallons of natural gas sales at this company for the 31 days (day 0 to day 31) of January 1993?

(A) $4,777,730$ (B) $4,617,930$ (C) $154,120$ (D) $148,965$ (E) $148,561$

2. $\int \cos(7t+3)dt =$

(A) $7\sin(7t+3)+C$ (B) $\sin(7t+3)+C$ (C) $\frac{1}{7}\sin(7t+3)+C$

(D) $-7\sin(7t+3)+C$ (E) $-\frac{1}{7}\sin(7t+3)+C$

NO CALCULATOR

3. If $f(x) = \sin x,\ g(x) = \cos(2x),\ and\ h(x) = f(g(x))$, what is $h'(\frac{\pi}{4})$?

(A) -2 (B) $-\sqrt{2}$ (C) 0 (D) $\sqrt{2}$ (E) 2

4. The position of a particle moving in a line is $s(t) = t^3 - 5t^2 + 2t - 13.$ What is the speed of the particle at t = 2?

(A) -21 (B) -6 (C) 6 (D) 10 (E) 32

Multiple Choice Set #19

CALCULATOR MAY BE USED

1. If $k > 1$, the area under the curve $y = kx^2$ from $x = 0$ to $x = k$ is

(A) $\frac{1}{3}k^4$ (B) $\frac{1}{3}k^3$ (C) $\frac{1}{4}k^4$ (D) $\frac{1}{3}k^3 - k$ (E) k^3

2. A continuous function $g(t)$ is defined in the closed interval [0,6] with values given in the table below. Using a midpoint Riemann sum with three subintervals of equal length, the approximate value of $\int_0^6 g(t)dt$ is

t	$g(t)$
0	4
1	7
2	8
3	12
4	15
5	22
6	26

(A) 68 (B) 82 (C) 89 (D) 94 (E) 153

NO CALCULATOR

3. $\int_1^6 \sqrt{x+3}dx =$

(A) $-\frac{5}{36}$ (B) 1 (C) $\frac{58}{5}$ (D) $\frac{38}{3}$ (E) 19

4. A young girl, 5 feet tall, is walking away from a lamppost which is 12 feet tall. She walks at a constant rate of 2 feet per second and notices that, as she moves away from the lamppost, the length of her shadow is increasing. How fast is

the length of her shadow increasing in feet per second when she is 20 feet from the post?

(A) $\frac{7}{10}\,ft/\sec$ (B) $\frac{10}{7}\,ft/\sec$ (C) $2\,ft/\sec$ (D) $\frac{34}{7}\,ft/\sec$ (E) $\frac{27}{10}\,ft/\sec$

5. If $f(x)=3x^3+5x$ and $g(x)=f^{-1}(x),$ what is $g'(8)$?

(A) $\frac{1}{14}$ (B) $\frac{1}{11}$ (C) $\frac{1}{8}$ (D) 11

(E) 14

Multiple Choice Set #20

CALCULATOR MAY BE USED

1. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

2. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line y = 3, and the vertical line x = 1. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 2.561 (B) 6.612 (C) 8.046 (D) 8.755 (E) 20.773

NO CALCULATOR

3. $\dfrac{d}{dx}\left(\displaystyle\int_0^{x^2} \sin\left(t^3\right) dt\right) =$

(A) $-\cos\left(x^6\right)$ (B) $\sin\left(x^3\right)$ (C) $\sin\left(x^6\right)$

(D) $2x\sin\left(x^3\right)$ (E) $2x\sin\left(x^6\right)$

4. Region Q is bounded by $y = \sin 2x, y = 0, x = 0 x = \frac{\pi}{2}$. Which of the following expressions gives the volume of a solid whose base in the xy-plane is region Q and whose cross sections, perpendicular to the x-axis, are squares with a side in the xy-plane?

(A) $\pi\int_0^{\frac{\pi}{2}}\left(1-\cos^2 2x\right)dx$ (B) $\int_0^{\frac{\pi}{2}}\sin^2 2x dx$ (C) $\int_0^{\frac{\pi}{2}}(1-\cos 2x)dx$

(D) $\int_0^{\frac{\pi}{2}}\left(1-\cos 2x^2\right)dx$ (E) $\pi\int_0^{\frac{\pi}{2}}\sin\left(2x\right)^2 dx$

5. If $f(x) = x^2 + 2x,$ then $\frac{d}{dx}(f(\ln x)) =$

(A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2x$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$

(E) $\frac{2x+2}{x}$

6. If the region enclosed by the y-axis, the line y = 2, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$

(E) π