32 AP Calculus AB Problems | Show work on margins or on separate paper. Justify completely.

1. $\lim_{x\to 3^-} \frac{|x-3|}{3-x} =$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

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2. Find the coordinates of the point where the line tangent to the parabola $y = x^2 - 4x - 5$ at $x = 4$ intersects the axis of symmetry of the parabola.

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3. If $f(2) = 7$ and $f'(2) = -3$, then the equation of the tangent to the curve $y = f(x)$ at $x = 2$ is

(A) $y = -3x + 13$
(B) $y = -3x + 23$
(C) $y = x$
(D) $y = 2x - 17$
(E) $y = 7x - 17$

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4. On the interval $1 < x < 2$, the curve $y = x^3 - 6x^2 + 9x + 1$ is

(A) increasing and concave up
(B) increasing and concave down
(C) decreasing and concave up
(D) decreasing and concave down
(E) horizontal

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5. The minimum value of the function $f(x) = \sqrt[3]{x^2 + 4ax + 12a^2}$, $a > 0$, is

(A) $-2a$ (B) $\sqrt[3]{6a^2}$ (C) $2\sqrt[3]{a^2}$ (D) $2a$
(E) none of the above

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6. A function is defined for all real numbers and has the following property: $f(x+h) - f(x) = 4x^2h + 2xh - 6x^3h^2$. Find $f'(-3)$.

7.■ For what positive value of k is the line $y = -9x + k$ tangent to the curve $y = x^3 - 6x^2$?

8.■ A rectangular field is to be fenced off along the bank of a river, and no fence is required along the river. If the material for the fence costs \$5.00 per foot for the two ends and \$7.50 per foot for the side parallel to the river, find the dimensions of the field of largest possible area that can be enclosed with \$9,000.00 worth of fence.

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9. $\lim\limits_{x \to -\infty} \dfrac{10 - 2^x}{10 + 2^{-x}} =$

(A) -1 (B) 0 (C) 1 (D) 10 (E) ∞

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10. The x-coordinate of the point where the tangent to the parabola $y = ax^2$ at $x = p$ (not a vertex) intersects the x-axis is

(A) $\dfrac{p}{2}$ (B) $\dfrac{p^2}{2}$ (C) $\dfrac{ap}{2}$ (D) $\dfrac{ap^2}{2}$ (E) $\dfrac{a}{p^2}$

11.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	–3	6	–5	1
4	0	3	–3	9
5	3	–2	4	5

The table above shows some of the values of two differentiable functions f and g and their derivatives. If $h(x) = f(x)g(x)$, then $h'(5) =$

(A) 2 (B) 7 (C) 14 (D) 20 (E) 26

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12. Using the values in the table from the previous problem, if $h(x) = f(g(x))$, then $h'(4) =$

(A) –45 (B) –27 (C) –15 (D) 0 (E) 25

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13. If $f(x)$ is a continuous function and $f(2)=7$ and $f'(2)=-3$, then $f(2.01)$ is approximately

(A) -6.03 (B) 6.92 (C) 6.97 (D) 7.01 (E) 7.03

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14. Consider the curve $y=2x^3-3(k+1)x^2+6kx$, $k>1$. On the interval $1<x<k$,

(A) y' is positive, and y'' is first positive, then negative
(B) y' is positive, and y'' is first negative, then positive
(C) y' is negative, and y'' is first positive, then negative
(D) y' is negative, and y'' is first negative, then positive
(E) Neither the sign of y' nor the sign of y'' can be determined without knowing the value of k.

15.■ A cube is expanding so that its surface area is increasing at a constant rate of $9\sqrt{2}$ in^2/sec. How fast is the volume increasing at the instant when the surface area is 108 in^2? Show units in your answer.

16.■ Runner B is 7.35 yards behind Runner A. Both are running at 9 yards/sec. At this point, A tires and decelerates at 0.2 yards/sec^2. B picks up speed, accelerating at 0.1 yards/sec^2. If they continue like this, how many more yards does Runner A cover before the two runners are side by side? (Give your answer rounded to the nearest tenth of a yard).

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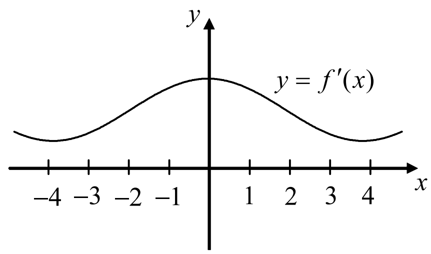
17. If $f(x)=2^x$ and $2^{3.03}\approx 8.168$, which of the following is closest to $f'(3)$?

(A) .168 (B) .97 (C) 1 (D) 3 (E) 5.6

18.



Pictured above is the graph of $f'(x)$. For what values of x is the graph of $f(x)$ concave down?

(A) $-2 < x < 2$
(B) $x < -4$ or $0 < x < 4$
(C) $-4 < x < 4$
(D) all values of x
(E) the graph of $f(x)$ is always concave up

19.

x	$g(x)$	$g'(x)$
1	3	4
2	8	3

If $g(x)$ and $g'(x)$ have the values shown in the table above, and $f(x) = g^2(x)$, then $f'(2) =$

(A) 12 (B) 16 (C) 23 (D) 24 (E) 48

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20. If $\int_0^4 f(x)\,dx = 10$, $\int_0^5 f(x)\,dx = 9$, and $\int_4^7 f(x)\,dx = 1$, then $\int_5^7 f(x)\,dx =$

(A) -1 (B) 1 (C) 2 (D) 3 (E) 4

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21. If $u = x^2 + 1$, then $\int_1^2 \frac{x^2}{x^2+1}\,dx =$

(A) $\int_1^2 \frac{u-1}{u}\,du$

(B) $\int_1^2 \frac{\sqrt{u-1}}{u}\,du$

(C) $\int_2^5 \frac{u-1}{u}\,du$

(D) $\int_2^5 \frac{\sqrt{u-1}}{u}\,du$

(E) $\int_2^5 \frac{\sqrt{u-1}}{2u}\,du$

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22. The average area of all circles with radii between 3 and 6 is

(A) $\frac{25}{2}\pi$ (B) $\frac{27}{2}\pi$ (C) 18π (D) 21π (B) $\frac{45}{2}\pi$

23.■ A rumor spreads continuously at the rate of $3t^2 + 6t$ (where t is measured in days). How many people hear the rumor on the third day?

(A) 21 (B) 34 (C) 44 (D) 45 (E) 54

24.■ Find the total area of all regions bounded by the graphs of $y = \sin x$ and $y = \tan\frac{x}{2}$ over the interval $-2\pi \le x \le 2\pi$.

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25. If $\lim\limits_{x\to 2}\left[\ln f(x)\right] = 1$, then $\lim\limits_{x\to 2} f(x) =$

(A) 0 (B) $\ln 2$ (C) 1 (D) 2 (E) e

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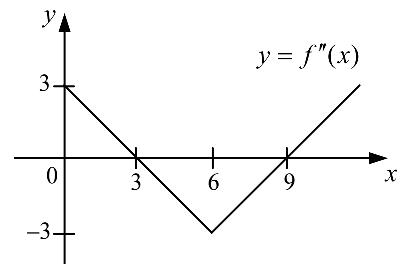
26. The point $A(a, b)$ is on the parabola $y = x^2$. Point V is the vertex of the parabola. Point $C(0, c)$ is the point where the perpendicular bisector of $\overline{AV}$ intersects the y-axis. Find $\lim\limits_{a\to 0} c$.

27.

The graph of $y = f''(x)$, shown to the right, consists of two straight line segments. If $f'(0) = 0$, then in the vicinity of which of the following values of *x* the curve $y = f(x)$ is falling and concave down?



(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

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28. If $f(x) = \sin 2x \cos 3x$ and *k* is an odd integer, then $f'(k\pi) =$

(A) –5 (B) –2 (C) –1 (D) 1 (E) 5

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29. If $F(x) = \int_1^x \frac{4}{1+\ln t}\,dt$, then $F'(e) =$

(A) $\frac{1}{e^2}$ (B) $\ln 2$ (C) 2 (D) $2e$ (E) e^2

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30. If the slope of the tangent to the curve at any point (x, y) on the curve equals $\frac{x}{y}$, what kind of curve can it be?

(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola
(E) none of the above

31.■ How much should be invested today to accumulate $20,000 in seven years at 5% annual interest, compounded continuously?

32.■ In the year 2000, the yearly consumption of oil throughout the world was approximately 25 billion barrels and increasing exponentially at a rate of 5%. Assuming the world's total oil reserves are one trillion barrels, in what year will the oil reserves be depleted?

SOLUTIONS

1. $x<3 \Rightarrow |x-3|=3-x \Rightarrow \lim_{x\to 3^-}\frac{|x-3|}{3-x}=\lim_{x\to 3^-}\frac{3-x}{3-x}=\boxed{1}$.

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2. At $x=4$, the slope of the tangent line is $2x-4\big|_{x=4}=4$, $y=4^2-4\cdot 4-5=-5$, an equation of the tangent line is $y+5=4(x-4)$. The line of simmetry is $x=2$. At the intersection, $y+5=4(2-4) \Rightarrow y=-13$. The coordinates of the intersection points are $\boxed{x=2,\ y=-13}$.

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3. $y=f(2)+f'(2)(x-2) \Rightarrow y=7-3(x-2) \Rightarrow \boxed{y=-3x+13}$.

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4. $y'=3x^2-12x+9=3(x-1)(x-3);\ y''=6x-12=6(x-2)$. On the interval $1<x<2,\ y'<0$ and $y''<0$. Therefore, the curve is $\boxed{\text{decreasing and concave down}}$.

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5. Since $\sqrt[3]{x}$ is increasing, $f(x)$ reaches the minimum at the same x as $x^2+4ax+12a^2$, that is, at $x=-2a$. $f(-2a)=\sqrt[3]{4a^2-8a^2+12a^2}=\sqrt[3]{8a^2}=\boxed{2\sqrt[3]{a^2}}$.

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6. A function is defined for all real numbers and has the following property: $f(x+h)-f(x)=4x^2h+2xh-6x^3h^2$. Find

$$f'(-3)=\lim_{h\to 0}\frac{f(-3+h)-f(-3)}{h}=\lim_{h\to 0}\frac{4(-3)^2h+2(-3)h+6(-3)^3h^2}{h}=$$

$4\cdot 9-6=\boxed{30}$.

7.■ $3x^2-12x=-9 \Rightarrow x=3$ or $x=1$. For $x=3,\ 3^3-6\cdot 3^2=-9\cdot 3+k \Rightarrow k=0$. For $x=1,\ 1^3-6\cdot 1^2=-9+k \Rightarrow \boxed{k=4}$.

8.■ $x\cdot 7.5+2y\cdot 5=9000 \Rightarrow y=900-.75x \Rightarrow xy=x(900-.75x)$ reaches maximum at $x=\dfrac{900}{1.5}=600 \Rightarrow y=450$. $\boxed{\text{600 by 450}}$.

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9. $\left.\begin{array}{l}\lim\limits_{x\to-\infty}(10-2^x)=10\\ \lim\limits_{x\to-\infty}(10+2^{-x})=\infty\end{array}\right\} \Rightarrow \lim\limits_{x\to-\infty}\dfrac{10-2^x}{10+2^{-x}}=\boxed{0}$.

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10. $y(p)=ap^2;\ y'(p)=2ap \Rightarrow$ an equation of the tangent line is $y=ap^2+2ap(x-p)$. Its x-intercept is when $ap^2+2ap(x-p)=0 \Rightarrow \boxed{x=\dfrac{p}{2}}$.

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11. $h'(5)=f(5)g'(5)+g(5)f'(5)=3\cdot 5+(-2)\cdot 4=\boxed{7}$.

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12. $h'(4)=f'(g(4))g'(4)=f'(3)g'(4)=(-5)\cdot 9=\boxed{-45}$.

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13. $f(2.01)\approx f(2)+f'(2)\cdot 0.01=7-3\cdot 0.01=\boxed{6.97}$.

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14. $y'=6x^2-6(k+1)x+6k=6(x-1)(x-k)$. $y''=12x-6(k+1)=12\left(x-\dfrac{k+1}{2}\right)$.

For $1<x<k,\ x-1>0;\ x-k<0 \Rightarrow y'<0$. If $1<x<\dfrac{k+1}{2},\ y''<0$. If $\dfrac{k+1}{2}<x<k,\ y''>0$. $\boxed{y' \text{ is negative, and } y'' \text{ is first negative, then positive}}$.

15.■ $V=\left(\sqrt{\dfrac{A}{6}}\right)^3=\left(\dfrac{A}{6}\right)^{\frac{3}{2}} \Rightarrow \dfrac{dV}{dt}=\left(\dfrac{1}{6}\right)^{\frac{3}{2}}\dfrac{3}{2}A^{\frac{1}{2}}\dfrac{dA}{dt}=\left(\dfrac{1}{6}\right)^{\frac{3}{2}}\dfrac{3}{2}108^{\frac{1}{2}}9\sqrt{2}=$

$\dfrac{27}{2}\left(\dfrac{108\cdot 2}{216}\right)^{\frac{1}{2}}=\boxed{\dfrac{27}{2}}$.

16.■ $x_A = 9t - \frac{0.2t^2}{2}$. $x_B = 9t + \frac{0.1t^2}{2} - 7.35$. $x_A = x_B \Rightarrow \frac{0.3t^2}{2} = 7.35 \Rightarrow$

$t^2 = 49 \Rightarrow t = 7 \Rightarrow x_A = 9 \cdot 7 - 0.2\frac{49}{2} = 63 - 4.9 = \boxed{58.1}$.

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17. $f'(3) \approx \frac{f(3.03) - f(3)}{0.03} = \frac{8.168 - 8}{0.03} = \frac{16.8}{3} = \boxed{5.6}$.

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18. The graph of $f(x)$ is concave down when $f'(x)$ is decreasing, that is when $\boxed{x < -4 \text{ or } 0 < x < 4}$.

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19. $f'(2) = 2g(2)g'(2) = 2 \cdot 8 \cdot 3 = \boxed{48}$.

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20. $\int_4^5 f(x)\,dx = \int_0^5 f(x)\,dx - \int_0^4 f(x)\,dx = 9 - 10 = -1$.

$\int_5^7 f(x)\,dx = \int_4^7 f(x)\,dx - \int_4^5 f(x)\,dx = 1 - (-1) = \boxed{2}$.

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21. $u = x^2 + 1 \Rightarrow u(1) = 2;\ u(2) = 5;\ du = 2x\,dx \Rightarrow$

$\int_1^2 \frac{x^2}{x^2+1}\,dx = \int_2^5 \frac{x^2}{u}\frac{du}{2x} = \int_2^5 \frac{x}{2u}\,du = \boxed{\int_2^5 \frac{\sqrt{u-1}}{2u}\,du}$.

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22. Average area $= \frac{\int_3^6 \pi r^2\,dr}{6-3} = \frac{1}{3} \cdot \frac{\pi r^3}{3}\Big|_3^6 = \frac{\pi}{3}(72 - 9) = \boxed{21\pi}$.

23.■ $N = \int_2^3 \left(3t^2 + 6t\right)dt = \left(t^3 + 3t^2\right)\Big|_2^3 = (27 + 27) - (8 + 12) = \boxed{34}$.

24.■ There are four regions of equal area. The total is

$$4\int_0^{\frac{\pi}{2}}\left(\sin x-\tan\frac{x}{2}\right)dx=4\left[-\cos x+2\ln\left|\cos\frac{x}{2}\right|\right]\Bigg|_0^{\frac{\pi}{2}}=4\left[\ln\left(\frac{1}{\sqrt{2}}\right)^2+1\right]=\boxed{4(1-\ln 2)}.$$

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25. Since e^x is a continuous function, $\lim_{x\to2}f(x)=\lim_{x\to2}e^{\ln f(x)}=e^{\lim_{x\to2}\ln f(x)}=\boxed{e}$.

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26. $C(0,\ c)$ is equidistant from $A\left(a,\ a^2\right)$ and $V(0,\ 0)$, so $(a-0)^2+\left(a^2-c\right)^2=c^2\ \Rightarrow$

$a^2+a^4-2a^2c+c^2=c^2\ \Rightarrow\ 1+a^2-2c=0\ \Rightarrow\ c=\dfrac{1+a^2}{2}\ \Rightarrow\ \lim_{a\to0}c=\boxed{\dfrac{1}{2}}$.

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27. We must have $f'(x)<0$ and $f''(x)<0$. $f'(x)=\int_0^x f''(t)\,dt\ \Rightarrow\ f'(x)<0$ for $x>6$.

$f''(x)<0$ for $3<x<9$. The only option that satisfies both conditions is $\boxed{x=8}$.

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28. If $f'(k\pi)=3\sin(2k\pi)\sin(3k\pi)+2\cos(2k\pi)\cos(3k\pi)=0+2\cdot1\cdot(-1)=\boxed{-2}$.

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29. By the Fundamental Theorem of Calculus, $F'(e)=\dfrac{4}{1+\ln x}\Bigg|_{x=e}=\dfrac{4}{1+1}=\boxed{2}$.

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30. $\dfrac{dy}{dx}=\dfrac{x}{y}\ \Rightarrow\ \int y\,dy=\int x\,dx\ \Rightarrow\ y^2=x^2+C\ \Rightarrow\ y^2-x^2=C$ — $\boxed{\text{a hyperbola}}$.

31.■ $Ae^{0.05\cdot7}=20000\ \Rightarrow\ A=20000e^{-0.35}=\boxed{14093.76}$.

32.■ $\int_0^y 25e^{0.05t}\,dt=1000\ \Rightarrow\ \dfrac{25}{0.05}e^{0.05t}\Bigg|_0^y=\dfrac{25}{0.05}\left(e^{0.05y}-1\right)=1000\ \Rightarrow\ e^{0.05y}-1=2\ \Rightarrow$

$y=20\ln 3\approx 21.97$. The oil reserves will be depleted in $\boxed{2022}$.
