Problem 1

a) $C'(5) \approxeq \frac{C(7)-C(3)}{7-3} = \frac{69-85}{4} = -4$ degrees Celcius per minute.

b) $\int_0^{12} C(t)\ dt \approxeq L_3 = f(0)(3-0) + f(3)(7-3) + f(7)(12-7) = 100 * 3 + 85 * 4 + 69 * 5 = 985.$

The expression $\frac{1}{12}\int_0^{12} C(t)\ dt$ is the average temperature of coffee during the 12 minutes.

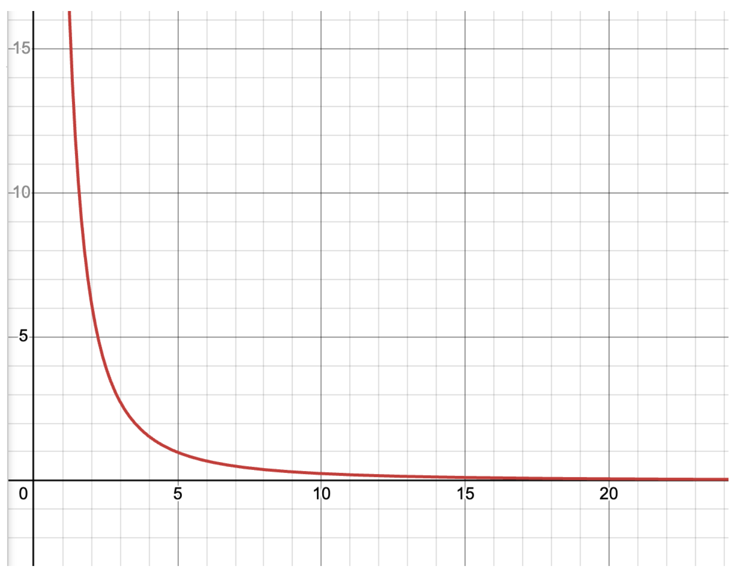
c) By the Fundamental Theorem of Calculus, $\int_{12}^{20} C'(t)\ dt = C(20) - C(12)$.

Therefore, $C(20) = C(12) + \int_{12}^{20} C'(t)\ dt \approxeq= 55 + (-14.6708) \approxeq 40.329$ degrees Celsius.

(Use graphing calculator to approximate the definite integral.)

d) The graph of $C''(t)$ is positive on the interval $12 < t < 20$, so the temperature of coffee is changing at an increasing rate.

$$C''(t) > 0 \rightarrow C'(t) \text{ is increasing.}$$

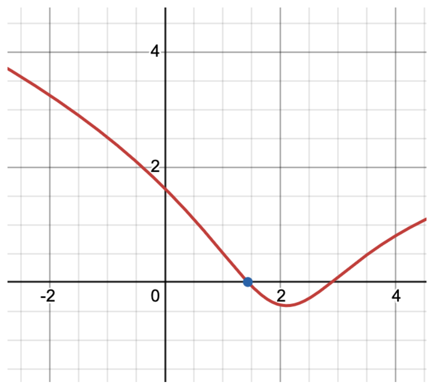


Problem 2

a) The particle is at rest when the velocity is zero.

$v(t) = \ln(t^2 - 4t + 5) - 0.2t = 0 \to t_R \approxeq 1.426.$

The velocity graph is positive on the interval $0 < t < t_R$, so the particle is moving to the right during this time.



b)

$a(t) = v'(t) = \frac{2t-4}{t^2-4t+5} - 0.2$

$a(1.5) = v'(1.5) = \frac{-1}{5/4} - 0.2 = -0.8 - 0.2 = -1$

$v(1.5) \approxeq -0.0768564 < 0$ (graphing calculator)

Because velocity and acceleration are both negative at $t = 1.5$, the particle is speeding up at this moment, which means its speed is increasing.

c)

By the Fundamental Theorem of Calculus:

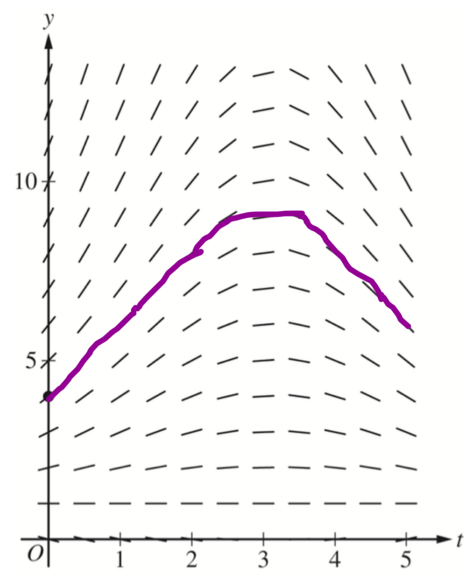
$x(4) - x(1) = \int_1^4 v(t)$ dt, so it follows that $x(4) = x(1) + \int_1^4 v(t)$ dt $\approxeq -2.803$

d)

Total distance is given by $\int_1^4$ speed $dt = \int_1^4 |v(t)| dt \approxeq 0.958$ units of distance.

Problem 3

a) Below is a solution curve that passes through the point (0,4).



b) A critical number occurs when $\frac{dH}{dt} = 0$ or is undefined. Becase $H(t) > 1$, it follows that $\cos\left(\frac{t}{2}\right) = 0$. Therefore, $\frac{t}{2} = \frac{\pi}{2} \rightarrow t = \pi$ is the solution we are looking for in the interval $0 < t < 5$. We need to study the sign change:

For $t < \pi \rightarrow \cos\left(\frac{t}{2}\right) > 0 \rightarrow \frac{dH}{dt} > 0$.

For $\pi < t \rightarrow \cos\left(\frac{t}{2}\right) < 0 \rightarrow \frac{dH}{dt} < 0$.

By the first derivative test for local extrema, the depth water has reached a local maximum at the given critical number. Note that we ignored the factor $(H - 1)$ when studying the sign since we know that $H(t) > 1$.

c) We first separate the variables before integrating:

$\frac{1}{H-1} dH = \frac{1}{2}\cos\left(\frac{t}{2}\right) dt$

$\int \frac{1}{H-1} dH = \int \frac{1}{2}\cos\left(\frac{t}{2}\right) dt$

$\ln|H - 1| = \sin\left(\frac{t}{2}\right) + C \leftarrow H(0) = 4$

$C = \ln 3$ so we have: $|H(t) - 1| = e^{\sin(t/2)} e^{\ln 3} = 3e^{\sin(t/2)}$

$H(t) = 1 \pm 3e^{\sin(t/2)}$. The option that satisfies our initial condition is: $H(t) = 1 + 3e^{\sin(t/2)}$.

Problem 4

a) $g(x) = \int_0^x f(t)\ dt$.

$g(-6) = \int_0^{-6} f(t)\ dt = -\int_{-6}^0 f(t)\ dt = -12$.

$g(4) = \int_0^4 f(t)\ dt = \frac{base*height}{2} = \frac{4*2}{2} = 4$ (the area of the right triangle in the first quadrant).

$g(6) = \int_0^6 f(t)\ dt = \int_0^4 f(t)\ dt + \int_4^6 f(t)\ dt = \frac{4*2}{2} - \frac{2*1}{2} = 3$.

b) The graph of $g(x)$ has a critical point whenever its derivative, $g'(x) = f(x)$ (by FTC) is zero or undefined. On the given interval, the only such number is $x = 4$, since $f(4) = 0$.

c) $h(x) = \int_{-6}^x f'(t)\ dt$. We use the Fundamental Theorem of Calculus in the computations below.

$h(6) = \int_{-6}^6 f'(t)dt = f(6) - f(-6) = (-1) - (0.5) = -\frac{3}{2}$.

$h'(x) = f'(x)$, therefore $h'(6) = f'(6) = \frac{rise}{run} = -\frac{1}{2}$.

$h''(x) = f''(x)$. Since the graph of f is a linear function, it follows that its second derivative is zero everywhere. Therefore, $h''(6) = 0$.

Problem 5

a) Plug $(2,4)$ into $\frac{dy}{dx} = \frac{-2x}{3+4y}$ to find a slope of $\frac{-4}{19}$. The equation of the tangent line at $(2,4)$ is $L(x) = 4 - \frac{4}{19}(x-2)$. Therefore, $f(3) \approxeq L(3) = 4 - \frac{4}{19} = \frac{72}{19}$.

b) For the line $y = 1$ to be tangent, we need to first check when the derivative is zero: $\frac{dy}{dx} = 0 \rightarrow x = 0$. If we plug this condition into the equation of the curve, we get $2y^2 + 3y = 48$. Since $y = 1$ does not satisfy this condition, we conclude that $y = 1$ is not tangent to the curve.

Alternatively, if we plug in $y = 1$ into the equation of the curve, we get $x^2 = 43$, which disagrees with the condition $x = 0$.

c) We evaluate the slope expression at the point $(\sqrt{48}, 0)$:

$$\frac{dy}{dx} = \frac{-2\sqrt{48}}{3 + 4 * 0} = \frac{-8\sqrt{3}}{3}$$

. The slope we found is a finite number, so the tangent line cannot be vertical at this point.

d) We differentiate the equation $y^3 + 2xy = 24$ implicitly with respect to time t.

$$3y^2\frac{dy}{dt} + 2x\frac{dy}{dt} + 2\frac{dx}{dt}y = 0$$

Plugging in the known information, we get:

$$3 * (2^2)(-2) + 2 * (4)(-2) + 2\frac{dx}{dt} * 2 = 0$$

$$\frac{dx}{dt} = 10$$

.

At the instant when the particle is at the point $(4,2)$, its $x-$coordinate is changing at a rate of 10 units per second.

Problem 6

a) $A(R) = \int_0^2 f(x) - g(x)\ dx$

b) A typical rectangle (cross-section) has area given by $A(x) = \frac{1}{2}(x^2 - 2x)^2 = \frac{1}{2}(x^4 - 4x^3 + 4x^2)$

The volume is given by the integral of the area of a typical cross section:

$V = \int_2^5 A(x)dx = \int_2^5 \frac{1}{2}(x^4 - 4x^3 + 4x^2)dx$

The anti-derivative of the area function is:

$F(x) = \frac{1}{2}\left(\frac{x^5}{5} - x^4 + \frac{4x^3}{3}\right)$

Therefore, the volume is:

$V = F(5) - F(2) = \frac{1}{2}\left(\left(\frac{5^5}{5} - 5^4 + \frac{4*5^3}{3}\right) - \left(\frac{2^5}{5} - 2^4 + \frac{4*2^3}{3}\right)\right) = \frac{1}{2}(\frac{500}{3} - \frac{16}{15}) = \frac{414}{5}$ cubic units.

c) We use the Washer Method with the following radii:

outer radius $= R(x) = 20 - 0$

inner radius $= r(x) = 20 - g(x)$

$$V_{\text{washer}} = \pi \int_2^5 (20)^2 - (20 - g(x))^2\ \text{dx}$$